

# Time-Efficient Genetic Algorithm for Peak Power Reduction of OFDM Signal

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*Abstract*— Orthogonal frequency division multiplexing (OFDM) is superior in spectral efficiency and is widely used in today's digital communication. One of the drawbacks of OFDM is that the peak-to-average power ratio (PAPR) of the transmitted signal tends to be high. In order to overcome this problem, peak power reduction methods based on tone injection have been proposed. The peak power reduction problem solved with tone injection is a combinatorial problem. In this paper, we apply genetic algorithm (GA) to the reduction method based on tone injection. The proposed method utilizes a tree-structured fitness table to reduce the computation time. The effectiveness of the GA method is demonstrated by numerical simulations in terms of PAPR, bit error rate (BER) and computation time. We also investigate effective fitness functions and crossover operators. It is shown that a uniform crossover operation is most effective. Further, it is shown that our GA method is superior in terms of computation time compared to a conventional GA method and a random search method.

*Keywords:* OFDM, PAPR, tone injection, genetic algorithm, tree-structured table

## 1 Introduction

Recent advance in digital signal processing technology demands faster wireless communication. Orthogonal frequency division multiplexing (OFDM) [1] is superior in spectral efficiency and is widely used in today's digital communication. One of the drawbacks of OFDM is that the peak-to-average power ratio (PAPR) of the transmitted signal tends to be high. In order to overcome this problem, various peak power reduction methods have been proposed[2, 3, 4, 5, 6, 7]. Tone injection is an effective technique to overcome the PAPR problem[6, 7]. The peak power reduction problem solved with tone injection is a combinatorial problem. In [6], a greedy searching algorithm has been proposed. Its main drawback is to easily get a stack shallow local minimum. In [7], the solution space is reasonably reduced by introducing some

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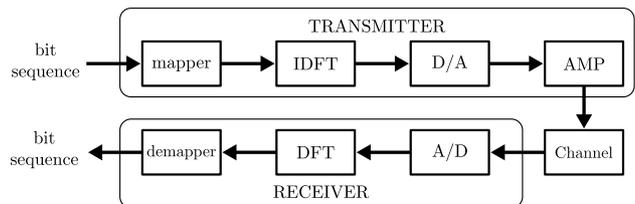


Figure 1: Flow of OFDM from a transmitting end to a receiving end.

constraints into the tone injection technique, and a peak power reduction method based on neural network (NN). However, the reduction performance is high.

In this paper, we apply genetic algorithm (GA) to the reduction method based on tone injection. The proposed GA method utilizes a tree-structured fitness table to reduce the computation time. The effectiveness of the GA method is demonstrated by numerical simulations in terms of PAPR and bit error rate (BER). We also investigate effective fitness functions and crossover operators. It is shown that a uniform crossover operation is most effective. Further, it is shown that our GA method is superior in terms of computation time compared to the conventional GA method and a random search method.

## 2 OFDM and Tone Injection

### 2.1 OFDM

OFDM uses multiple subcarriers that are orthogonal to each other. Let  $T$  be the OFDM symbol time. The subcarriers are spaced  $1/T$  Hz apart from each other. The flow of OFDM from a transmitting end to a receiving end is shown in Fig.1.

At the transmitter side, a bit sequence  $b_{L-1}, b_{L-2}, \dots, b_0$  to be transmitted is converted into a sequence of complex symbols  $X_0, X_1, \dots, X_{N-1}$  by mapper. In this conversion, each  $M$ -bit subsequence in  $L$ -bit sequence is mapped to a complex number according to the used digital modulation scheme such as phase-shift keying (PSK) and quadrature amplitude modulation (QAM). When using  $M$ -QAM,  $X_n$  represents  $\log_2 M$ -bit subsequence  $b_{L-m \cdot n-1}, b_{L-m \cdot n-2}, \dots, b_{L-m \cdot n-M}$ , where

$L = M \log_2 M$  and  $m = \log_2 M$ . Samples of OFDM signal  $Y_0, Y_1, \dots, Y_{N-1}$  are generated by IDFT (Inverse Discrete Fourier Transform) as follows:

$$Y_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi}{N} nk}. \quad (1)$$

The IDFT operation can be performed by IFFT (Inverse Fast Fourier Transformation) in  $O(N \log N)$  steps. The samples are converted to an analog signal, and then the signal is amplified and is fed to the transmission channel.

