A Geometric Postprocessing Method Using Joint Positions of a 5-Axis Machine

Jae-Deuk Yun, Yoong-Ho Jung*, Dong-Been Tae, and Jin-Gyu Lee

Abstract— **We present a postprocessing algorithm for 5-axis machines, which can be applied to 2 rotary axes (2R-3L) types and 3 rotary axes (3R-2L) types. Five-axis machining requires a postprocessor for converting cutter-location (CL) data to machine-control (NC) data. The existing methods for postprocessing use inverse kinematics equations from the forward kinematics. However, for 5-axis machines with three rotary axes, the inverse kinematics equations cannot be induced directly, because the forward kinematics equations are nonlinear. To derive the joint values from the forward kinematics equations, previous algorithms use numerical methods for the postprocessing; this requires a search algorithm with much computation time and may fail to obtain a solution. We propose a general method for the postprocessing that can be applied to both 2-rotary and 3-rotary 5-axis machines. Our algorithm has three advantages: first, the forward kinematics equations are not required; second, the method is reliable and eliminates the need for numerical methods for the inverse kinematics, which results in exact solution; and finally, the proposed algorithm can also be applied to 2R-3L 5-axis machines.**

Index Terms—**5-axis machining, inverse kinematics, NC data, 3 rotary.**

I. Introduction

Five-axis milling is used for machining impellers, turbine blades, and artificial joints, and is extending its application to the die and mold industry, including tire mold parts. With its two additional rotational axes, 5-axis machining enables cutting that is considered impossible by 3-axis machining, along with reduced setup time and high surface quality. It is well known in the die and mold industry that the gain of 5-axis machining is up to 20 times on 3-axis machining [1], [2].

Five-axis machines have various configurations according to the kinematics chain of the translation and rotation axes. The appropriate machine type is selected for the shape of the workpiece. For example, 3R-2L (three rotation axes with two linear translation axes) machines are used for ships' propellers, because 2R-3L (two rotation axes with three linear translation axes) machines must be larger to perform the same task.

Meanwhile, 5-axis machining requires machine-control (NC) postprocessing to convert the cutter-location (CL) data that define the tool path data with a CAM system into the NC data that the machine can read. This process calculates the machine tool's joint values from tool position and orientation data defined in the workpiece's coordinate system.

While only simple postprocessing is required for 3-axis machining involving simple transformation between coordinate systems, 5-axis machining requires complicated calculations to benefit from the additional rotation axes. Five-axis NC postprocessing requires an inverse kinematics equation that transforms workpiece coordinates into machine coordinates, which are driven from the forward kinematics equation.

Meanwhile, 5-axis machining must need NC postprocessing in order to convert the CL data that define the tool path data with CAM system into NC data with which the machine can read. This process calculates machine tool's joint values from tool position and orientation data defined in workpiece coordinate system. While 3-axis machining needs simple postprocessing by simple transformation between coordinate systems, 5-axis machining needs complicated calculation due to additional rotation axes. In order for the 5-axis NC postprocessing, it is necessary to have an inverse kinematic equation that transforms the work-piece coordinates into the machine coordinates, which are driven from the forward kinematic equation.

Various studies have addressed the issue of developing postprocessors for 5-axis machine tools. Tutunea-Fatan and Feng [3] proposed a generalized relationship between coordinate systems for 5-axis machines with two rotation axes. Lee and She [4] presented an inverse kinematics equation for three representative models of 5-axis machines. She et al. [5], [6] suggested inverse kinematics that could be applied to general 5-axis machine tools, consisting of four rotation transformations for the coordinate transformation from table to tool. Jung et al. [7] presented an inverse kinematics equation for table tilting models in their work on phase reversal problems. Zaidman [8] suggested a generalized method for coordinate transformation that can be applied to machines with nonorthogonal rotation axes, while Sørby [9] proposed a postprocessing algorithm for machines with rotation axes that are inclined at 45°. All these methods

Manuscript received July 20, 2009. This work was financially supported by the Ministry of Education, Science Technology (MEST) and Korea Institute for Advancement of Technology (KIAT) through the Human Resource Training Project for Regional Innovation and by grants-in-aid for the National Core Research Center Program from MEST/KOSEF (No.R15-2006-022-03002-0).

