Multi-Objective Optimization of Multi-commodity, Multi-Period Facility Location within Distribution Network Design

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Abstract— In this paper an approach to optimize facility location problem in distribution system is presented. The contribution is to minimize total establishment, transportation and inventory management costs through simultaneously maximizing customer satisfaction. Including dynamic view to the problem will lead to better facility location within distribution network design. This paper reviews an optimizing approach for a multi-objective multi-commodity multi-period distribution system and is supported by a case study to be implemented and followed the solutions. The demand of customer is assumed to be deterministic. The results show that the new model with components to cover inventory decisions and multiple periods will increase the efficiency of the system significantly.

Index Terms—Facility location, Multi-Commodity, Multi-Objective, Multi-Period, Optimizing Distribution System

I. INTRODUCTION

Supply chain management can be defined as a set of approaches utilized to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide cost while satisfying service level requirements[20].

Distribution decisions play major role in satisfaction of customers by on time delivery at the lowest shipping costs. These decisions affect distribution channels, the number of channels, the location of warehouses, transportation modes, and the consolidation of goods. There are more works in literature considering concepts of SCM in variant areas which, some of them are stated as follows.

As describe by [10], the generic facility location problem in logistic systems is defined by taking simultaneous decisions regarding design, management and control of generic distribution network (2), [7], [16], [18] and [21]). [17] presented a review of the literature on facility location and supply chain management. According to this review, important decisions in facility location are:

1. Location of new supply facilities in a given set of demand points. The demand points correspond to existing customer locations.

2. Demand flows to be allocated to available or new suppliers (i.e. production and/or distribution facilities).

3. Configuration of a transportation network, i.e. design of paths from suppliers to customers, management of routes and vehicles in order to supply demand needs simultaneously.

In fact facility location problem, in many papers and researches, is introduced to be strongly associating with effective management of multi-stage production and distribution networks. Many papers in recent years have studied the facility location (e.g. [5], [10], [13], [16], [23], [28], [30]) and all of them have brought contributions to the main concept.

In traditional supply chain management, the focus of the integration of SCN is usually on single objective such as minimum cost or maximum profit. For example, [4], [11], [14], [15], [24], [25], [27] and [29] had considered total cost of supply chain as an objective function in their studies. However, there are no design tasks to be single objective problems. The design/planning/scheduling projects are usually involving trade-offs among different incompatible goals. Recently, multi objective optimization of SCNs has been considered by different researchers in literature. [19] developed an integrated multi-objective supply chain model for strategic and operational supply chain planning under uncertainties of product, delivery and demand. While cost, fill rates, and flexibility were considered as objectives, e-constraint method had been used as a solution methodology. [6] proposed a multi-objective genetic optimization procedure for the order distribution problem in a demand driven SCN. They considered minimization of total cost of the system, total delivery days and the equity of the capacity utilization ratio for manufacturers as objectives. [7] developed a multi-product, multi-stage, and multi-period scheduling model for a multi-stage SCN with uncertain demands and product prices. As objectives, fair profit distribution among all participants, safe inventory levels and maximum customer service levels, and robustness of decision to uncertain demands had been considered, and a two-phased fuzzy decision-making method was proposed to solve the problem. [8] proposed a model which assigns suppliers to warehouses and warehouses to customers. They used a multi-objective optimization modeling framework for minimizing cost and maximizing customer satisfaction. [12] formulated the SCN design problem as a multi-objective stochastic mixed integer linear programming model, which was solved by e-constraint method, and branch and bound techniques. Objectives were SC profit over the time horizon.
and customer satisfaction level. [6] developed a hybrid approach based on genetic algorithm and Analytic Hierarch Process (AHP) for production and distribution problems in multi-factory supply chain models. Operating cost, service level, and resources utilization had been considered as objectives in their study.

Some of the authors in recent years have developed multi-period models. [9] presented a model for simultaneously optimizing inventory and designing a distribution network. They explored the single sourcing version of the problem by using a branch-and-price optimal solution procedure. [3] proposed linear models to solve the simultaneous warehouse location-inventory management-routing problem. [22] formulate the stochastic version of the joint location-inventory management problem, by introducing the likelihood of occurrence of each cost factor into the objective function. [26] proposed a mixed integer linear model for the design and planning of a production-distribution system. [10] presented a non-linear model supporting strategic, tactical, and operational choices of decision makers in the field of facility location, inventory, and production management is formulated in a multi-period perspective. [1] introduced a multi-objective model for optimizing multi-commodity distribution facility location problem. This model improved inventory decisions in distribution network design.

