Simple Methods for Stability Analysis of Nonlinear Control Systems

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Abstract — **Three methods for stability analysis of nonlinear control systems are introduced in this contribution: method of linearization, Lyapunov direct method and Popov criterion. Since stability analysis of nonlinear control systems is difficult task in engineering practice, these methods are made easier and tabulated. Method of linearization: The table includes the nonlinear equations and their linear approximation. Lyapunov direct method: The table contains Lyapunov functions for usually used equations second order. Popov criterion: The table will allow us to directly determine the stability of the nonlinear circuit with the transfer function $G(s)$ and the nonlinearity that satisfies the slope k.**

Index Terms — **Global asymptotic stability (GAS), Phase-plane trajectory, Modified frequency response.**

I. Method of Linearization

Consider the nonlinear autonomous n-order system. This system might be described by one nonlinear n-order equation or by a set on n first-order nonlinear differential equations [1]

$$\begin{aligned} x_1' &= f_1(x_1, x_2, \dots, x_n) \\ x_2' &= f_2(x_1, x_2, \dots, x_n) \\ &\dots\dots\dots\dots \\ x_n' &= f_n(x_1, x_2, \dots, x_n) \end{aligned} \tag{1}$$

or matrix equation $\mathbf{x}' = \mathbf{f}(\mathbf{x})$ (2)

The solution of the system (1) is given phase-plane trajectory in the n-dimensional state space. The points of the space in which is $f_1(\mathbf{x}) = f_2(\mathbf{x}) = \dots f_n(\mathbf{x}) = 0$ are singular points of the system because in the equilibrium points are speeds $x_1' = x_2' = \dots x_n' = 0$. The matrix representation for the linear system (1), where $\mathbf{f}(\mathbf{x})$ is linear function $\mathbf{x}$ we can write $\mathbf{x}' = \mathbf{A}\mathbf{x}$. On supposing that $\det \mathbf{A} \neq 0$ is solution $\mathbf{x} = 0$. The linear time-invariant system has an equilibrium point at the origin.

A nonlinear system can have more an equilibrium points because $\mathbf{f}(\mathbf{x}) = 0$ can have more solutions – more singular points. The equilibrium points can be stable or unstable. It depends on the phase-plane trajectory. They are stable if the trajectory approaches the equilibrium point as t tends to infinity and they are unstable if the trajectory recedes.

Stability theory plays a central role in systems theory and engineering. Stability of an equilibrium points we can find out by linearization of the equations (1) in the neighbourhood of each equilibrium point and then we have to find out stability of a surrogate system. If the linearization is allowable then the nonlinear system behaves similarly as the linearized system in the neighbourhood of equilibrium point.

If we can express function f_i (i = 1,2, ..., n) in a set (1) in Taylor series in the neighbourhood of each singular point then we can write for this singular point

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_i - x_{i0}) = \left(\frac{\partial f_i}{\partial x_1}\right)(x_1 - x_{10}) + \dots + \left(\frac{\partial f_i}{\partial x_n}\right)(x_n - x_{n0}) \tag{3}$$

or matrix way

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{x} - \mathbf{x}_0) = \mathbf{J}(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0) \tag{4}$$

where

$$\mathbf{J}(\mathbf{x}_0) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{5}$$

is Jacobian matrix that is defined as the matrix of partial derivatives with numerical values given singular point. The equation (3) is a set of linear differential equations that substitute the original set (1).

A necessary and sufficient condition of stability of system is that the characteristic equation has all the roots in the left half-plane. If the characteristic equation has one or more roots in the right half-plane, the system is unstable. If the single or multiple roots are located on the imaginary axis, we can't find out stability using linearization. But this stability that was found out by linearization is applicable only in a small enough region in the neighbourhood of equilibrium point.

Now we will accomplish the practical linearization of second order system assumed the equation

$$y'' + g(y') + f(y) = 0 \tag{6}$$

If this is rearranged as two first-order equations, choosing the phase variables as the state variables, that is $x_1 = y$; $x_2 = y'$, then equation (1) can be written as

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -g(x_2) - f(x_1) \end{aligned} \tag{7}$$

Singular points of this system we will obtain by solving $x_1' = x_2' = 0$ and they are the points on the real axis $[x_{10}\ ;0]$. When we introduce for conciseness the symbol

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Table I: Linearized equations of nonlinear systems

