Proceedings of the World Congress on Engineering and Computer Science 2010 Vol I
WCECS 2010, October 20-22, 2010, San Francisco, USA

Design of a Robust Adaptive Control (RAC) Of Robotic Manipulators for Trajectory Tracking With Structured and Unstructured Uncertainties

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Abstract−**This paper deals with design of a robust adaptive control (RAC) of robotic manipulators for trajectory tracking with structured and unstructured uncertainties. This controller has been used for evaluation of tracking error and time of settlement of output and desired output has been compared. Simulation results on a two-link SCARA manipulator are shown for trajectory tracking in the presence of the impulse disturbance.**

***Index Terms*— Robust Adaptive Control (RAC), Robot Manipulator, Uncertainty.**

I. INTRODUCTION

Robust Adaptive control [1]-[4] are widely accepted as a powerful methods of tackling uncertain non-linear systems. Model-based adaptive controller has received more attention in the last years because it makes it possible to cope with the above variations using linear like techniques do to the linear parameterization property of the mode. In general, the adaptive control laws designed use a non-linear term having the same structure as the regressor of the model in which appear the actual state variables, their desired values or both. (Slotine & Li, 1986; Ortega& Spong, 1989; Whitcomb, Rizzi & Koditschek , 1993; Alonge, D'Ippolito & Raimondi, 1999). The adaptive control laws are based on a plant model that is free of noise, disturbances and unmodeled dynamics.

These schemes are to be implemented on actual plants that most likely deviate from the plant models on which their design is based. In the adaptive approach, one designs a controller that attempts to "learn" the uncertain parameters of the system and, if properly designed, will eventually be a "best" controller for the system in question. In the robust approach, the controller has a fixed structure that yields "acceptable" performance for a class of plants which include the plant in question. In general, the adaptive approach is applicable to a wider range of uncertainties, but robust controller is simpler to implement and no time is required to "tune" the controller to the particular plant. The typical structure of a robust control is composed of a nominal part, similar to a feedback linearization or inverse control law, and of additional terms aimed at dealing with model uncertainty [5].

In this paper, section 2 gives a brief review on robust adaptive control with uncertain parameter and unmodeled dynamics and bounded measurement noise. Then in section 3, this way is applied on a *two-link SCARA manipulator* as a plant to show its ability and merits.

II. MATHEMATICAL MODEL OF ROBUST ADAPTIVE CONTROLLER WITH UNMODELED DYNAMICS AND PARAMETER UNCERTAINTY

One of the attractive features of the adaptive controllers is that the control implementation does not require a priori knowledge of unknown constant parameters such as payload masses or friction coefficients. Two disadvantages of the adaptive controllers are that large amounts of on-line calculation are required, and the lack of robustness to additive bounded disturbances.

Two of the attractive features of the robust controllers are that on-line computation is kept to a minimum and their inherent robustness to additive bounded disturbances. One of the disadvantages of the robust control approach is that these controllers require *a priori* known bounds on the uncertainty. In general, calculations of the bounds on the uncertainty can be quite a tedious process since this calculation involves finding the maximum values for the mass and friction related constants for each link of the robot manipulator. Another disadvantage of the robust control approach is that even in the absence of additive bounded disturbances, we cannot guarantee asymptotic stability of the tracking error. In general, it would be desirable to obtain

Manuscript received July 7, 2010.

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ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

at least a "theoretical" asymptotic stability result for the tracking error.

The adaptive robust controller can be thought of as combining the best qualities of the adaptive controller and the robust controller. This control approach has the advantages of reduced online calculations (compared to the adaptive control method), robustness to additive bounded disturbances, no *a priori* knowledge of system uncertainty, and asymptotic tracking error performance.

For purposes of control design, we assume that the robotic manipulator is a revolute manipulator with dynamics given by

$$\tau = M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d \quad (1)$$

Where F_d is a $n \times n$ positive definite, diagonal matrix that is used to represent the dynamic coefficients of friction, $F_s(\dot{q})$ is a $n \times 1$ vector containing the static friction terms, T_d is a n×1 vector representing an unknown bounded disturbance.