In this paper, we present a multi-objective mixed integer programming formulation for location within network distribution problem. Objectives are to minimize total cost including establishment and transportation cost and to maximize customer satisfaction. The problem describes two location layers in multiple periods. We determine the volume of the inventory in both stocks and middle warehouses. For better understanding of presented model, a case in automobile spare parts distribution is studied and the model was implemented in real environment. The results would be meaningful for verification of presented model.

The left parts of the paper are as follows. Model description is stated in section II. In Section III, mathematical model is formulated, computational results are indicated in section IV and conclusions are discussed in section V.

II. MODEL DESCRIPTION

Components of supply chain are described as below by explanation of including parts.

Central warehouses: the main stocks of supply chain that demands are supplied here. There are L potential locations for central warehouses.

Regional warehouses: stocks between central warehouses and customers that demands are distributed there. There are M potential locations for regional warehouses that they are in the capital of provinces.

Customers: there are N customers that are located in the cities of the provinces.

Goods: O types of commodities can be supplied for the customers demanding O families of cars.

Assumptions of problem are as follows:
- There are limited capacities for both central and regional warehouses,
- Transportation cost per unit is as a coefficient of distance between central and regional warehouses and between regional warehouses and customers,
- There is a minimum level of customer satisfaction.

There are two objectives for supply chain, minimizing total cost including establishment, transportation and inventory management cost and maximizing customer satisfaction.

III. MODEL FORMULATION

A. Sets and indices

L Sets of central warehouses (\( |L| = l \), \( l \in L \)),
\( M \) Sets of regional warehouses (\( |M| = m \), \( j \in M \)),
\( N \) Sets of customers (\( |N| = n \), \( i \in N \)),
\( O \) Sets of good types (\( |O| = o \), \( i \in O \)),
\( F \) Sets of periods (\( |F| = f \), \( f \in F \))

B. Variables

\( v_k \) = \begin{cases} 1, & \text{If the potential point of } k \text{ for} \\ 0, & \text{central warehouses is located} \end{cases} \\
\( u_j \) = \begin{cases} 1, & \text{If the potential point of } j \text{ for} \\ 0, & \text{regional warehouses is located} \end{cases} \\
\( x_{pij} \) Percentage of demand customer \( i \) for commodity \( t \) that is supplied by regional warehouse \( j \) in period \( p \) ,
\( y_{pjk} \) Percentage of demand regional warehouse \( j \) for commodity \( t \) that is supplied by central warehouse \( k \) in period \( p \).

C. Parameters

\( a_{pit} \) Demand of customer \( i \) for commodity \( t \) in period \( p \),
\( b_{pjt} \) Capacity of regional warehouse \( j \) for commodity \( t \) in period \( p \),
\( c \) Cost of transportation per unit,
\( d_{ij} \) Distance between regional warehouse \( j \) and customer \( i \),
\( d'_{jk} \) Distance between regional warehouse \( j \) and central warehouse \( k \),
\( e_{pkt} \) Capacity of central warehouse \( k \) for commodity \( t \) in period \( p \),
\( s_{it} \) Minimum level of customer satisfaction \( i \) for commodity \( t \),
\( q_k \) Cost of installation central warehouse \( k \),
\( w_j \) Cost of installation regional warehouse \( j \),
\( h_w \) Warehousing cost per unit goods in warehouses
\( h_s \) Warehousing cost per unit goods in stocks
\( \pi \) Back ordered cost per unit goods
\( d_{wspk} \) The demand of regional warehouse \( j \) from commodity \( t \) to central warehouse \( k \) in period \( p \)
D. Mathematical Model