No	Equation of nonlinear system	ψ	Singular points	Linearized equation
1	$y'' + by^r y' + y = 0$	$-bx_1^r x_2 - x_1$	$[0;0]$	$y'' + y = 0$
2	$ay'' + by^r y' + cy = 0$	$-\frac{b}{a}x_1^r x_2 - \frac{c}{a}x_1$	$[0;0]$	$y'' + \frac{c}{a}y = 0$
3	$ay'' + by' + cy + dy^2 = 0$	$-\frac{b}{a}x_2 - \frac{c}{a}x_1 - \frac{d}{a}x_1^2$	$[0;0]$	$y'' + \frac{b}{a}y' + \frac{c}{a}y = 0$
			$\left[-\frac{c}{d};0\right]$	$y'' + \frac{b}{a}y' - \frac{c}{a}y + \frac{c}{a} = 0$
4	$ay'' + by' + cy + dy^t = 0$	$-\frac{b}{a}x_2 - \frac{c}{a}x_1 - \frac{d}{a}x_1^t$	$[0;0]$	$y'' + \frac{b}{a}y' + \frac{c}{a}y = 0$
5	$ay'' + by' + cy^r y' + dy = 0$	$-\frac{b}{a}x_2 - \frac{c}{a}x_1^r x_2 - \frac{d}{a}x_1$	$[0;0]$	$y'' + \frac{b}{a}y' + \frac{d}{a}y = 0$
6	$ay'' + by' + cy'^r + dy + ey^2 = 0$	$-\frac{b}{a}x_2 - \frac{c}{a}x_2^r - \frac{d}{a}x_1 - \frac{e}{a}x_1^2$	$[0;0]$	$y'' + \frac{b}{a}y' + \frac{d}{a}y = 0$
			$\left[-\frac{d}{e};0\right]$	$y'' + \frac{b}{a}y' - \frac{d}{a}y = 0$
7	$ay'' + by' + cy'^r + dy + ey^t = 0$	$-\frac{b}{a}x_2 - \frac{c}{a}x_2^r - \frac{d}{a}x_1 - \frac{e}{a}x_1^t$	$[0;0]$	$y'' + \frac{b}{a}y' + \frac{d}{a}y = 0$
8	$ay'' + b(m + ny^r)y' + cy = 0$	$-\frac{b}{a}(m + nx_1^r)x_2 - \frac{c}{a}x_1$	$[0;0]$	$y'' + \frac{b}{a}my' + \frac{c}{a}y = 0$
9	$ay'' + b(my^p + ny^r)y'^q + cy = 0$	$-\frac{b}{a}(mx_1^p + nx_1^r)x_2^q - \frac{c}{a}x_1$	$[0;0]$	$y'' + \frac{c}{a}y = 0$

$$\psi = -g(x_2) - f(x_1) \tag{8}$$

then the Jacobian matrix of this system is (where we have to give for $x_1\,;x_2$ the coordinate of singular point $x_1 = x_{10}\,;\;\; x_2 = 0$)

$$\mathbf{J}(\mathbf{x}_0) = \begin{bmatrix} 0 & 1 \\ \frac{\partial\psi}{\partial x_1} & \frac{\partial\psi}{\partial x_2} \end{bmatrix} \tag{9}$$

and then following equations represents the linearization of the nonlinear equations about the equilibrium point

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_1 - x_{10}) = x_2$$

$$\frac{\mathrm{d}}{\mathrm{d}t}x_2 = \frac{\partial\psi}{\partial x_1}(x_1 - x_{10}) + \frac{\partial\psi}{\partial x_2}x_2 \tag{10}$$

$$\Rightarrow \begin{aligned} x_1' &= x_2 \\ x_2' &= \frac{\partial\psi}{\partial x_1}x_1 - \frac{\partial\psi}{\partial x_1}x_{10} + \frac{\partial\psi}{\partial x_2}x_2 \end{aligned}$$

These equations correspond to the linearized second order equation (11)

$$y'' - \frac{\partial \psi}{\partial x_2} y' - \frac{\partial \psi}{\partial x_1} y + \frac{\partial \psi}{\partial x_1} = 0 \tag{11}$$

We can substitute the origin nonlinear equation (6) this equation (11) and to find out stability of the nonlinear system like stability of the linear system, but only in a small enough region in the neighbourhood of equilibrium point.

Notice that the absolute member in the equation (11) is constant that doesn't influence stability. As long as a singular point lies at the origin $x_{10} = x_{20} = 0$, the constant is zero and the equation (11) is

$$y'' - \frac{\partial \psi}{\partial x_2} y' - \frac{\partial \psi}{\partial x_1} y = 0 \tag{12}$$

The results of this linearization are in the table I.