J. D. Yun is with the Pusan National University, Busan, 609-735 S. Korea (e-mail: yjdeuk@pusan.ac.kr).

Y. H. Jung* is with the School of Mechanical Engineering, Pusan National University, Busan, 609-735 S. Korea. (phone: +82-51-510-2469; fax: +82-51-512-1722; e-mail: yhj@pusan.ac.kr).

D. B. Tae is with the Pusan National University, Busan, 609-735 S. Korea (e-mail: taebin99@pusan.ac.kr).

J. G. Lee is with the Pusan National University, Busan, 609-735 S. Korea (e-mail: wlsrb02@pusan.ac.kr).

can be used in the postprocessing for 2R-3L 5-axis machines. In addition, all these methods are based on the common algorithm that derives an inverse kinematics equation from the forward kinematics equation. A previous method has addressed the postprocessing method for 3R-2L 5-axis machines based on the previous algorithm. However, an inverse kinematics equation for 3R-2L machines cannot be derived from the forward kinematics equation directly, because the forward kinematics equation is nonlinear for this type. To resolve this problem, a numerical approach has been used for solving the nonlinear equation [10], which may cause large computation times and give erroneous results according to the numerical method. The numerical method is also deficient because it cannot find exact solutions.

Unlike the existing methods, we propose a geometric method for postprocessing that does not require the forward and inverse kinematics equation. The proposed method can be applied not only to 2-rotary, but also to 3-rotary 5-axis machines without resorting to numerical methods, thus saving much computation time over previous methods.

II. Postprocessing Method for 5-Axis Machines

Existing methods for postprocessing for 5-axis machines require the forward and inverse kinematics equations. For example, for the 2R-3L 5-axis machine shown in Fig. 1, existing methods first set local coordinate systems on each rotation and translation joint, as shown in Fig. 2. Then, they obtain the transformation matrix from the table for the tool as shown in (1), where X, Y, and Z represent the joint values for the three linear motions, A and C are joint values for the two rotary motions, and DY and DZ are offset distances from the center of the table to the origin of the machine coordinate system. Next, they obtain the forward kinematics equation, shown in (2), from the transformation equation (1), where $[x \quad y \quad z \quad i \quad j \quad k]^T$ represents the tool tip position and orientation for the CL data.

Finally, they derive the inverse kinematics equation, shown in (3), from the forward kinematics equation (2), which results in the joint values $[X \quad Y \quad Z \quad A \quad C]^T$ of the machine.

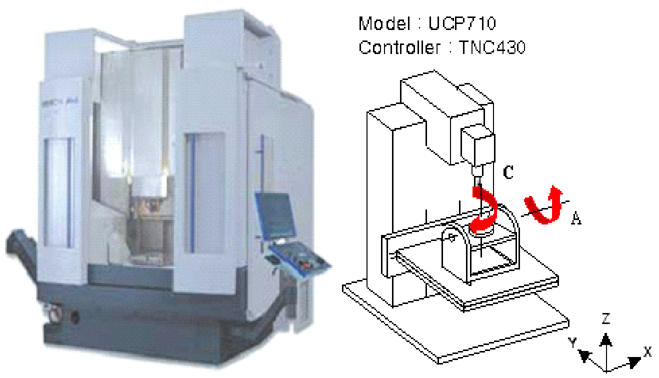


Fig. 1 2R-3L 5-axis machine (MIKRON UCP710 [11])