The demodulation process is performed in reverse order of transmitter's operations. From the received OFDM signal,  $N$  samples  $\tilde{Y}_0, \tilde{Y}_1, \dots, \tilde{Y}_{N-1}$  are extracted with  $T/N$  sampling interval. Symbols  $\tilde{X}_0, \tilde{X}_1, \dots, \tilde{X}_{N-1}$  are restored by DFT (Discrete Fourier Transform) as follows:

$$\tilde{X}_n = \sum_{k=0}^{N-1} \tilde{Y}_k e^{-j \frac{2\pi}{N} nk}. \quad (2)$$

The restored symbol sequence  $\tilde{X}_0, \tilde{X}_1, \dots, \tilde{X}_{N-1}$  is converted to a bit sequence  $\tilde{b}_{L-1}, \tilde{b}_{L-2}, \dots, \tilde{b}_0$ . If  $\tilde{X}_n = X_n$  for all  $n \in \{0, 1, \dots, N-1\}$ , the receiver retrieves the transmitted bit sequence  $b_{L-1}, b_{L-2}, \dots, b_0$  with no error, that is, for all  $l \in \{0, 1, \dots, L-1\}$ ,  $\tilde{b}_l = b_l$ .

The OFDM signal often has a very high peak power compared to its average power, because the signal is produced as a synthetic signal of a number of subcarriers. The degree of the peak power is evaluated by peak-to-average power ratio (PAPR) defined as follows:

$$\text{PAPR} = \frac{\max_{0 \leq k < N} |Y_k|^2}{E\{|Y_n|^2\}}, \quad (3)$$

where  $E\{|Y_n|^2\}$  is the average power of the OFDM signal.

## 2.2 Tone Injection

Tone injection suppresses the peak power by relocating some complex symbols from their original position to other position[6, 7]. Assume that the used digital modulation scheme is  $M$ -QAM with an  $\sqrt{M} \times \sqrt{M}$  constellation grid, and the minimum distance between constellation points is  $d$ . The original position of a complex symbol, which will be referred as original symbol, is located at a grid point of the  $\sqrt{M} \times \sqrt{M}$  grid, and the destination of relocation is located outside of the grid. In Fig.2, the original symbols for 16-QAM are shown as 16 black circles. Let the original position of a complex symbol representing a bit sequence  $\mathbf{b}_n = (b_{L-1}, b_{L-2}, \dots, b_0)$  be  $X_n$ . In [6], the set of complex symbols  $\hat{X}_n$  representing  $\mathbf{b}_n$  is given by the following equation:

$$\hat{X}_n = X_n + p_n D + j q_n D, \quad (4)$$

where  $D = \rho d M$ ,  $\rho \geq 1$  and  $p_n$  and  $q_n$  are any integer numbers. For  $p_n = q_n = 0$ ,  $\hat{X}_n$  is the original symbol

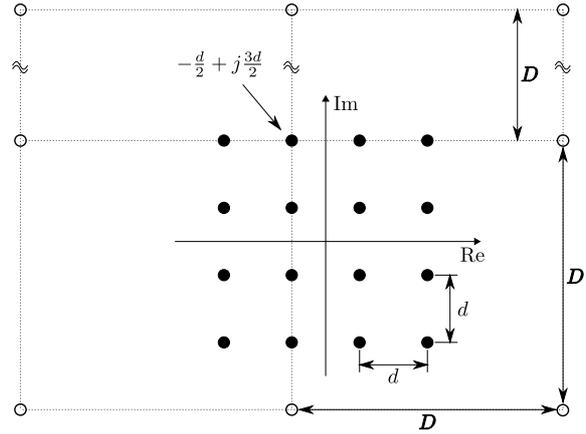


Figure 2: Symbol relocation of tone injection for 16-QAM and  $\rho = 1$ .

