Control Volume Method Applied To Simulation Of Hydrodynamic Lubrication Problem

Mobolaji H. Oladeinde and John A. Akpobi

***Abstract*—**In this paper, a control volume formulation is used to obtain the pressure distribution in a finite slider bearing with slip surfaces. The domain of the finite slider bearing is discretized into control volumes and the discretized form of the governing equation obtained by integrating the governing equation over each control volume. The derivatives which result are approximated using finite difference method. The values of the film thickness at the boundaries were evaluated by computing the harmonic mean as a means of ensuring continuity. The system of equations obtained exhibits diagonal dominance and is solved using Gauss Seidel iterative scheme. Comparison of the results obtained using the present method with that obtained using FEHYDROLUB, a finite element based software shows good agreement. The method has been shown to be suitable for simulating hydrodynamic lubrication problems

***Keywords*—** Control volume, hydrodynamic lubrication, pressure, FEHDYDROLUB.

I. Introduction

The solution of the hydrodynamic lubrication problem requires obtaining an approximate numerical solution to Reynolds equation. Several numerical techniques have been proposed to provide the solution of the fluid film lubrication problem The complexity, non linearity and absence of close form solution to the full Reynolds renders it unsolvable by known analytical methods. Researchers have therefore resorted to numerical means to solve the problem.

A number of researchers have investigated obtained the solution of different slider bearing configurations using different numerical schemes. In recent times, most numerical work in hydrodynamic lubrication has involved the use of the Reynolds equation and the finite difference method [1]. A finite difference multigrid approach was used to investigate the squeeze film behavior of poroelastic bearing with couple stress fluid as lubricant by [2]. In [3], the modified Reynolds equation extended to include couple stress effects in lubricants blended with polar additives was solved using the Finite difference method with a successive over relaxation scheme. The conjugate Method of iteration was used to build up the pressure generated in a finite journal bearing lubricated with a couple stress fluids in [4]. Reference [5] provided a numerical solution for a mathematical model for hydrodynamic lubrication of misaligned journal bearings with couple stress fluids as lubricants using the Finite Difference Method. Reference [6] calculated the steady and perturbed pressures of a two dimensional plane inclined slider bearing incorporating a couple stress fluids using the conjugate gradient method. In [7], the problem of finite hydrodynamic journal bearing lubricated by magnetic fluids with couple stresses was investigated using the finite difference method. The finite element method has been used prominently for some years to continuum and field problems Reference [8] presented the finite element solution for incompressible lubrication problems of complex geometries without the loss of accuracy as the finite difference method. In [9], a velocity-pressure integrated, mixed interpolation, Galerkin finite element method for the Navier-Stokes equations was reported. A finite element method was used to analyze the electromechanical field of a hydrodynamic-bearing (HDB) spindle motor of computer hard disk drive at elevated temperature in [10]. .The finite element method to solve the modified Reynolds equation governing the pressure distribution in an parabolic slider bearing with couple stress fluids in [11].. Reference [12] reported the steady state characteristics of an infinitely wide inclined slider bearingobtained using the finite element method.

The open literature is replete with slider bearing design using finite difference and finite element methods as the numerical tool for analysis as can be deduced from the literature cited above. Previous researchers seem not to have exploited the applicability of control volume methods in slider bearing design. . It is this gap that the present paper seeks to fill. In particular, this work centers on the use of control volume method for solving the modified Reynolds equation governing the pressure distribution in a finite slider bearing with lip surfaces

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II. GOVERNING EQUATION

The equation governing the pressure distribution in a finite slider bearing with slip surfaces is given by (1)

$$\frac{\partial}{\partial X}\left(H^3\frac{\partial p}{\partial X}\left(1+\frac{3A}{H+A}\right)\right)+L^2\frac{\partial}{\partial Y}\left(H^3\frac{\partial p}{\partial Y}\left(1+\frac{3A}{H+A}\right)\right)=U\frac{\partial}{\partial X}\left(H\left(1+\frac{H}{H+A}\right)\right)$$

$$H=1+k-kx \tag{1}$$

The boundary conditions are given by the specification of the pressure at the perimeter of the bearing which is equal to atmospheric pressure.