The adaptive robust controller is very similar to the robust control strategies, in that an auxiliary controller is used to "bound" the uncertainty. The robust controllers bounded the uncertainty by using a scalar function that was composed of tracking error norms and positive bounding constants. The uncertainty for a given robot controller can be shown in this form:

$$\omega = M(q)(\ddot{q}_d + \dot{e}) + V_m(q,\dot{q})(\dot{q}_d + e) + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d \quad (2)$$

That is, the dynamics given by (2) are uncertain in that payload masses, coefficients of friction, and disturbances are not known exactly. It is assumed; however, that a positive scalar function ρ can be used to bound the uncertainty as follows:

$$\rho \geq \|\omega\| \quad (3)$$

As delineated in [Dawson et al. 1990], the physical properties of the robot manipulator can be used to show that the dynamics given by (2) can be bounded as

$$\rho = \delta_0 + \delta_1\|e\| + \delta_2\|e\|^2 \geq \|\omega\| \quad (4)$$

Where

$$e = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \quad (5)$$

And δ_0, δ_1, and δ_2 are positive bounding constants that are based on the largest possible payload mass, link mass, friction coefficients, disturbances, and so on.

Similar to the general development presented in [Corless and Leitmann 1983], the adaptive robust controller has the form

$$\tau = K_v r + v_R \quad (6)$$

Where K_v is a n×n diagonal, positive-definite matrix, r is the filtered tracking error and v_R is a n×1 vector representing an auxiliary controller. The auxiliary controller v_R in (6) is defined by

$$v_R = \frac{r\hat{\rho}^2}{\hat{\rho}\|r\| + \varepsilon} \quad (7)$$

Where

$$\dot{\varepsilon} = -k_\varepsilon \varepsilon \quad ; \varepsilon(0) > 0 \quad (8)$$

k_ε Is a positive scalar control constant, $\hat{\rho}$ is a scalar function defined as

$$\hat{\rho} = \hat{\delta}_0 + \hat{\delta}_1\|e\| + \hat{\delta}_2\|e\|^2 \quad (9)$$

And $\hat{\delta}_0, \hat{\delta}_1, \hat{\delta}_2$ are the dynamic estimates of the corresponding bounding constants δ_0, δ_1, and δ_2 defined in (4). The bounding estimates denoted by "^" are changed on-line based on an adaptive update rule. Before giving the update rule, we write (9) in the more convenient form

$$\hat{\rho} = h\hat{x} \quad (10)$$

Where

$$h = [1 \ \ \|e\| \ \ \|e\|^2] \ \ and \ \ \hat{x} = \left[\hat{\delta}_0 \ \ \hat{\delta}_1 \ \ \hat{\delta}_2\right]^T \quad (11)$$

The actual bounding function ρ given in (3) can also be written in the matrix form

$$\rho = hx \quad (12)$$

The bounding estimates defined in (10) are updated on-line by the relation

$$\dot{\hat{x}} = \gamma h^T\|r\| \quad (13)$$

Where r is defined in (6), h is defined in (10), and γ is a positive scalar control constant.

We now analyze the stability of the error system with the Lyapunov-like function

$$V = \frac{1}{2}r^T M(q) r + \frac{1}{2}\tilde{x}^T\gamma^{-1}\tilde{x} + k_\varepsilon^{-1}\varepsilon \quad (14)$$

With differentiating and simplifying we obtain

$$\dot{V} \leq -r^T K_v r \quad (15)$$

The right side of (15) is negative, so with respect to the Lyapunov theory the stability of the system has been ensured.