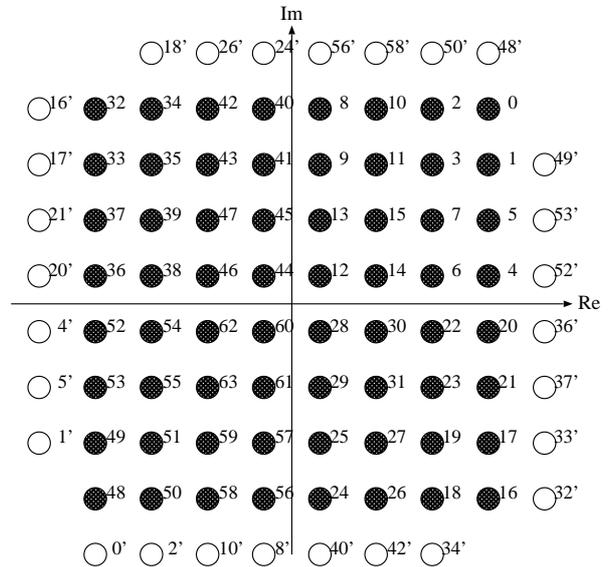


Figure 3: The used constellation map of 64-QAM.

$X_n$ . For  $p_n \neq 0$  and/or  $q_n \neq 0$ ,  $\hat{X}_n$  is a relocated symbol, which is located outside of the  $\sqrt{M} \times \sqrt{M}$  grid. In Fig.2, the relocated symbols from  $X_n = -\frac{d}{2} + j \frac{3d}{2}$  for  $\rho = 1$ ,  $|p_n| \leq 1$  and  $|q_n| \leq 1$  are shown as 9 white circles.

The PAPR reduction task based on tone injection is to determine  $p_n$  and  $q_n$ , for all  $n$ , so as to minimize PAPR. That is, the task is a combinatorial optimization problem. However, the number of solutions are enormous even if the range of  $p_n$  and  $q_n$  are limited. Therefore, A greedy algorithm proposed in [6] is as follows:

(Step 1) Find the complex symbol  $X_n$  or  $\hat{X}_n$  that most contributes to the peak power.

(Step 2) Find the most effective  $p_n$  and  $q_n$  for PAPR reduction by evaluating all the combinations. Move the symbol  $X_n$  or  $\hat{X}_n$  according to the found  $p_n$  and  $q_n$ .

(Step 3) Repeat steps 1 and 2 until obtaining a sufficient PAPR reduction.

The following drawbacks of this method have been noted[6, 7]: (1) any average power increases results in a reduction in SNR margin, (2) unnecessary power increases can lead to higher secondary peaks, and (3) the greedy searching algorithm tends to get a stack shallow local minimum, which will result in a poor PAPR reduction performance.

In [7], two constraints, which are effective to relax the above drawbacks (1) and (2), have been introduced in peak power reduction based on tone injection. The constraints are as follows: (a) movable symbols are limited to the symbols located along the outer circumference of the original constellation, and (b) for a movable symbol the destination of relocation is limited to one place, which is an almost symmetrical position of the original position with respect to the origin. The constraints (a) and (b) will reduce the peak powers of subcarriers compared with the without the constraints, because the magnitude of any complex symbol with the constraints is not larger than the one without the constraints. Further, the constraint (a) is validated by the observation that the outer circumferential symbols will contribute more to the peak power than the inner symbols.

In this paper, we also introduce the above constraints and use the constellation map of 64-QAM as shown in Fig.3. In Fig.3, the original symbols are shown as 64 black circles and the relocated symbols are shown as 28 white circles. In the figure, the black circle with a number  $m \in \{0, 1, \dots, 63\}$  is the original symbol  $X^{(m)}$ , and the white circle with a number  $m'$  is the replacement of the original symbol  $X^{(m)}$ .

Although the above constraints reduce the solution spaces, the number of solutions is still enormousness. Assuming that  $N$  symbols to be transmitted randomly occur, the average number of solutions is  $2^{\frac{7}{16}N}$ , which is approximately  $5.2 \times 10^{33}$  and  $2.7 \times 10^{67}$  for  $N = 256$  and  $512$ , respectively. As a solution against the drawback (3), we will present a searching algorithm based on GA in the next section.