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III. CONTROL VOLUME DESCRITIZATION

Fig. 1 shows a typical control volume around a central node P. Control volume discretization is used to discretize the governing equation shown in (1). The control volume consists of a rectangular volume whose sides' passes through the point's n, s, e and w.

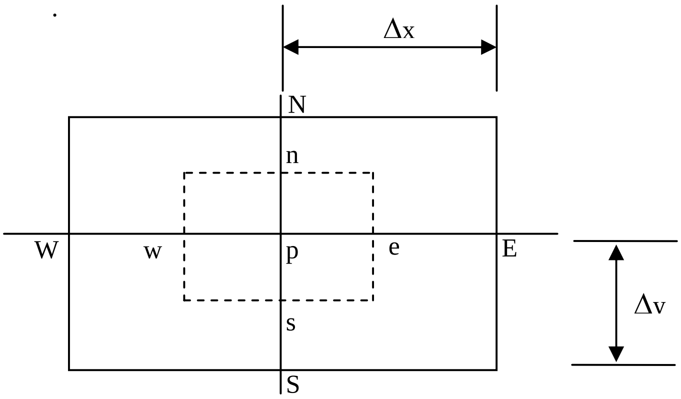

Fig. 1 A control volume around a node P

The control volume is used to solve for the pressure at the point O in terms of the pressures at the points N, E, S and W. Integrating (1) over the control volume, (2) is obtained

$$\int_s^n\int_e^w \frac{\delta y}{\delta x}\left(K\frac{\delta y}{\delta x}\right)dxdy + L^2\int_s^n\int_e^w \frac{\delta y}{\delta y}\left(K\frac{\delta y}{\delta y}\right)dxdy = U\int_s^n\int_e^w \frac{\delta}{\delta x}(C)\,dxdy \tag{2}$$

Integrating (2), (3) is obtained

$$\left(K\frac{\delta P}{\delta x}\right)_e \Delta y - \left(K\frac{\delta P}{\delta x}\right)_w \Delta y + L^2\left(K\frac{\delta P}{\delta y}\right)_n \Delta x - L^2\left(K\frac{\delta P}{\delta y}\right)_s \Delta x$$
$$= U(C)_e\,\Delta y - U(C)_w\,\Delta y \tag{3}$$

Equation (2) was obtained by substituting the expressions in (4) into (1).

$$K = H^3\left(1+\frac{3A}{H+A}\right) \qquad C = H\left(1+\frac{A}{H+A}\right) \tag{4}$$

It can be observed that (3) contains derivatives which can be approximated using the finite difference method. The derivatives in (3) can be approximated using (5) and (6)

$$\left(\frac{\delta P}{\delta x}\right)_e = \frac{P_E - P_P}{\Delta x} \qquad \left(\frac{\delta P}{\delta x}\right)_w = \frac{P_p - P_w}{\Delta x} \tag{5}$$

$$\left(\frac{\delta P}{\delta y}\right)_n = \frac{P_N - P_P}{\Delta y} \qquad \left(\frac{\delta P}{\delta y}\right)_s = \frac{P_P - P_S}{\Delta y} \tag{6}$$

Substituting (5) and (6) into (3) and noting (7), the final discretized form of (1) is obtained.

$$a_E = \frac{K_e}{\Delta x}\Delta y \qquad a_W = \frac{K_w}{\Delta x}\Delta y \qquad a_N = L^2\frac{K_n}{\Delta y}\Delta x \quad a_S = L^2\frac{K_s}{\Delta y}\Delta x$$
$$k_e = \frac{2K_E K_P}{K_E + K_P} \qquad k_w = \frac{2K_W K_P}{K_W + K_P} \qquad k_s = \frac{2K_S K_P}{K_S + K_P}$$
$$k_n = \frac{2K_N K_P}{K_N + K_P} \qquad a_p = a_N + a_S + a_E + a_W$$
$$S_C = U\left(C_w - C_e\right)\Delta y \tag{7}$$