III. SIMULATION

In order to verify the peculiarities of the previously discussed control laws, an application example is proposed. A robot manipulator is shown in Fig.1. The arm dynamics are given by two coupled nonlinear differential equations:

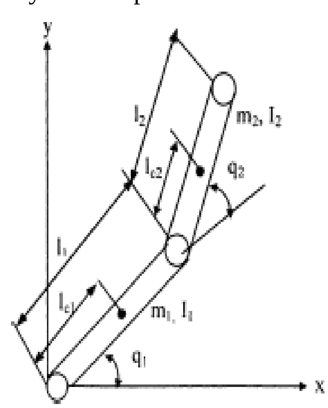

Fig.1. Robot manipulator

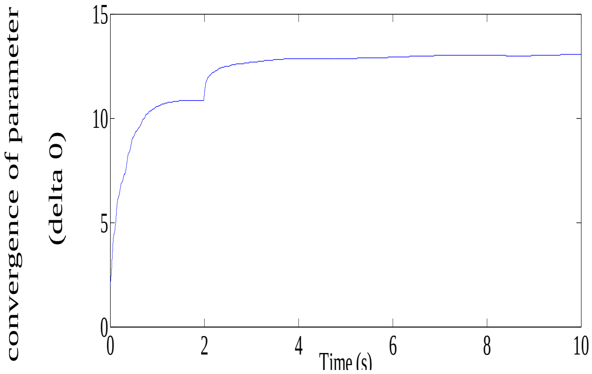

Fig.2. It shows the first parameter (δ_0) that is converging to the constant value.

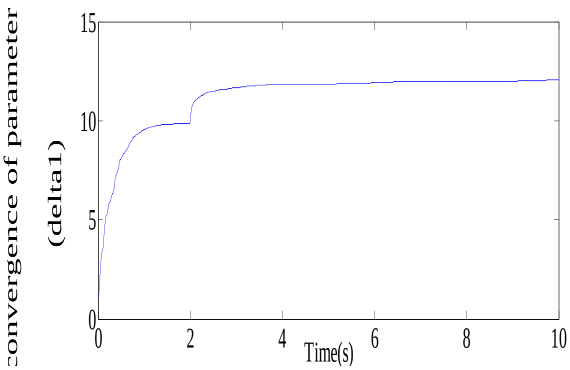

Fig.3. It shows the first parameter (δ_1) that is converging to the constant value.

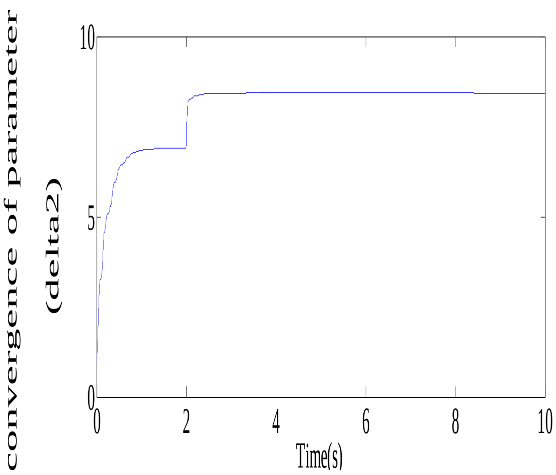

Fig.4. It shows the first parameter (δ_2) that is converging to the constant value.

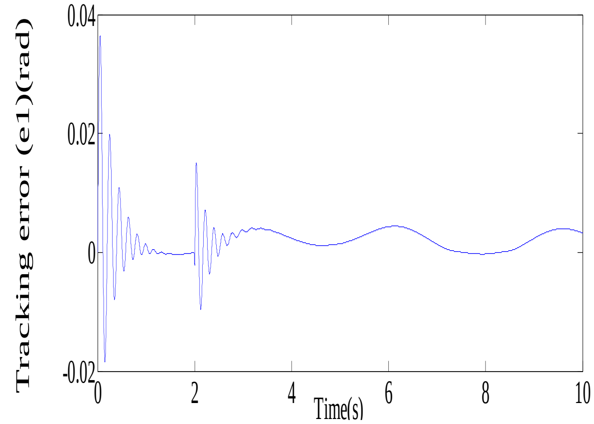

Fig.5. It shows the tracking error of joint 1.