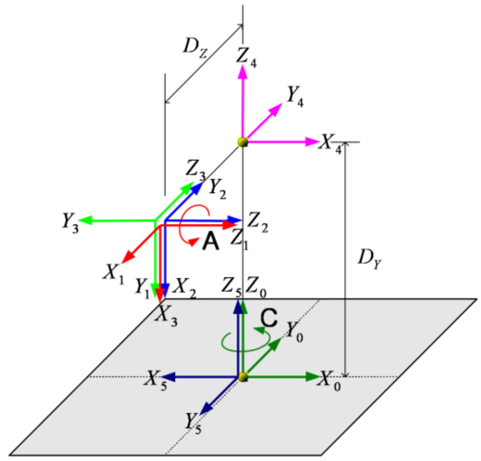


Fig. 2 Local coordinate systems for a 2R-3L 5-axis machine

$$\begin{aligned}\mathbf{T}_T^W &= \mathbf{T}_5^0 \cdot \mathbf{T}_3^5 \cdot \mathbf{T}_2^3 \cdot \mathbf{T}_1^2 \\ &= Rot(Z, C) \cdot Trans(0, -D_Y, D_Z) \\ &\quad \cdot Rot(X, A) \cdot Trans(X, Y + D_Y, Z - D_Z)\end{aligned} \tag{1}$$

$$\begin{bmatrix} x \\ y \\ z \\ i \\ j \\ k \end{bmatrix} = \begin{bmatrix} X \cos C - \sin C \cos A(Y + D_Y) + \sin C \cos A(Z - D_Z) + D_y \sin C \\ X \sin C + \cos C \cos A(Y + D_Y) - \cos C \cos A(Z - D_Z) - D_Y \cos C \\ \sin A(Y + D_Y) + \cos A(Z - D_Z) + D_Z \\ \sin C \sin A \\ -\cos C \sin A \\ \cos A \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ A \\ C \end{bmatrix} = \begin{bmatrix} x \cos C + u \sin C \\ y \cos A / \cos C - X \cos A \sin C / \cos C + (Z - D_Z) \sin A - D_Y (1 - \cos C) \\ (z - Y \sin A - D_Y \sin A - D_Z) / \cos A + D_Z \\ \cos^{-1}(k) \\ \tan^{-1}(i / -j) \end{bmatrix} \tag{3}$$

A typical 3R-2L 5-axis machine has three rotation axes (A, B, C) and two linear axes (Y, Z) as shown in Fig. 3. For the postprocessing for these machines with the existing methods, the forward kinematics equation must be established. However, the forward kinematics equation of this type of machine is complicated because of the complex configuration with three rotation axes. In addition, the rotational joint values (A, B, C) cannot be obtained directly from the forward kinematics equation because it is nonlinear, as shown in (4).

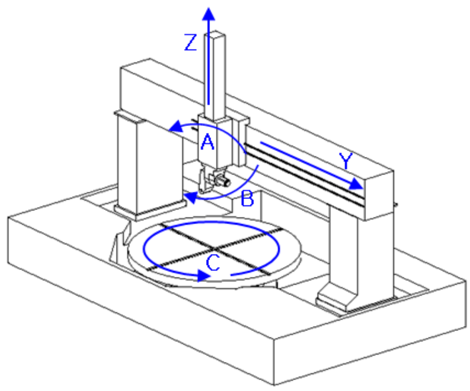


Fig. 3 3R-2L 5-axis machine

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} \cos C \sin A \sin B - \cos B \sin C \\ \cos B \cos C + \sin A \sin B \sin C \\ -\cos A \sin B \end{bmatrix} \quad (4)$$

To find the solution of this nonlinear equation, existing methods use numerical analysis: after representing the nonlinear equation as two simultaneous equations with variables *B* and *C*, an iterative algorithm is used to search for an acceptable solution for *B* and *C*. The numerical method may require large computation times to find an acceptable solution, which may be approximate. Furthermore, the search method may fail to find a solution.