### 3 Proposed Method

We apply genetic algorithm (GA) to the peak power reduction method based on tone injection. The GA, which is a search heuristic based on the process of natural evolution, can find a good solution for optimization problems by evolving the population of solutions with genetic operators such as selection, mutation and crossover[8]. GAs have been employed for solving many combinatorial optimization problems in various fields[9, 10, 11, 12], and it has been shown that GAs can find a near-optimal solution in a much shorter time compared to the conventional

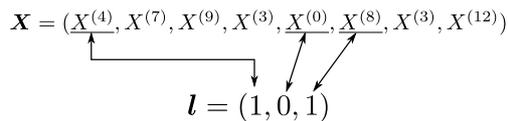


Figure 4: An example of chromosome for  $N = 8$  and  $S = 3$ .

methods such as random search and exhaustive one, especially in large solution spaces. The proposed method uses GA to find an effective combination of moved symbols for PAPR reduction.

#### 3.1 Genetic Representation

In GA, a solution is coded as a string, called chromosome. Let  $\mathcal{A} = \{X^{(0)}, X^{(1)}, \dots, X^{(M-1)}\}$  be the set of all the original complex symbols. Let  $\mathcal{M}$  be the set of the movable original complex symbols. That is, for Fig.3,  $\mathcal{M} = \{X^{(0)}, X^{(1)}, X^{(2)}, X^{(4)}, X^{(5)}, X^{(8)}, X^{(10)}, X^{(16)}, X^{(17)}, X^{(18)}, X^{(20)}, X^{(21)}, X^{(24)}, X^{(26)}, X^{(32)}, X^{(33)}, X^{(34)}, X^{(36)}, X^{(37)}, X^{(40)}, X^{(42)}, X^{(48)}, X^{(49)}, X^{(50)}, X^{(52)}, X^{(53)}, X^{(56)}, X^{(58)}\}$ .

Let  $\mathbf{X} = (X_0, X_1, \dots, X_{N-1})$  be the vector of original complex symbols to be transmitted. Let  $\mathcal{S} = \{X_n \in \mathcal{M} | 0 \leq n \leq N-1\}$  and  $S = |\mathcal{S}|$  be the set of movable symbols in  $\mathbf{X}$  and the size of  $\mathcal{S}$ , respectively. Let  $n(s)$  be the function of integer  $s \in \{1, 2, \dots, S\}$  such that  $|\{X_i \in \mathcal{M} | 0 \leq i \leq n(s)\}| = s$  and  $X_{n(s)} \in \mathcal{M}$ . That is, the function  $n(s)$  returns the  $s$ -th smallest index number  $n$  of  $X_n$  among  $\{X_n \in \mathcal{M} | 0 \leq n \leq N-1\}$ .

A solution (chromosome) for  $\mathbf{X}$  is coded as an  $S$ -dimensional binary vector

$$\mathbf{l} = (l_1, l_2, \dots, l_S), \quad (5)$$

where  $l_s \in \{0, 1\}$ , and  $l_s = 1$  and  $l_s = 0$  mean that the symbol  $X_{n(s)}$  is moved and is not moved, respectively. Fig.4 shows an example of the chromosome coding. Note that the length of a chromosome depends on  $\mathbf{X}$ . When the symbols to be transmitted randomly occur, the average length is  $\frac{7}{16}N$ .

#### 3.2 Fitness Function

When PAPR by Eq.(3) is used as evaluation function, a smaller PAPR value means a better solution. In order that a better solution has a larger evaluation value, we use the following evaluation function as GA's fitness function:

$$f = \left( \frac{1}{\text{PAPR}} \right)^\alpha, \quad (6)$$

where PAPR is calculated by Eq.(3) and  $\alpha > 0$  is a constant value that controls the degree of convergence. A larger  $\alpha$  makes the convergence faster. An effective value for  $\alpha$  is given by simulation in the next section.