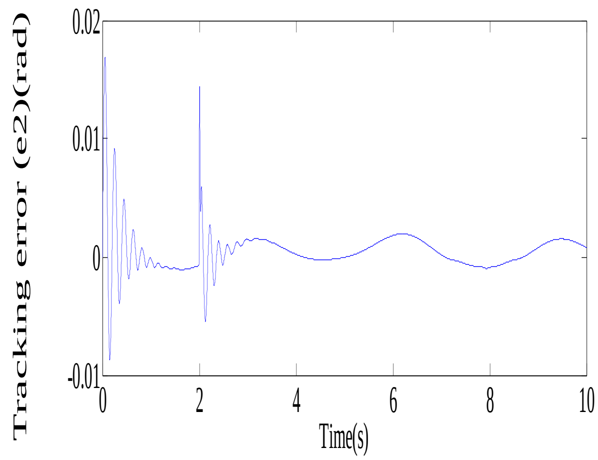

Fig.6. It shows the tracking error of joint 2.

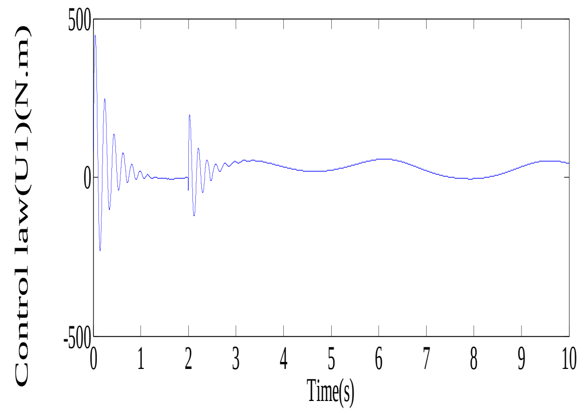

Fig.7. It shows the control law of joint 1.

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

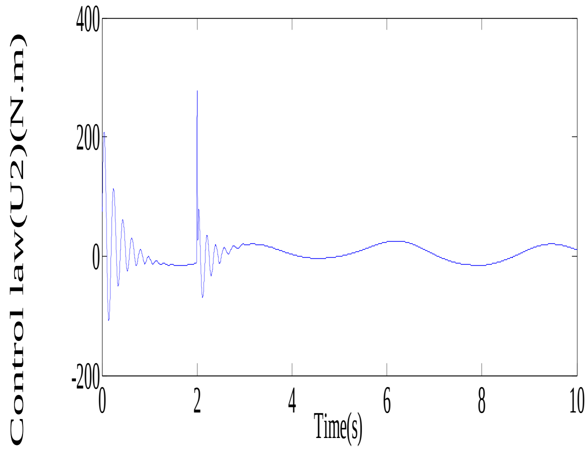

Fig.8. It shows the control law of joint 2.

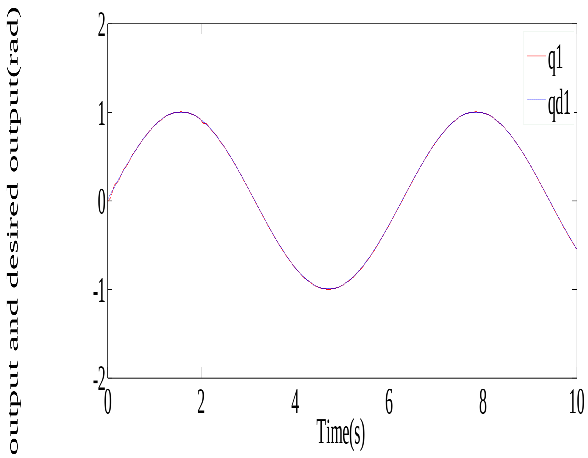

Fig.9. It shows output and desired output of joint 1 in a workspace (the first data (red color) is denoted as output and next (blue color) is denoted as desired output).

