A Practical Optimization Techniques for M ^{X}/D/1 Queuing Model

Pradeep K. Joshi, Chitaranjan Sharma

ABSTRACT - In the present paper the problem of minimizing total queuing time for single server queuing system with batch arrival has been considered. The customers arrive for service in batches of different sizes. For different categories of customers the arrival pattern follows the Poisson distribution. The service time distribution is based on deterministic model.

I. INTRODUCTION

Many researchers have been attracted towards practical optimization techniques for the study of queuing problems of day to day life. But much more work is need in the way of demonstrating the scheduling of available time of a service facility by using optimization techniques. [6]-[7] studied the problem of scheduling of out patients clinic having different categories of patients. [1] considered the problem of scheduling total time of service facility between different types of customers on the priority basis for M/D/I queuing model. They extended their work in [2]. There exist a considerable literature bulk/ batch queues see [3]. [4] developed non-linear programming techniques for a queuing system with Pareto service time distribution. [8] prepared an attempt in this direction by developing a non-linear programming approach for M^x/M/I queuing model.

In the present analysis, an optimization technique has been applied to more general M^x/D/I queuing model. In this model, we consider the problem of scheduling of a service facility between different categories of customers in order to minimize the total queuing time.

Pradeep K. Joshi, Reader, School of Computers and Electronics, IPS Academy, Indore,M.P. (India).
Chitaranjan Sharma, Asst. Professor, Holkar Science Coollege, Indore, M.P. (India).

II .MATHEMATICAL MODEL AND ANALYSIS

Let us assume that the mean and square coefficient of variation of the batch size $\left(\overline{a}_i, C_a^2\right)$, the service time $\left(\frac{1}{\mu_i}, C_s^2\right)$ and mean arrival rate $\frac{1}{\lambda_i}$ for customers of categories i (i = 1, 2, 3, 4, ……….. N). The probability distribution for the group size customers of ith categories is prob. (x = k) = a_{ki} ,(k = 1, 2, ……N, i = 1, 2, 3 ………. N).

For statistical equilibrium the average waiting time for the ith categories of the customers is given by [3] as

$$W_i = \frac{\overline{a_i}(c_a^2 + 1) + l_i - 1}{2(1 - l_i)\mu_i} \qquad (1)$$

where $l_i = \frac{\lambda_i \overline{a_i}}{\mu_i} < 1$

Let $x_i = \mu_i - \lambda_i \overline{a_i} > 0 \qquad (2)$

Substituting values from (2) in (1), we have

$$w_i = \frac{\overline{a_i}(c_a^2 + 1) - x_i \frac{\lambda_i \overline{a_i}}{x_i + \lambda_i \overline{a_i}} - 1}{2x_i} \qquad (3)$$

Our problem is to share the excess time between N types of the customers, so that their time can be globally minimized. Therefore we have the problem:

$$E = T - \sum_{i=1}^{N} \lambda_i \sigma_i \overline{a_i} \qquad (4)$$

where σ_i is the service time of customers of ith categories.

Therefore, we have to

$$\text{Min} \quad \sum_{i=1}^{N} l_i \lambda_i \overline{a_i} w_i \qquad (5)$$

$$\text{Such that } E = \sum_{i=1}^{N} \sigma_i x_i \qquad (6)$$

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Substituting the value of w_i from (3) in the (5) and applying Kuhn-Tucker condition. The customers of i^{th} categories (i = 1, 2, ………………N) is served with a priority with weight age l_i .

$$\frac{\lambda_i \overline{a_i} l_i}{2x_i^2}\left[\frac{-\overline{a_i}(c_a^2+1)+1-\lambda_i\overline{a_i}\lambda_i\overline{a_i}\left[1+\frac{2x_i}{\lambda_i\overline{a_i}}\right]}{\left(\lambda_i\overline{a_i}\right)^2\left(1+\frac{x_i}{\lambda_i\overline{a_i}}\right)^2}\right]+z\sigma_i=0 \quad (7)$$

where z is an unknown real number.

From (7), we obtain

$$z\sigma_i=\frac{-\lambda_i\overline{a_i}l_i}{2x_i^2}\left[\frac{-\overline{a_i}(c_a^2+1)+1-(\lambda_i\overline{a_i})^2\left[1+\frac{2x_i}{\lambda_i\overline{a_i}}\right]}{\left(\lambda_i\overline{a_i}\right)^2\left(1+\frac{x_i}{\lambda_i\overline{a_i}}\right)^2}\right] \quad (8)$$

(8) can be written as

$$z\sigma_i=\frac{-\lambda_i\overline{a_i}l_i}{2x_i^2}\left[-\overline{a_i}(c_a^2+1)\right] \quad (9)$$

Since $x_i << \lambda_i a_i$ therefore $\left(1+\frac{x_i}{\lambda_i\overline{a_i}}\right)^2$ can be

approximated by $\left(1+\frac{2x_i}{\lambda_i\overline{a_i}}\right)$.

Now (9) gives

$$x_i=\left[\frac{l_i\ \lambda_i\overline{a_i}\left\{\overline{a_i}(c_a^2+1)\right\}}{2z\sigma_i}\right]^{\frac{1}{2}} \quad (10)$$

Now by (6) and (10), we have

$$E=\sum_{i=1}^{N}\sigma_i x_i=\sum_{i=1}^{N}\left[\frac{\sigma_i l_i\ \lambda_i\overline{a_i}\left\{\overline{a_i}(c_a^2+1)\right\}}{2z}\right]^{\frac{1}{2}} \quad (11)$$

Therefore

$$z'=(2z)^{\frac{1}{2}}=\left(\frac{1}{E}\right)\sum_{i=1}^{N}\left[\sigma_i l_i\ \lambda_i\overline{a_i}\left\{\overline{a_i}(c_a^2+1)\right\}\right]^{\frac{1}{2}} \quad (12)$$

Now (10) becomes

$$x_i=\frac{1}{z'}\left[\frac{l_i\ \sigma_i\overline{a_i}\left\{\overline{a_i}(c_a^2+1)\right\}}{\sigma_i}\right]^{\frac{1}{2}} \quad (13)$$

By using (2), (12) and (13), we can obtain service time parameters μ_i for (i = 1, 2, …………. N) for different categories of customers.

III. DISCUSSION

Our approach for study of scheduling of total time between N types of customers who can arrive in batches of different sizes provides an easy computable formula for unknown service parameters. This queuing situation can occur in many practical situations e.g. restaurants, clinics etc.

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