### 3.3 Mutation and Crossover Operators

The used mutation operator operates on each locus of chromosomes. Given a chromosome  $\mathbf{l} = (l_1, l_2, \dots, l_S)$ , each locus  $l \in \{l_1, l_2, \dots, l_S\}$  is updated as follows:

$$l \leftarrow \begin{cases} (l + 1) \bmod 2 & \text{with probability } P_m \\ l & \text{with probability } 1 - P_m. \end{cases} \quad (7)$$

Another approach is to perform the mutation for each chromosome. However, according to our preliminary simulation, this approach is not good.

We consider four types of crossover operator: one-point, two-point, three-point and uniform. For every types, the crossover points are randomly selected.

### 3.4 The Algorithm

The evaluation of a solution involves IFFT operation, which requires  $O(N \log N)$  steps. In order to reduce the computation time, the proposed algorithm utilizes a tree-structured fitness table. Once a solution is evaluated, its fitness value is registered in the table. When thereafter the same solution appears, its fitness value is obtained by referring to the table instead of by calculation.

If the search time is much smaller than the one of IFFT, a significant reduction of the computation time will be achieved. The fitness table also should be efficient in terms of memory. Therefore, we use a binary tree as the fitness table. The binary tree is as follows: (1) the height is  $S$ , (2) an edge between levels  $s - 1$  and  $s$  ( $1 \leq s \leq S$ ) is associated with  $l_s$ , (3) for each node at level  $s - 1$ , the left and right edges are associated with  $l_s = 0$  and  $l_s = 1$ , respectively, (4) the existence of the path corresponding to a solution  $\mathbf{l} = (l_1, l_2, \dots, l_S)$  means that the fitness value of  $\mathbf{l}$  is already registered, and (5) the fitness value of a solution  $\mathbf{l}$  is memorized in the leaf node of the path corresponding to  $\mathbf{l}$ . An example of the tree-structured table is shown in Fig.5. Since the search and the registration of the fitness value of a solution  $\mathbf{l}$  trace from the root node to the corresponding leaf node, they completes in at most  $S$  steps. As mentioned before, the average of  $S$  is  $\frac{7}{16}N$ . Therefore, the search and the registration of a fitness value completes in  $O(N)$  steps.

The algorithm of the proposed GA is shown below.

#### Algorithm

**Step 1:** Set the current generation  $g \leftarrow 1$ . Set the best fitness  $f_{\text{best}} \leftarrow 0$ .

**Step 2:** Randomly generate  $K$  individuals  $\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_K$  in the form of Eq.(5).

**Step 3:** For each  $k \in \{1, 2, \dots, K\}$ , search for the fitness  $f_k$  in the tree-structured fitness table. If  $f_k$  is not found, calculate  $f_k$  according to Eq.(6), and register  $f_k$  into the fitness table.

Let  $f_{\text{max}} = \max_{k \in \{1, 2, \dots, K\}} f_k$ . Let  $k_{\text{max}} \in \{1, 2, \dots, K\}$  such that  $f_{k_{\text{max}}} = f_{\text{max}}$ . If  $f_{\text{max}} > f_{\text{best}}$ , then  $f_{\text{best}} \leftarrow f_{\text{max}}$  and  $\mathbf{l}_{\text{best}} \leftarrow \mathbf{l}_{k_{\text{max}}}$ .

**Step 4:** If  $g > G$ , then go to Step 10.

**Step 5:** For each  $k \in \{1, 2, \dots, K\}$ , calculate the selection probability

$$P_k = \frac{f_k}{\sum_{i \in \{1, 2, \dots, K\}} f_i}.$$

Draw  $K$  samples  $\mathbf{l}'_1, \mathbf{l}'_2, \dots, \mathbf{l}'_K$  with replacement from  $\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_K$  with probabilities  $P_1, P_2, \dots, P_K$ .

**Step 6:** For each pair  $\mathbf{l}'_{2k+1}$  and  $\mathbf{l}'_{2k+2}$  ( $0 \leq k \leq \frac{K}{2} - 1$ ), perform the crossover operation with probability  $P_c$ .

**Step 7:** For each locus  $l_s$  in  $\mathbf{l}'_1, \mathbf{l}'_2 \dots \mathbf{l}'_K$ , perform the mutation operation with probability  $P_m$ .