IV NUMERICAL EXAMPLE

In the following section, we implement the method described above to the solution of the equation governing the pressure distribution in a plane inclined slider bearing with slip velocity. The following data are used for the control volume discretization. Slip velocity (A) = 50, Slider velocity = 100, film thickness ratio = 1.5. The length of the bearing in the x direction is equal to the length of the bearing in the y direction such that the ratio $\mathrm{L_x} / L_y = 1$.

A Discretization of Solution Domain

The grid spacing in the x direction Δx and that in the y direction Δy are choose to be equal in length. Since the governing equation has been presented in dimensionless form, the grid spacing is obtained by specifying the number of grids in each direction and the grid spacing is computed by finding the reciprocal of the number of grids.

B Solution Using a 2x2 grid

The discretized form of the governing equation for the three nodes in the control volume is given by the system of equations below.

$$37.9586P_1 - 6.87753P_2 = 0.5625$$
$$-6.87753P_1 + 21.09730P_2 - 3.50101P_3 = 0.5625$$
$$-3.50101P_2 + 10.1446P_3 = 0.5625$$

In matrix form, the system of equations above can be written as shown below

$$\begin{bmatrix} 37.9586 & -6.87753 & 0 \\ -6.87753 & 21.09730 & -3.50101 \\ 0 & -3.50101 & 10.1446 \end{bmatrix}\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 0.5625 \\ 0.5625 \\ 0.5625 \end{pmatrix}$$

Solving the system of equations above we obtain the following solution.

$P_1 = 0.02316 \qquad P_2 = 0.04605 \qquad P_3 = 0.07134$

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C Solution Using a 4X4 Grid

For a 4x4 grid the grid spacing Δx and Δy are equal to 0.25. The system of equations governing the pressure distribution in all the internal nodes in the control volume is shown below.

$$47.766P_1 - 19.191P_2 - 3.439P_4 = 0.28125$$
$$-19.191P_1 + 47.766P_2 - 19.191P_3 - 3.439P_5 = 0.28125$$
$$-19.191P_2 + 47.766P_3 - 3.439P_6 = 0.28125$$
$$-3.439P_1 + 26.627P_4 - 10.719P_5 - 1.751P_7 = 0.28125$$
$$-3.439P_2 - 10.719P_4 + 26.627P_5 - 10.719P_6 - 1.751P_8 = 0.28125$$
$$-3.439P_3 - 10.719P_5 + 26.627P_6 - 1.751P_9 = 0.28125$$
$$-1.751P_4 - 12.871P_7 - 5.199P_8 = 0.28125$$
$$-1.751P_5 - 5.199P_7 + 12.871P_8 - 5.199P_9 = 0.28125$$
$$-1.751P_6 - 5.199P_8 + 12.871P_9 = 0.28125$$

The system of equations above is observed to exhibit diagonal dominance and was solved using Gauss Seidel iteration method with a convergence criterion of 10^{-5}. On pressure convergence the solution below is obtained

$$P_1 = 0.01798 \;\; P_2 = 0.02373 \;\; P_3 = 0.01798 \;\; P_4 = 0.03558$$
$$P_5 = 0.04714 \;\; P_6 = 0.03558 \;\; P_7 = 0.05657 \;\; P_8 = 0.07396$$
$$P_9 = 0.05657$$