III. GEOMETRIC POSTPROCESSING METHOD

The proposed geometric postprocessing method consists of the following three steps. First, it sets each 'joint point' at the center of each joint of the machine in turn as shown in the left figure of Fig. 4. In addition, the distances between the fixed joint points are determined at this step. In the second step, it sets the position of the first joint point P_0 of the tool end point to (x_0 , y_0 , z_0) of the CL data, while setting the second joint point P_1 on the direction of (i_0, j_0, k_0) of the CL data. Then, it determines each position of the remaining joint points in turn considering the kinematics constraints of the 5-axis machine as in the middle figure of Fig. 4. Finally, it calculates the joint values for linear and rotational displacements as in the right figure of Fig. 4. When the position of each joint point is determined, the joint value for the linear displacement is determined as the distance between the current and initial positions of the joint. In the same way, the joint values for rotational displacements are also determined as the angular difference between the current and initial angles, which are generated by two vectors connecting joint points at the rotation joint. A more detailed explanation is given in the following subsections.

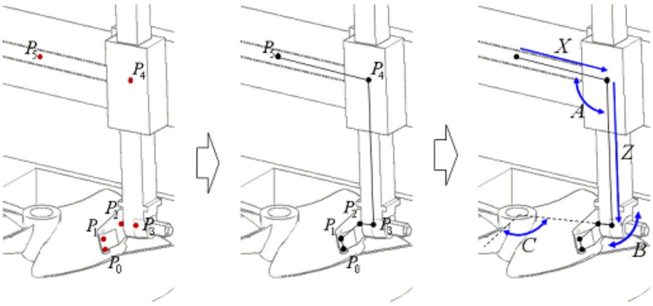


Fig. 4 Process of the proposed geometric method

A. Locate joint points

To foster understanding the proposed method, we will explain our algorithm with the 3R-2L 5-axis machine shown in Fig. 3. For the first step of the proposed method using geometric constraints, it locates each joint point using the kinematic constraints of the 5-axis machine and determines the distances between the fixed joint points, as shown in Fig. 5. In other words, it locates the first joint point P_0 at the tool tip and the remaining joint points at the center of each kinematic joint in turn. Although P_1 is not a kinematic joint, it is necessary to represent the direction of the tool axis.

Before proceeding to the next step, it determines the distance between the fixed joint points. In more detail, it calculates the length of each line $\overrightarrow{P_0P_1}, \overrightarrow{P_1P_2}, \overrightarrow{P_2P_3}$ and the height of P_5 from the center of the machine coordinate system as L_0, L_1, L_2 and L_3 .

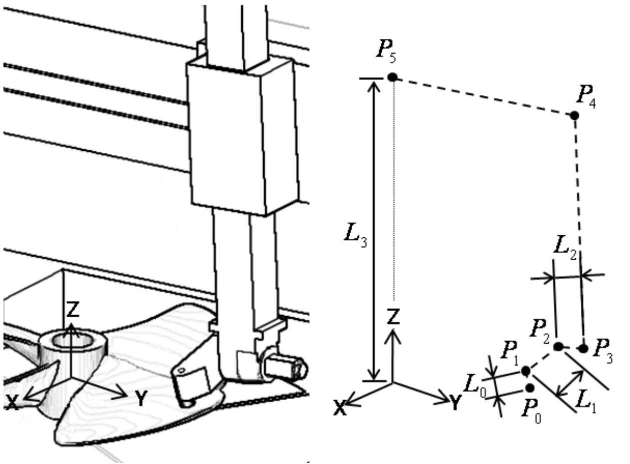


Fig. 5 Joint points of 3R-2L 5-axis machine

B. Calculate positions of joint points for the CL data

The second step of the proposed method calculates the position of every joint point so that the first joint point P_0 copies the position data (x_0 , y_0 , z_0) of the CL data and the direction vector from P_0 to P_1 has the direction (i_0, j_0, k_0) of the CL data.

Next, it calculates the position of the second joint point P_1 by offsetting from P_0 in the direction (i_0, j_0, k_0) by the distance L_0 , as shown in Fig. 6.