**Step 8:** Copy  $\mathbf{l}'_1, \mathbf{l}'_2, \dots, \mathbf{l}'_K$  to  $\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_K$ , respectively.

**Step 9:**  $g \leftarrow g + 1$  and go to Step 3.

**Step 10:** Return  $\mathbf{l}_{\text{best}}$  as the final solution. □

In GA, as the generation progresses, the large portion of the population converges to particular solutions. This property will make it possible for Step 3 to reduce the computation time.

In Step 6, one of the four types of crossover operator is used. The best type will be shown in the next section.

## 4 Numerical Simulations

The simulations assume an AWGN (Additive White Gaussian Noise) channel, where a white noise with SNR 20dB is added to the signal from the amplifier. The used

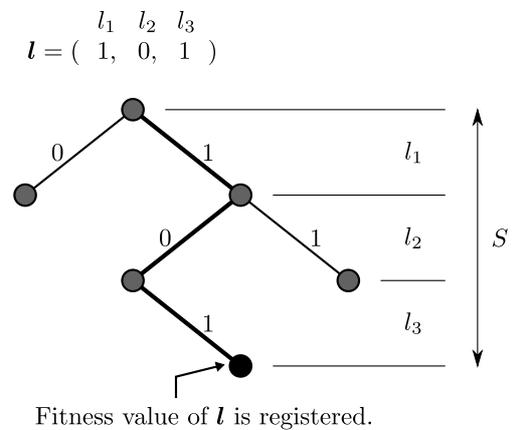


Figure 5: An example of the tree-structured fitness table for  $S = 3$ : the bold lines show the path of the solution  $\mathbf{l}$ .

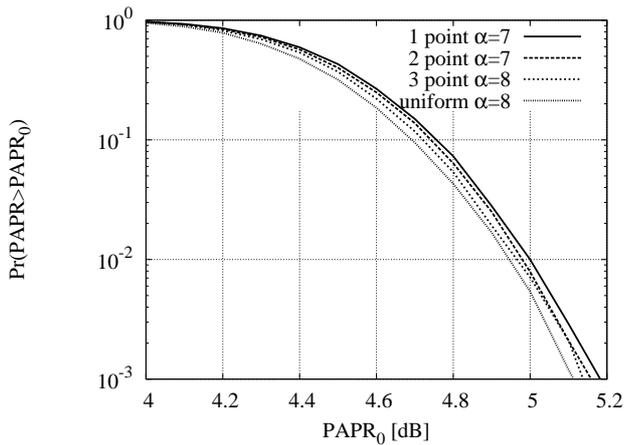


Figure 6: PAPR for different crossover methods.

nonlinear power amplifier model has the following input-output characteristic.

$$F[\rho] = \frac{\rho}{(1 + \rho^6)^{1/6}}, \quad (8)$$

where  $F[\rho]$  is the gain of the amplifier and  $\rho$  is the ratio of the mean input amplitude to the saturation amplitude.

The number of subcarriers is  $N = 128$ , the population size is  $K = 30$ , the maximum generation number is  $G = 50$ , and the probabilities for crossover and mutation are  $P_c = 0.9$  and  $P_m = 0.01$ , respectively. Each result is calculated from 100000 trials.

#### 4.1 PAPR Reduction Performance

At first, for each crossover operator type, the effective value of parameter  $\alpha$  in the fitness function is investigated in terms of PAPR. From the results, for 1-point, 2-point, 3-point and uniform crossovers, the best values for  $\alpha$  are  $\alpha = 7$ ,  $\alpha = 7$ ,  $\alpha = 8$  and  $\alpha = 8$ , respectively.

The results with the best  $\alpha$  are summarized in Fig.6.  $PAPR_0$  at the horizontal axis is the upper limit of the linear amplification range, and  $\Pr(PAPR > PAPR_0)$  at the vertical axis is the probability that PAPR exceeds the limit  $PAPR_0$ . This figure shows a tendency that the PAPR reduction performance improves with the number of crossover points and the uniform crossover operator provides the best performance. Therefore, in the following, the proposed method assumes to use uniform crossover and  $\alpha = 8$ .