For example to find out stability of the system

$$y'' + 10y' + 5y + 2{,}5y^3 = 0$$

Substitute into this $x_1 = y$; $x_2 = y'$ the equation will be

$$x_1' = x_2$$

$$x_2' = -10x_2 - 5x_1 - 2{,}5x_1^3$$

The system has a unique equilibrium point at the origin $[0\,;0]$. In this point is

$$\frac{\partial \psi}{\partial x_1} = \frac{\partial\left(-10x_2 - 5x_1 - 2{,}5x_1^3\right)}{\partial x_1} = -5 - 7{,}5x_1^2\Big|_{[0;0]} = -5$$

$$\frac{\partial \psi}{\partial x_2} = \frac{\partial\left(-10x_2 - 5x_1 - 2{,}5x_1^3\right)}{\partial x_2} = -10$$

and the linearized equation (12) is

$$y'' + 10y' + 5y = 0$$

The characteristic equation has roots $s_1 = -9{,}47$; $s_2 = -0{,}53$. The roots are real negative, the equilibrium point is stable. The system is stable in the neighbourhood of this equilibrium point. We could find out that the equilibrium point is knot.

In table I are the commonly used the equation of the second order, their singular points and the linearized equation for the neighbourhood of equilibrium point. The table helps us to find out immediately if the equilibrium point is stable or unstable.

The example above had the equation

$$y'' + 10y' + 5y + 2{,}5y^3 = 0$$

correspond to No. 4, where is $a = 1$; $b = 10$; $c = 5$; $d = 2{,}5$; $t = 3$. This equation has one singular point $[0\,;0]$ and in the neighbourhood of equilibrium point is valid the equation.

The table can help us to solve stability of the nonlinear second order systems.

II. Lyapunov direct method

Lyapunov's method for stability analysis is in principle very general and powerful. The major drawback, which seriously limits its in practise, is the difficulty often associated with constructions of the Lyapunov function or V-function required by the method.

Consider again the nonlinear autonomous n-order system. This system might be described by one nonlinear n-order equation or by a set on n first-order nonlinear differential equations (1) or matrix equation (2).

The vector **x** is the state vector, and its elements are state varibles. The origin $\mathbf{x} = \mathbf{0}$ ($x_1 = \ldots = x_n = 0$) of the state space will be assumed to be an equilibrium solution.

The Lyapunov function $V(x_1, x_2, \ldots, x_n)$ is a scalar function of the state variables and it is positive definite. Now let

$$V' = \frac{dV}{dt} = \frac{\partial V}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial V}{\partial x_2}\frac{dx_2}{dt} + \ldots + \frac{\partial V}{\partial x_n}\frac{dx_n}{dt} \tag{13}$$

be calculated by substituting (1). If V' were to be found to be always negative, then apparently V decreases continuously, and the state must end up in the origin of the state space, implying asymptotic stability.

To develop these concepts, the following definitions are used for the sign of V and V':

- A system is globally asymptotically stable if V' is negative definite.
- It is globally stable if V' is negative semidefinite.
- It is unstable if V' is positive definite or semidefinite.

(If V' is indefinite then it is not possible to decide about stability.)

Lyapunov's method is a very powerful tool for studying the stability of equilibrium points. However, there are two drawbacks of the method that we should be aware of. First, there is no systematic method for finding a Lyapunov function for a given system. Second, the conditions of the method are only sufficient; they are not necessary. Failure of a Lyapunov function candidate to satisfy the conditions for stability or asymptotic stability does not mean that the origin is not stable or asymptotically stable.

We would like to eliminate the first drawback of Lyapunov's method. The table of Lyapunov's functions for second-order systems is the table II and for third-order systems is the table III.

For example: Consider the second-order system

$$y'' + \left(1 + y^2\right)y' + y = 0$$

Taking x_1 and x_2 as the state variables, we obtain the state equation

$$x_1' = x_2$$

$$x_2' = -\left(1 + x_1^2\right)x_2 - x_1$$

The system has evidently an equilibrium point at the origin. Using the table 2 we will choose Lyapunov's function number
4 ($a = b = c = m = n = q = t = 1$; $r = 0$; $s = 2$)

$$V(x_1, x_2) = x_1^2 + x_2^2$$

The derivative of V along the trajectories of the system is given by

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Table II: Lyapunov´s functions for second-order systems