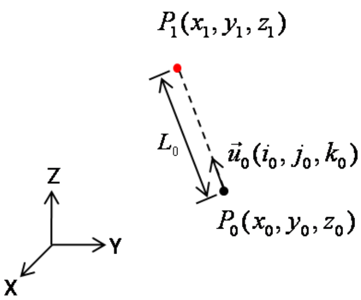


Fig. 6 Locating the first joint point P_0

The third joint point P_2 is on the *B*-axis of the machine. In addition, P_2 is separated from P_1 by the distance of L_1 in the direction normal to the line $\overline{P_0P_1}$. Therefore, the point P_2 must be on a circle that has radius L_1 , center P_1, and its normal in the tool axis, as shown in Fig. 7. Meanwhile, the

B-axis of the machine is designed to be on a plane that passes through the *Z*-axis of the machine coordinate system, while the positions of P_2 and P_3 are not yet known. As a result, P_2 is the intersection of the circle and the plane that passes through the *Z*-axis of the machine as shown in Fig. 7.

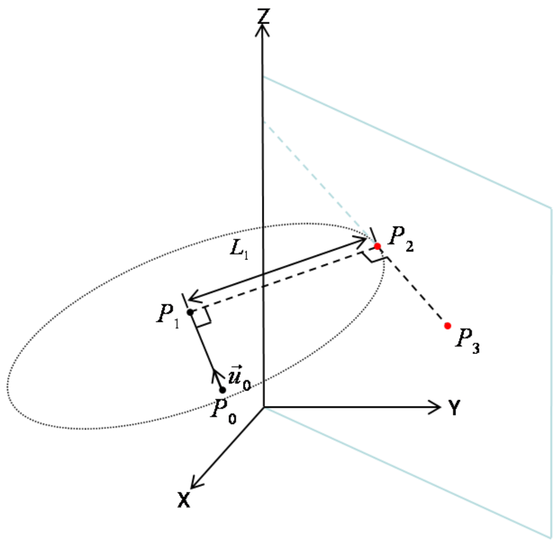
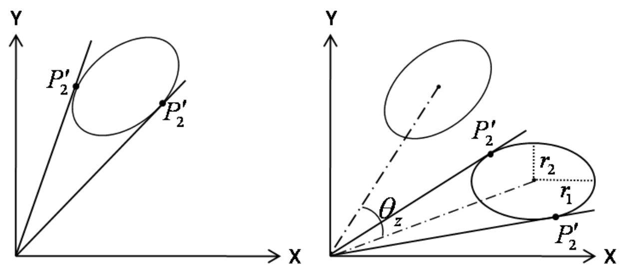


Fig. 7 Locating the third joint point P_2

To calculate the position of P_2, we first project the circle and the plane onto the *X*–*Y* plane of the machine coordinate system as an ellipse and a line, as shown in Fig. 8(a). In Fig. 8(a), there are two lines, because there may be two planes that contact the circle passing through the *Z*-axis. Once we find the intersection point P_2' of the ellipse and the line, we can determine the position of P_2. For the ellipse with axes parallel to the *X*-*Y* axes of the projected plane, we use (5) to calculate the inclined angle θ_{X-Y} of the axis $\vec{u}_0$ with respect to the *Z*-axis and the rotation angle θ_Z of the ellipse with respect to the *X*-axis, as shown in Fig. 8(b).



(a) (b)

Fig. 8 Ellipse on the *X*-*Y* plane and tangent lines to the ellipse

Then, we can calculate the new center (x', y') of the ellipse rotated by $-\theta_Z$ with (6), and the longer and shorter axes of the ellipse with (7). As a result, the equation of the new rotated ellipse is (8) and of the line is (9).

$$\begin{aligned}\theta_{X-Y} &= \arctan 2(\sqrt{i_0^2 + j_0^2},\ k_0)\\ \theta_Z &= \arctan 2(j_0, i_0)\end{aligned} \tag{5}$$

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \begin{bmatrix} \cos(-\theta_z) & -\sin(-\theta_z) \\ \sin(-\theta_z) & \cos(-\theta_z) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \tag{6}$$

$$r_1 = L_1, \quad r_2 = L_1 \cos\theta_{X-Y} \tag{7}$$

$$\frac{(x - x_1')^2}{r_1^2} + \frac{(y - y_1')^2}{r_2^2} = 1 \tag{8}$$

$$y = ax \tag{9}$$

Two solutions can be obtained from (8) and (9), which means there are two intersection points, as shown in Fig. 8(b). When both solutions are valid, we choose the better one, which is the one that will not cause any collision problems within the limit of machine operation.