Next, the proposed method is compared with a conventional method based on neural network (NN)[7] and a random search. The result is shown in Fig.7. In the figure, “NN”, “Random” and “Proposed” are the method based on NN (NN method), a random search and the proposed method, respectively. All the methods use the same constellation map shown in Fig.3. The random

search randomly generates 1500 solutions and returns the best solution as the final solution. According to the result, the proposed method achieves the best performance among them.

We also measure the computation time for the random search and two types of GA methods. The first type of GA method is the proposed method. The other type of GA method is the conventional one that does not use the tree-structured fitness table and that calculates the fitness for all the individuals. The computation time is calculated in terms of CPU time. The random search, the conventional GA method and the proposed GA method take 35.16ms, 36.18ms and 22.78ms, respectively. Our proposed method achieves about 1/3 reduction in the computation time. The total number of unique individuals occurring through all the generations of GA is approximately 1000. Since the total number of individuals through all the generations is 1500, the amount of reduction in the computation time is proportional to the amount of the duplicated individuals.

#### 4.2 Bit Error Rate Performance

The proposed GA method is compared with the conventional NN method and the random search in terms of bit error rate (BER).

$$BER = \frac{(\text{Total number of error bits})}{(\text{Total number of transmitted bits})} \quad (9)$$

The result is shown in Fig.8, where IBO (Input Back-Off) defines the degree of nonlinearity of an amplifier and is given by the following equation.

$$IBO = \frac{(\text{Saturation amplitude})}{(\text{Mean input power})} \quad (10)$$

In Fig.8, “Original” is the case of no PAPR reduction. The result shows that the proposed method achieves the best BER performance.

### 5 Conclusions

In this paper, we applied GA to the peak power reduction method based on tone injection. The proposed GA method utilized a tree-structured fitness table to reduce the computation time. We tested four types of crossover operators: one-point, two-point, three-point and uniform crossovers. Our results showed that the PAPR reduction performance improves with the number of crossover points and the uniform crossover operator provides the best performance. Our proposed method was compared with the conventional NN method and a random search in terms of PAPR and BER. The results showed that our GA method is superior compared to the conventional NN method and the random search. Further, our GA method is superior in terms of PAPR and BER compared to the conventional GA method and the random search.

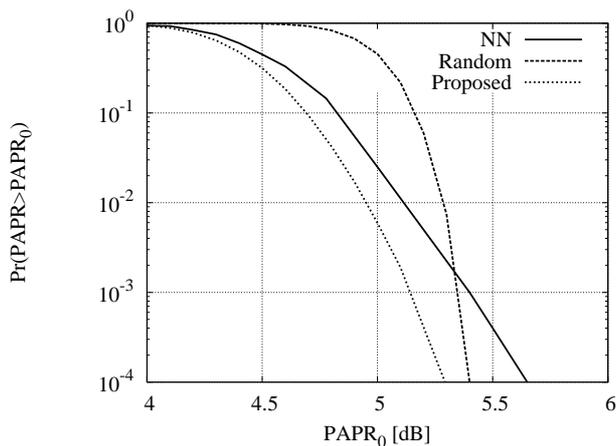


Figure 7: Performance comparison of PAPR reduction with conventional methods.

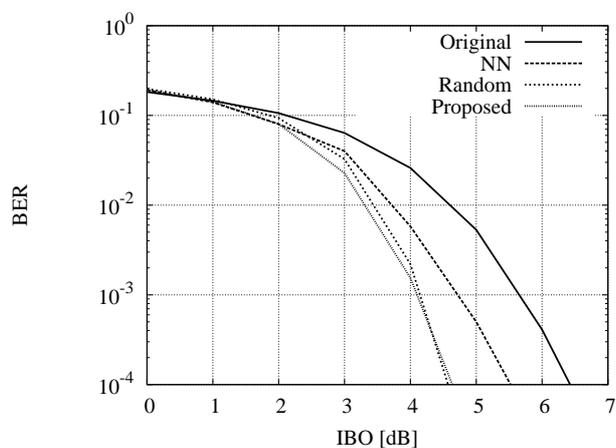


Figure 8: BER performance.

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