No	Equation of system	Restriction		Lyapunov function V
1	$ay'' + by' + cy = 0$	-		$V = \frac{c}{a} x_1^2 + x_2^2$
2	$ay'' + by^r y'^q + cy = 0$	r even	q odd	
3	$ay'' + by^r y'^q + cy^t = 0$	r even	q,t odd	$V = 2\frac{c}{a}\frac{1}{t+1} x_1^{t+1} + x_2^2$
4	$ay'' + b\left(my^r + ny^s\right)y'^q + cy^t = 0$	r,s even	q,t odd	
5	$ay'' + b\left(my^r + ny^s\right)\left(ey'^q + fy'^k\right) + cy^t = 0$	r,s even	k,q,t odd	

Table III: Lyapunov´s functions for third-order systems

No	Equation of system	Restriction	Lyapunov function V
1	$y''' + ay''^m + by'^r = 0$	a,b>0 m,r odd	$V = \frac{2b}{r+1} x_2^{r+1} + x_3^2$
2	$y''' + ay''^m y'^n + by''^r y^t + ey'^h = 0$	a,b,e>0 m,r odd n,t,h even	$V = \frac{2e}{h+1} x_2^{h+1} + x_3^2$
3	$y''' + ay''^m y'^n y^p + by''^r y'^s y^t + ey'^h = 0$	a,b,e>0 h,m,r odd n,p,s,t even	$V = \frac{2e}{h+1} x_2^{h+1} + x_3^2$
4	$y''' + ay''^m y'^n + by''^p y^r + cy''^s y'^t y^u +$ $+ dy''^{-1} y' y^z = 0$	a,b,c,d>0 m,p,s,z odd n,r,t,u even	$V = \frac{2d}{z+1} x_2^{z+1} + x_3^2$
5	$y''' + ay''^m y'^n + by''^p y^r + cy''^s y'^t y^u +$ $+ ey'^h = 0$	a,b,c,e>0 m,p,s,h odd n,r,t,u even	$V = \frac{2e}{h+1} x_2^{h+1} + x_3^2$
6	$y''' + ay''^m y'^n + by''^p y^r + cy''^s y'^t y^u +$ $+ dy''^{-1} y' y^z + ey'^h = 0$	a,b,c,d,e>0 h,m,p,s,z odd n,r,t,u even	$V = \frac{2d}{z+1} x_1^{z+1} +$ $+ \frac{2e}{h+1} x_2^{h+1} + x_3^2$
7	$y''' + ay''^m y'^n + by''^p y^r + cy''^s y'^t y^u +$ $+ y''^{-1} y' y + y' - y = 0$	a,b,c>0 m,p,s odd n,r,t,u even	$V = \left(x_1 - x_2\right)^2 + x_3^2$
8	$y''' + ay''^m y'^n + by''^p y^r + cy''^s y'^t y^u +$ $+ dy''^{-1} y' y + ey' + fy = 0$	a,b,c,d,e>0 f<0 $e = \frac{f^2}{d}$ m,p,s odd n,r,t,u even	$V = \left(\sqrt{d}\, x_1 + \frac{f}{\sqrt{d}} x_2\right)^2 + x_3^2$

$$V'\left(x_1, x_2\right) = \frac{\partial V}{\partial x_1} x_1' + \frac{\partial V}{\partial x_2} x_2' =$$
$$= 2x_1 x_2 + 2x_2\left[-\left(1 + x_1^2\right)x_2 - x_1\right] = -2\left(1 + x_1^2\right)x_2^2$$

The function V' is negative semidefinite (because for $x_2 = 0$ the function V' is for arbitrary x_1 equals zero) and the system is globally stable.

III. Popov criterion

The Popov criterion is considered as one of the most appropriate criteria for nonlinear systems and it can be compared with the Nyquist criterion for linear systems [4]. However there are reservations that relate to the very essence, correctness and reliability of the criterion. It is necessary to emphasise that this criterion is reliable, but the conditions of its appl. should be clearly specified in advance.