\[
\begin{align*}
\text{Min } Z_1 &= \sum_{p} \sum_{k} \sum_{j} \sum_{t} c \cdot d_{jk} b_{pjkt} + \sum_{j} w_{j} u_{j} + \sum_{k} q_{k} v_{k} \\
&+ h_{w} \sum_{p} \sum_{j} \left( \sum_{k} b_{pjkt} y_{pjk} - \sum_{j} a_{p} x_{pjkt} \right) \\
&+ h_{s} \sum_{p} \sum_{j} \sum_{k} \left( e_{p} v_{k} - \sum_{j} y_{pjk} \right) \\
\text{Min } Z_2 &= \frac{1}{n \cdot o \cdot f} \sum_{p} \sum_{t} \sum_{j} \sum_{i} x_{pjkt} \\
&\leq \sum_{p} e_{pkt} \\
&\leq \sum_{p} b_{pjkt} y_{pjk} \geq \sum_{i} a_{p} x_{pijt} \\
&\forall j, p \quad (8.1) \\
b_{pjkt} u_{j} \geq \sum_{k} d_{w_{p} jkt} \\
&+ \sum_{j} \sum_{k} \left( b_{(p+1)kt} y_{(p+1)jk} \right) \\
&\forall j, p \quad (8.2) \\
&> 1, t \\
d_{w_{p} jkt} \leq b_{pjkt} u_{j} \\
&\sum_{j} \sum_{k} \left( b_{(p+1)kt} y_{(p+1)jk} \right) \\
&\forall j, p \quad (8.3) \\
&> 1, t \\
d_{w_{p} jkt} \leq \sum_{j} \sum_{k} a_{p} x_{pjkt} \\
&\sum_{j} \sum_{k} \left( b_{(p+1)kt} y_{(p+1)jk} \right) \\
&\forall j, p \quad (8.4) \\
&> 1, t, k \\
e_{p} v_{k} \geq \sum_{j} d_{w_{p} jkt} u_{j} \\
&\sum_{j} \sum_{k} \left( b_{(p+1)kt} y_{(p+1)jk} \right) \\
&\forall j, p \quad (8.5) \\
u_{j} \in \{0, 1\} \\
&\forall j \\
v_{k} \in \{0, 1\} \\
&\forall k \\
&\forall k \quad (10)
\end{align*}
\]

First objective function \(Z_1\), which is multiplied by weighting coefficient of \(P\), is summation of:

- Transportation cost between central and regional warehouses, \(\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t} c \cdot d_{ij} a_{p} x_{pijt}\).
- Transportation cost between regional warehouses and customer, \(\sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{j=1}^{n} c \cdot d_{jk} b_{pjkt}\).
- Installation cost for central warehouses, \(\sum_{k} q_{k} v_{k}\)
- Installation cost for regional warehouses, \(\sum_{k} q_{k} v_{k}\)
- Warehousing costs, for commodities in all periods in regional and central warehouses.

Second objective, \(Z_2\), is the summation of the level of the customer satisfaction that is multiplied by \((1 - P)\).

Constraints (1), (3), (5) and (6) states if regional warehouse \(j\) or central warehouse \(k\) satisfy the demand in period \(p\), it has been installed. Constraints (2) and (7) show capacity restriction for each regional warehouse for each commodity in all periods. Constraint (4) implies that there is a minimum level of customer satisfaction \(t\) for commodity \(t\). Constraint (8) considers that amount of supply should be greater than amount of demand. Constraints (9) and (10) are related to integer programming.
IV. COMPUTATIONAL RESULTS

This model was solved with data from the mentioned case. This case includes 2 nominated central warehouses, 8 nominated regional warehouses, 5 groups of commodities and 4 periods for implementation. According to the multi-objective nature of the model, a weighting mechanism is designed to cope with different values of weighting parameter, $P$, in the objective function. So the model is solved with different quantitative of “$P$” and the results can be shown in Table 1. To indicate the accuracy of the model, a graph is depicted in $s=0.1$ for objective function values versus each other (see Fig. 1).

Another important variable which is added to model to control the minimum initial satisfaction level plays an essential role in optimal selection of location decisions. Table 1 shows the Computational results for different $P$ values at different $s_{th}$. To highlight the importance Fig. 2 is presented which shows that how much it could be critical to adjust parameters and the necessity of wisely solution selection after parameter tuning.

To cope with these difficulties a method is presented. This method consists of a new function which includes the percentage summation of satisfaction and total costs divided by maximum cost in each $P$ value. The new function shows how to get desired aim in minimum costs and minimum customer dissatisfaction. Fig. 3 demonstrates new function values vs. different $s_{th}$.