V. DISCUSSION OF RESULTS

The governing equation has been solved using the control volume method for two different control volume meshes namely a 2x4 control volume and a 4x4 control volume. In the 2x4 control volume solution, the pressure obtained at the node at (0.25, 0.25) is equal to 0.02316 while that obtained using the 4x4 control volume is 0.02376. At the point (0.5,0.25) the 2x4 control volume produces a pressure of 0.04605 whereas the 4x4 control volume produces a pressure equal to 0.04714. At point (0.75,0.25) the 2x4 control volume gives a pressure of 0.07134 while the 4x4 control volume produces a pressure of 0.07396. It can be observed that as the size of the control volume decreases from 2x4 to 4x4, the pressure at the nodes converges towards the exact nodal pressure. In order to have comparative values from different method of simulation to compare the present results with, FEHYDROLUB, finite element based software developed by the authors was used to solve the same problem. It was observed that the exact value of the pressure at the middle of the bearing corresponding to the point (0.5, 0.25) is equal to 0.04914. This value was obtained when a series of simulations were made with progressively smaller elements until the results obtained became mesh independent. The pressure obtained by the present method at the middle of the bearing for a 4x4 control volume is 0.04714. With smaller control volumes, it is expected that the control volume solution will converge asymptotically to the exact. A control volume based software for solving hydrodynamic lubrication problem is being developed by the authors and will be presented in subsequent articles.

VI. CONCLUSION

A control volume method has been used to solve the hydrodynamic lubrication problem of slider bearings. The solution obtained is stable and converges with increase in the number of control volumes in the domain. It has been shown that the present method can be used to model hydrodynamic lubrication problems successfully.

REFERENCES

[1] B. P. Mitidierri, "Advanced modeling of elastohydrodynamic lubrication ", Doctoral Thesis, Tribology Section and thermo fluids section, Department of Mechanical Engineering, Imperial College, London, pp 19 .2005

[2] N. M. Bujurke and B.R. Kudenati, " Multigrid solution of modified Reynolds' equation incorporating poroelasticity and couple Stress", Journal of porous medis, Vol. 10, issue 2, pp 125 – 136 , 2007

[3] M. Senangi, B.C. Majumda, and A.S Sekhar,, " Elastohydrodynamically lubricated ball bearings with couple stress fluids, Part 1: Steady state analysis ", Tribology transactions, Vol 48, issue 3, pp 404 – 414, 2005

[4] R. Linj, " Squeeze film characteristics of finite journal bearings; couple stress fluid model Tribology International , Vol 31, issue 4, pp 201 – 207 ,1998

[5] A.A. Elsharkawy, "Effects of misalignment on the performance of finite journal bearings lubricated with couple stress fluids " , International Journal of Computer Applications in Technology (IJCAT), Vol. 21, No. 3, 2004

[6] J. R. Lin, " Derivation of dynamic couple-stress Reynold's equation of sliding-squeezing surfaces and numerical solution of plane inclined slider bearings", Tribology International , Vol 36, issue 9, pp 679 – 685 ,2003.

[7] G.S.Nada, and T.A. Osman, , "Static performance of finite hydrodynamic Journal bearings lubricated by magnetic fluids with couple stresses ", Tribology letters, Vol 27, number 3,, pp 261 – 268 ,2007.

[8] O.C. Zienkiewicz, " The Finite Element Method", 2nd edition, 1970, McGraw-Hill,1970.

[9] S.W.,Kim " A velocity-pressure integrated, mixed interpolation, Galerkin finite element method for high Reynolds number laminar flows, Final Report Universities Space Research Association, Huntsville, AL. Systems Dynamics Lab, 1988

[10] G.H. Jang, S.J Park and S.H. Lee,, "Finite-element analysis of electromechanical field of a HDB spindle motor at elevated temperature", IEEE Transactions, Volume 40, Issue 4, July 2004 pp: 2083 – 2085

[11] M.H. Oladeinde, and J.A. Akpobi, “Parametric characterization of load capacity of an infinitely wide parabolic slider bearing with couple stress fluids”, Proceedings of the World Congress of Science, Engineering and Technology, Vol. 57, pp. 565 – 569 ,2009.

[12] M.H .Oladeinde, and J.A Akpobi, , “Finite element simulation of performance characteristics of infinitely wide plane pad slider bearing”. Advanced Materials Research, Vol. 62 – 64, pp 637 – 642. 2009.

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