Fig. 1 shows the system configuration with one nonlinear element and a linear part with the transfer function $G(s)$ that can include all linear elements. It shows the nonlinearity that is single-valued, time–invariant and constrained to a hatch sector bounded by slopes k that is assumed to satisfy: for the case when all poles of $G(s)$ are inside the left-half plane

$$0 \le \frac{f(e)}{e} \le k < \infty \,,$$

for the case when $G(s)$ has poles on the imaginary axis (the so-called critical case)

$$0 < \frac{f(e)}{e} \le k < \infty$$

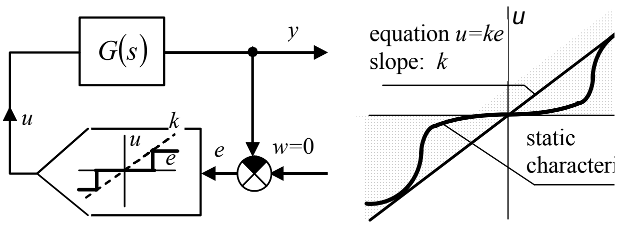


Fig.1: Non-linear control circuit and characteristic of nonlinear element

For the case that $G(s)$ has poles only inside the left-half s-plane, the static characteristic can be zero both in the beginning and out of the beginning. For the case when $G(s)$ has poles on the imaginary axis, it must not be zero out of the beginning.

A nonlinear circuit with a transfer function of the linear part $G(s)$ and with the nonlinear element (with the above described nonlinearity) is globally asymptotically stable when an arbitrary real number q (> 0 or = 0 or < 0) exists where for every $\omega \ge 0$ the following inequality is completed

$$\mathrm{Re}\left[(1 + j\omega q)G(j\omega)\right] + \frac{1}{k} > 0 \qquad (14)$$

The Popov criterion can be – for more convenience – applied graphically in the $G(j\omega)$-plane. Let us apply a modified frequency response function $G^*(j\omega)$, defined

$$\begin{aligned} \mathrm{Re}\, G^*(j\omega) &= \mathrm{Re}\, G(j\omega) \\ \mathrm{Im}\, G^*(j\omega) &= \omega . \mathrm{Im}\, G(j\omega) \end{aligned} \qquad \text{for } \omega \ge 0 \qquad (15)$$

and we obtain the graphical interpretation of the Popov criterion for global asymptotic stability (GAS): The sufficient condition for GAS of nonlinear circuit is that the plot of $G^*(j\omega)$ should lie entirely to the right of the Popov line which crosses the real axis at $-1/k$ at a slope $1/q$ (q is an arbitrary real number) .

In this contribution table III shows the commonly used nonlinear circuits (with stability being solved). The table has been constructed for the circuits with different transfer function $G(s)$.There is an algebraic solution to the inequality (14) on condition that:

$0 \le k < \infty$ (only for poles $G(s)$ inside the left-half of s-plane);
$0 < k < \infty$ (also for poles $G(s)$ on the imaginary axis); $a, b > 0$; ω... for every value from 0 to ∞ ; q...arbitrary.

In the table IV, the frequency response function of the linear part of the circuit is presented in the form inserted to the inequality (14), and the resultant relation is presented after modifications. If the inequality is satisfied for k, q, a, b, ω, the circuit in question is globally asymptotically stable (GAS)

Table IV: Modified frequency response plots

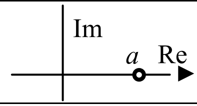
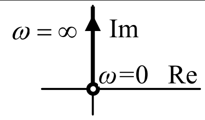
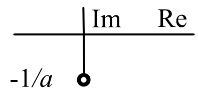
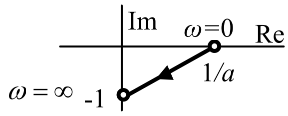
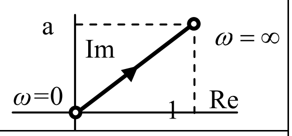
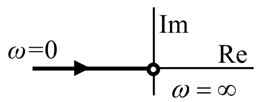
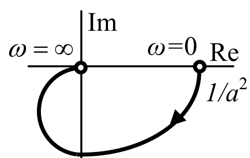
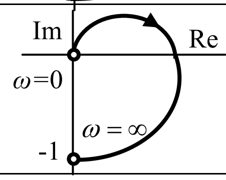
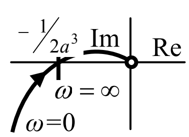
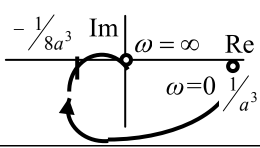
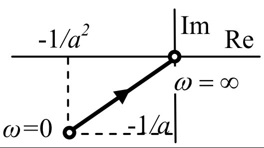
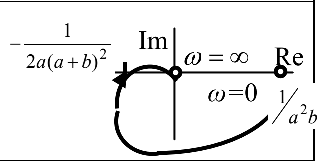
No	$G(s)$	$G^*(j\omega)$
1	a	
2	as	
3	$\frac{1}{as}$	
4	$\frac{1}{s+a}$	
5	$\frac{s}{s+a}$	
6	$\frac{1}{as^2}$	
7	$\frac{1}{(s+a)^2}$	
8	$\frac{s}{(s+a)^2}$	
9	$\frac{1}{s(s+a)^2}$	
10	$\frac{1}{(s+a)^3}$	
11	$\frac{s+b}{s(s+a)^2}$	
12	$\frac{1}{(s+b)(s+a)^2}$	