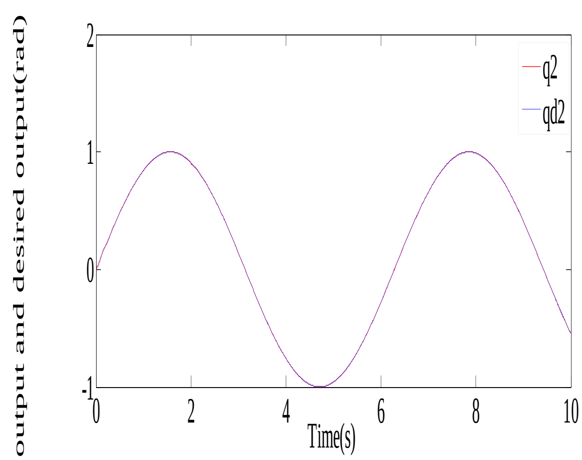

Fig.10. It shows output and desired output of joint 2 in a workspace (the first data (red color) is denoted as output and next (blue color) is denoted as desired output).

$$\begin{aligned}\tau_1 = {} & [(m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 cos\theta_2]\ddot{\theta}_1 \\ & + [m_2 l_2^2 + m_2 l_1 l_2 cos\theta_2]\ddot{\theta}_2 \\ & - m_2 l_1 l_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right)sin\theta_2 \\ & + (m_1 + m_2)g l_1 cos\theta_1 \\ & + m_2 g l_2 \cos(\theta_1 + \theta_2) \qquad (16)\end{aligned}$$

$$\begin{aligned}\tau_2 = {} & \left[m_2 l_2^2 + m_2 l_1 l_2 cos\theta_2\right]\ddot{\theta}_1 + m_2 l_2^2\ddot{\theta}_2 \\ & + m_2 l_1 l_2 \dot{\theta}_1^2 sin\theta_2 \\ & + m_2 g l_2 \cos(\theta_1 + \theta_2) \qquad (17)\end{aligned}$$

To model friction and disturbances, the dynamics

$2\dot{q}_1 + 0.5sgn(\dot{q}_1) + 0.2\sin(3t) + T_d$

$2\dot{q}_2 + 0.5sgn(\dot{q}_2) + 0.2\sin(3t) + T_d$

That T_d is bounded disturbance. Figs.2, 3 and 4 Show that all of the parameters have a nice convergence to desired values, that is requested. In figs.5, 6 tracking error of the system has converged after 3 seconds with using of this controller for both of joints approximately. The bounded disturbance is imposed in ***t=2***. As you see, in this time, all of the diagrams have a sharp pick in their curves, but this value is controlled with robust adaptive controller. After this time, all of the figures are converging asymptotically, nearly.

In equation (6) control law has been extracted. Performance of this control law is suitable. (Figs. 7 and 8)

Figs.9 and 10 show that the joint variables of robot (joint angles) are following the desired output in a workspace. As you see, there is a good convergence after 3 seconds in the system.

Likewise, in ***t=2*** the bounded disturbance is imposed and robust adaptive controller could control the system in presence of uncertainties.

It should be noted that from the theoretical development, we are only guaranteed that the position tracking error is asymptotically stable while all other signals remain bounded.

IV. CONCLUSION

In this paper, an approach is considered for designing a control law that includes parameter uncertainty and bounded disturbance. This approach is based on a detailed, though approximate, modeling of the dynamic of robot

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

manipulator starting from which the design of a robust adaptive control law is affected in presence of bounded disturbance and parameter uncertainty. Results showed that although there are four parts of friction and disturbances on the system, but using of this controller is suitable and tracking error converging globally asymptotically to zero. Likewise parameters converge to their real value.

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ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)