Finally, after finding the *x*- and *y*-components of P_2 by rotating the selected P_2' by θ_Z, we obtain the *z*-component of P_2 with (10), using that the line $\overrightarrow{P_0P_1}$ is normal to the line $\overrightarrow{P_1P_2}$.

$$z_2 = z_1 - \frac{(x_1 - x_0)(x_2 - x_1) + (y_1 - y_0)(y_2 - y_1)}{z_1 - z_0} \tag{10}$$

The fourth point P_3 is determined as a point separated from P_2 by the distance L_2; it is binormal to $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_1P_2}$ for the machine of Fig. 3, as shown in Fig. 9.

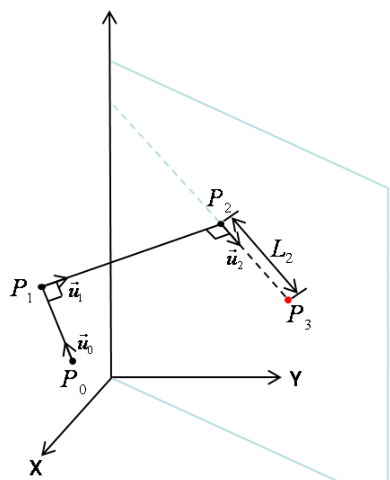


Fig. 9 Locating the fourth joint point P_3

The 5-axis machine of Fig. 3 has a horizontal upper column along which the *A*-axis moves. Therefore we first define a horizontal line (11) parallel with the horizontal upper column from point (0 , 0 , L_3) to point (x_3 , y_3 , L_3), and a plane (12) with a normal vector $\vec{u}_2$ (i_2 , j_2 , k_2) from the joint point P_3, as shown in Fig. 10. As a result, the fifth joint point P_4 is the intersection point of the line (11) and the plane (12).

$$\frac{x}{x_3} = \frac{y}{y_3}, \quad z = L_3 \tag{11}$$

$$i_2(x - x_3) + j_2(y - y_3) + k_2(z - z_3) = 0 \tag{12}$$

The final joint point P_5 is the reference point for the previous joint point P_4. It is located at the point that is separated from the origin of the machine coordinate system by L_3 in the Z-direction.

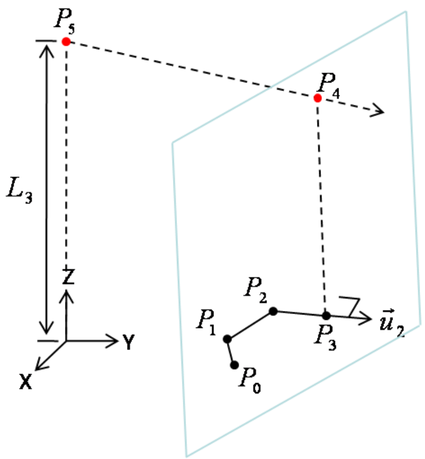


Fig. 10 Locating the fifth and the final joint points

C. Calculate joint values

In the last step of the proposed method, it calculates translational and rotational joint values for moving joints with the angles and distances between joint points determined in the previous step. For the translational joint values, it calculates the distance between P_4 and P_5 as the joint value *Y*, while the distance $L_3 - \left|\overrightarrow{P_3P_4}\right|$ as the joint value *Z*.

For the angular joint value *A*, it calculates the angle between $\overrightarrow{P_3P_4}$ and $\overrightarrow{P_4P_5}$, as shown in Fig. 11 when looking along the (-) *X*-axis at P_4. As a result, it calculates the joint value *A* as shown in (13).