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Table V: Results of stability analysis by Popov´s criterion

No	$G(s)$	$G(j\omega)$ in inequality (14) $\mathrm{Re}\left[\left(1+j\omega q\right)G\left(j\omega\right)\right]+\frac{1}{k}>0$	Solution of (14)	Conditions of stability
1	a	a	$ka+1>0$	GAS for every k
2	as	$aj\omega$	$1 - k\omega^3 q > 0$	GAS for every k
3	$\frac{1}{as}$	$-j\frac{1}{a\omega}$	$qk + a > 0$	GAS for every k
4	$\frac{1}{s+a}$	$\frac{a}{a^2+\omega^2}-j\frac{\omega}{a^2+\omega^2}$	$k\left(a+\omega^2 q\right)+a^2+$ $+\omega^2>0$	GAS for every k
5	$\frac{s}{s+a}$	$\frac{\omega^2}{a^2+\omega^2}+j\frac{a\omega}{a^2+\omega^2}$	$k\omega^2\left(1-aq\right)+a^2+$ $+\omega^2>0$	GAS for every k
6	$\frac{1}{as^2}$	$-\frac{1}{a\omega^2}$	$k<a\omega^2$	not stable for arbitrary k
7	$\frac{1}{\left(s+a\right)^2}$	$\frac{a^2-\omega^2}{\left(a^2+\omega^2\right)^2}-j\frac{2a\omega}{\left(a^2+\omega^2\right)^2}$	$k\omega^2\left(2aq-1\right)+ka^2+$ $+\left(a^2+\omega^2\right)^2>0$	GAS for every k
8	$\frac{s}{\left(s+a\right)^2}$	$\frac{2a\omega^2}{\left(a^2+\omega^2\right)^2}+j\frac{a^2\omega-\omega^3}{\left(a^2+\omega^2\right)^2}$	$k\omega^2\left[2a+q\left(\omega^2-a^2\right)\right]+$ $+\left(a^2+\omega^2\right)^2>0$	GAS for every k
9	$\frac{1}{s\left(s+a\right)^2}$	$\frac{-2a}{\left(a^2+\omega^2\right)^2}+j\frac{\omega^2-a^2}{\omega\left(a^2+\omega^2\right)^2}$	…	GAS for $k < 2a^3$
10	$\frac{1}{\left(s+a\right)^3}$	$\frac{1}{\left(j\omega+a\right)^3}=......$	…	GAS for $k < 8a^3$
11	$\frac{s+b}{s\left(s+a\right)^2}$	$\frac{j\omega+b}{j\omega\left(j\omega+a\right)^2}=...$	$k>-\frac{4a^6-12a^5b+6a^4b^2}{4a^4-10a^3b+8a^2b^2-2ab^3}$	
12	$\frac{1}{\left(s+b\right)\left(s+a\right)^2}$	$\frac{1}{\left(j\omega+b\right)\left(j\omega+a\right)^2}=...$	$k<\frac{2a^5+8a^4b+12a^3b^2+8a^2b^3+2ab^4}{\left(a+b\right)^2}$	

for every value k. If it is not the case, then it is possible to determine for which k circuit is GAS or that it is not for any k (this calculation is illustrated in the table). Therefore the table will allow us to directly determine the stability of the nonlinear circuit with the transfer function $G(s)$ and the nonlinearity that satisfies the slope k. Table V illustrates the modified frequency response plots enabling us graphic solutions to stability.

IV. Conclusion

Stability analysis of nonlinear control systems is difficult task in engineering practice. This paper can help to solve the problem. The describe methods creates tables and graphs for three methods of stability analysis.

First method is method of linearization. The table includes the nonlinear equations and directs their linear approximation. Second method is Lyapunov direct method. The table contains Lyapunov functions for usually used control circuits second and third-order. The last method is Popov criterion. The table will allow us to directly determine the stability of the nonlinear circuit with the transfer function $G(s)$ and the nonlinearity that satisfies the slope k.

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