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**Table 1. Computational results for different $P$**

<table>
<thead>
<tr>
<th>$s_{th}$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>2.9E+10</td>
<td></td>
</tr>
<tr>
<td>$Z_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$Z_3$</td>
<td>1.48E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>1.51E+10</td>
<td>2.9E+10</td>
<td></td>
</tr>
<tr>
<td>$Z_4$</td>
<td>0.99</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$Z_5$</td>
<td>1.45E+10</td>
<td>1.46E+10</td>
<td>1.46E+10</td>
<td>1.47E+10</td>
<td>1.48E+10</td>
<td>1.49E+10</td>
<td>1.49E+10</td>
<td>1.5E+10</td>
<td>1.51E+10</td>
<td>2.9E+10</td>
</tr>
<tr>
<td>$Z_6$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>1</td>
</tr>
<tr>
<td>$Z_7$</td>
<td>1.39E+10</td>
<td>1.4E+10</td>
<td>1.42E+10</td>
<td>1.43E+10</td>
<td>1.45E+10</td>
<td>1.46E+10</td>
<td>1.47E+10</td>
<td>1.49E+10</td>
<td>1.5E+10</td>
<td>2.9E+10</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$Z_9$</td>
<td>1.33E+10</td>
<td>1.35E+10</td>
<td>1.39E+10</td>
<td>1.41E+10</td>
<td>1.43E+10</td>
<td>1.45E+10</td>
<td>1.46E+10</td>
<td>1.48E+10</td>
<td>1.5E+10</td>
<td>2.9E+10</td>
</tr>
<tr>
<td>$Z_{10}$</td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>1</td>
</tr>
</tbody>
</table>

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**Fig. 2. Total Costs vs. Min Service Level at $P=0.1$ to $1$**

**Fig. 3. New Function Value vs. Min Service Level**

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Based on Fig 3, first this decision must be taken that what could be the minimum service level and then by reviewing Fig 3, the decision maker can find the minimum function value in relevant service level. By this method the proper coefficient in total objective value will be obtained. Table 2 demonstrates calculation results of model for mentioned case studied at \( s_{it} = 0.7 \) and \( P = 0.5 \).

<table>
<thead>
<tr>
<th>Variable Title</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs ((z_i))</td>
<td>1.46E+10</td>
</tr>
<tr>
<td>Customer Satisfaction((z_4))</td>
<td>98%</td>
</tr>
<tr>
<td>( s_{it} )</td>
<td>0.7</td>
</tr>
<tr>
<td>V(1,2)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>U(1,2,3,4,5,6,7,8)</td>
<td>(1,1,1,1,1,1,0)</td>
</tr>
</tbody>
</table>

As mentioned in previous parts, this model was implemented in a automobile part distribution corporation. So the results of implementation can be compared with before implementation situation. Some performance measurement indexes were defined for comparing these. Table 3 could be referred for the proof of the efficiency of new model.

<table>
<thead>
<tr>
<th>Index</th>
<th>Before</th>
<th>After</th>
<th>Improvement (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Satisfaction</td>
<td>65%</td>
<td>98%</td>
<td>50%</td>
</tr>
<tr>
<td>Total Cost</td>
<td>4.57E+09</td>
<td>3.66E+09</td>
<td>19%</td>
</tr>
<tr>
<td>Total Revenue</td>
<td>2.89E+12</td>
<td>3.91E+12</td>
<td>35%</td>
</tr>
<tr>
<td>Profit</td>
<td>2.85E+12</td>
<td>2.89E+12</td>
<td>36%</td>
</tr>
</tbody>
</table>

As is shown in above table, the implementation of model can improve the system performance considerably.

V. CONCLUSION

Application of supply chain concept to real problems has caused the issue to be more interesting. New problems include more variable and parameters. Also it is expected that the designed models fulfill more objectives. This paper is an embodiment of this issue.

This model is presented for multi-objective multi-commodity multi-period location problems. Nonconformity between objective functions is a concern and just weighting wouldn’t be sufficient to satisfy the aims. Therefore, some contribution is needed.

In this paper a mix integer model is introduced and solved with LINGO software. According to acquisition of model parameters from a real case, the model outcomes were used for decision making. According to extending the model for multiple periods, the efficiency of the presented model was improved comprehensively. There are similarities between the model and the studied case. Hence the model can be a good representative of the case. Therefore we can claim that the model is practical and useful.

VI. ACKNOWLEDGEMENT

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REFERENCES