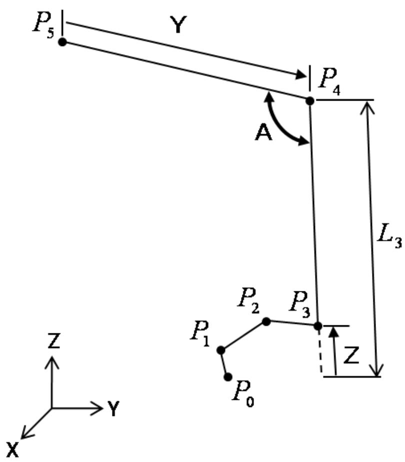


Fig. 11 Determination of joint angle A

$$\text{A} = \cos^{-1}\left(\frac{\overrightarrow{P_3P_4} \cdot \overrightarrow{P_4P_5}}{\left|\overrightarrow{P_3P_4}\right|\left|\overrightarrow{P_4P_5}\right|}\right) \tag{13}$$

For angular joint value *B*, it calculates the angle between $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_3P_4}$ as shown in Fig. 12(a) when looking along the direction $\overrightarrow{P_3P_2}$. As a result, it calculates the joint value *B* as shown in (14). In the same way, it calculates the joint value *C* as the angle between the *X*-axis and the line $\overrightarrow{P_5P_3}$, as shown in Fig. 12(b). Therefore, it calculates the joint value *C* as shown in (15).

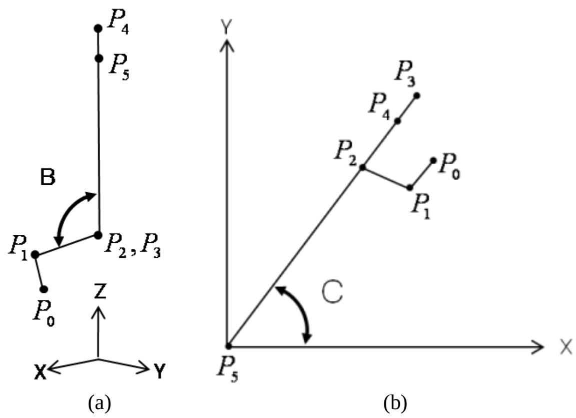


Fig. 12 Determination of joint angle B and joint angle C

$$\text{B} = \cos^{-1}\left(\frac{\overrightarrow{P_1P_2} \cdot \overrightarrow{P_3P_4}}{\left|\overrightarrow{P_1P_2}\right|\left|\overrightarrow{P_3P_4}\right|}\right) \tag{14}$$

$$\text{C} = \tan^{-1}\left(\frac{y_3}{x_3}\right) \tag{15}$$

IV. Application

We have developed a postprocessing program based on the proposed geometric method. To verify the proposed algorithm, we have generated postprocessed NC data using the developed program for finishing a propeller with a 3R-2L 5-axis machine, as shown in Fig. 13. Figure 14 shows the simulated result of the machining using the machining simulation program VERICUT [12], which shows that the tool trajectory of the CL data coincides exactly with the postprocessed NC data from the proposed algorithm.

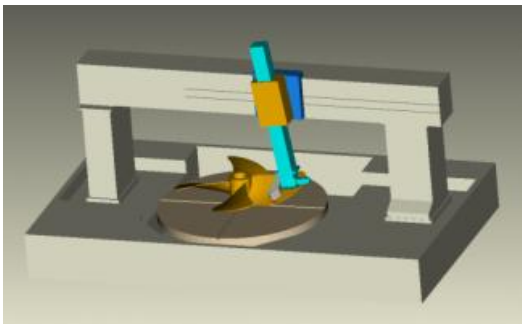


Fig. 13 Machining simulation for a 3R-2L machine

ISBN:978-988-18210-2-7

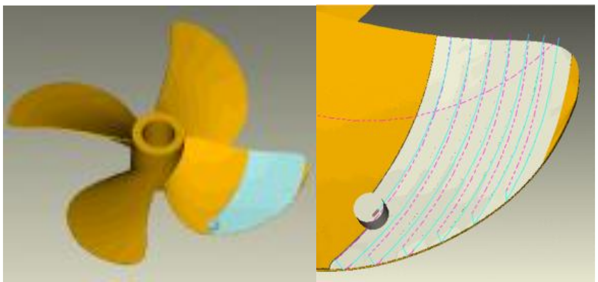


Fig. 14 Tool trajectory with the developed postprocessor

V. Concluding Remarks

The existing methods for postprocessing of 5-axis machines use a common approach that solves the inverse kinematics equation from the forward kinematics equation of each machine. However, when previous methods are applied to 3R-2L 5-axis machines, they generate complex and nonlinear forward kinematics equations, which results in failure to derive the inverse kinematics equation directly from the forward kinematics equation. To resolve the nonlinear equation, they use numerical methods to find a solution, which may cause large computation times and erroneous results.

We have proposed a geometric postprocessing algorithm using the joint points and the kinematic constraints of the 5-axis machine. The proposed method has the following advantages. First, no forward or inverse kinematics equation is required. Second, no numerical analysis is required to calculate the inverse kinematics of 3R-2L 5-axis machines, so efficient, exact solutions can be obtained. Finally, it is a general algorithm that can be applied to both 2R-3L and 3R-2L 5-axis machines.

Acknowledgment

This research was financially supported by the Ministry of Education, Science Technology (MEST) and Korea Institute for Advancement of Technology (KIAT) through the Human Resource Training Project for Regional Innovation and by grants-in-aid for the National Core Research Center Program from MEST/KOSEF (No.R15-2006-022-03002-0).

References

[1] P. Gray, S. Bedi, F. Ismail, N. Rao, and G. Morphy, “Comparison of 5-axis and 3-axis finish machining of hydroforming die inserts,” *International Journal of Advanced Manufacturing Technology*, vol. 17, 2001, pp. 562–569.
[2] R. Baptista and J.F. Antune Simoes, “Three and five axes milling of sculptured surfaces,” *Journal of Materials Processing Technology*, vol. 103, 2000, pp. 398–403.
[3] O. R. Tutunea-Fatan and H. Y. Feng, “Configuration analysis of five-axis machine tools using a generic kinematic model,” *International Journal of Machine Tools & Manufacture*, vol. 44, 2004, pp. 1235–1243.
[4] R. S. Lee and C. H. She, “Developing a postprocessor for three types of five-axis machine tools,” *International Journal of Advanced Manufacturing Technology*, vol. 13, 1997, pp. 658–665.
[5] C. H. She, C. C. Chang, and R. S. Lee, “A postprocessor based on the kinematics model for general five-axis machine tools,” *Journal of Manufacturing Processes*, vol. 2, no. 2, 2000, pp. 131–141.
[6] C. H. She and C. C. Chang, “Design of a generic five-axis postprocessor based on generalized kinematics model of machine tool,” *International Journal of Machine Tools & Manufacture*, vol. 47, 2007, pp. 537–545.
[7] Y. H. Jung, D. W. Lee, J. S. Kim, and H. S. Mok, “NC post-processor for 5-axis milling machine of table-rotating/tilting type,” *Journal of Materials Processing Technology*, vol. 130–131, 2002, pp. 641-646.
[8] E. G. Zaidman, “Development of a five-axis postprocessor system with a nutating head,” *Journal of Materials Processing Technology*, vol. 187–188, 2007, pp. 60–64.
[9] K. Sørby, “Inverse kinematics of five-axis machines near singular configurations,” *International Journal of Machine Tools & Manufacture*, vol. 47, 2007, pp. 299–306.
[10] B. K. Choi, J. W. Park, and C. S. Jun, “Cutter-location data optimization in 5-axis surface machining,” *Computer-Aided Design*, vol. 25, no. 6, 1993, pp. 377–386.
[11] Mikron Technology Group, http://www.mikron.com
[12] Vericut 5.2, http://www.vericut.com