Proposed Concept of Signals for Unit Step Functions

Satyapal Singh

***Abstract*— There are several elementary signals which play vital role in the study of signals. These elementary signals serve as basic building blocks for the construction of more complex signals. In fact, these elementary signals may be used to model a large number of signals which occur in nature. One of these elementary signals on which the article is based is Unit step function. This paper explains a new approach to explain UNIT STEP FUNCTION hence it is named as PROPOSED CONCEPT OF SIGNALS, which, if recognized may be known as '*SP's ANGLES BASED UNIT STEP FUNCTION*'.**

***Index Terms* — SP's – Satyapal's, Angle – The angles at which shape of the elementary signal changes, Clockwise – The direction of watch, Anticlockwise – the opposite direction of watch.**

I. Introduction

When we are asked to construct a shape from a given equation, then normally we are provided with an equation that usually contains basic or elementary signals. Most of the students and engineers may be unaware to what to do for a given equation even after learning the existing theory. For this I have tried to develop "Concept of Angles" theory that may be helpful in constructing the shapes from the given equation and in understanding the basic signals. Let us take the unit step signal function to explore the concept of angles.

II. SP's Concept of Angles

Signals can be represented by using angles also. This representation gives more clarity to understand the signals. Generally, signals are represented in equation form [1], [2], [3]. For example –

a. $u(t) = \begin{cases} 1, \text{ when } t >= 0 \\ 0, \text{ otherwise (that is for } t < 0) \end{cases}$

b. $r(t) = \begin{cases} t, \text{ if } t >= 0 \\ 0, \text{ otherwise (that is for } t < 0) \end{cases}$

When these elementary signals are put in equation form, then this form of equation representation may be difficult to understand by a student and it may become more tedious task when the student is asked to draw the shape. For better understanding, the concept of angles is tried to develop[4]. The concept of angles says that these signals can be represented by using angles too. In this method, the signal is broken into different angles as per the given signal. This concept does not change the original shape of the signal but it simplifies the process. With the help of concept of angles, complex signal equations can be broken into simple steps and can be plotted on the paper. This concept explores step by step procedure to how to draw the elementary signals.

III. Unit Step Signal

Unit step signal u(t) states that the signal will start from time zero and instantly will take unit height (amplitude) and depending upon given time characteristics (i.e. either positive or negative, here positive) the signal will follow the straight path either towards right or left, here towards right. Thus, the unit step function is a type of elementary function u(t) which exists only for positive side and is zero for negative. Also, the unit step function is discontinuous at t = 0. The continuous time unit step function is denoted by u(t) and may be represented in equation form as shown below. This equation is pictorially depicted as in figure 1[1], [2], [3]. In other words, the unit step function is a type of elementary function which exists only positive side and is zero for negative side. Also the unit step function is discontinuous at t = 0^3.

The continuous-time unit-step function is denoted by u(t) and it is mathematically expressed as –

$$u(t) = \begin{cases} 1, \text{ when } t >= 0 \\ 0, \text{ otherwise (that is for } t < 0) \end{cases}$$

To understand this, let us understand the example of u(t). It can be depicted as –

Satyapal Singh, Al-Falah School of Engineering and Technology, Dhauj, Faridabad, Haryana, India where the author is pursuing M.Tech. (Electronics & Communication) IV[th] semester (final) and Priyadarshini College of Computer Sciences, Greater Noida, UP, India where the author has gained teaching experience, e-mail: satyapalsingh67@yahoo.co.in; Registration No.: 1278146363; Paper No.: ICCSA_43; Contact No.: 09968554717, 09868347416; Qualification: M.Tech. (Computer Science & Engineering), Master of Computer Management, Master of Management Science (Marketing), B.Tech. (Electronics & Telecommunication), B.Sc. (PCM), Diploma in Electronics & Communication, Diploma in Materials Management, Diploma in Business Management; Address: H.No. 460, Near Deepak Public School, Sec 9, Vijay Nagar, Shivpuri, District – Ghaziabad, State – UP, Country – India Pin(Zip) : 201009

let x(t) = u(-t)

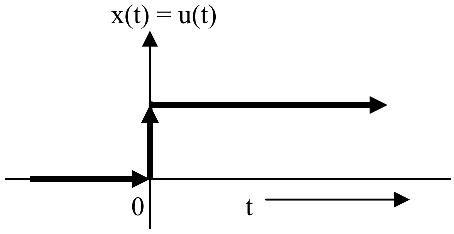

Figure 1. Unit step function u(t) based on existing theory.

Now if someone asks to depict the signal of u(t) – u(t-2) + u(t-3) – 2u(t-5) then it creates ambiguities that is when complex equations are given to draw then it becomes complex to draw.

Now if someone asks to depict the signal u(t) – 2u(t-2) + 4u(t-3) – 2u(t-5) then it creates ambiguities that is when complex equations are given to draw then it becomes complex to draw.

Here, I will try to present the logic regarding each and every elementary signal. My theory[4] says if we are provided with a set of unit step signals in the form of equations and are asked to depict on paper then it will be very easy to depict the diagram if we use the *concept of angles*. Here, for unit step signals remember to have 90 degree angle shift concept. How? Solution – first we learn how to draw u(t) for which shape is given in figure 1 and then we will learn how to draw u(-t), -u(t) and lastly –u(-t).

IV. Angles in Unit Step Function[4]

Unit step signal uses 90^0 Concept. How is it used, we will see in the coming paragraphs.

Drawing of Different Unit Step Functions Using Angles[4].

A. Drawing of u(t)

Figure 5 shows that there are actually two 90 degrees shifts in unit step function, it is explained with the help of first by taking a unit step function u(t) –

For u(t) case, u(t) can be represented in terms of angles as shown below –

$$u(t) = \begin{cases} 0, \text{ when } t < 0 \\ 1, 90^0 \text{ anticlockwise at } t = 0 \\ 1, 90^0 \text{ clockwise for } t > 0 \end{cases}$$

1. First assume that, in idle case when no signal is there, then signal u(t) is assumed to come on x-axis from negative infinity to origin. This condition is shown in figure 2.

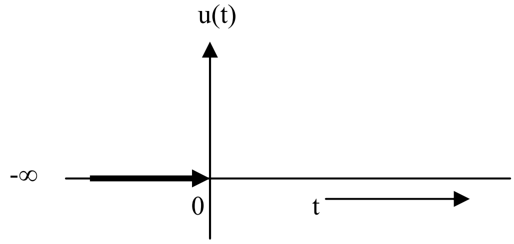

Figure 2. Sketching function u(t) using angle theory, step 1.

2. As soon as the signal u(t) comes/appears then the signal takes a 90 degree shift in anticlockwise direction and takes one straight line in u(t) direction i.e. in positive y-axis direction line. This condition is shown in figure 3.

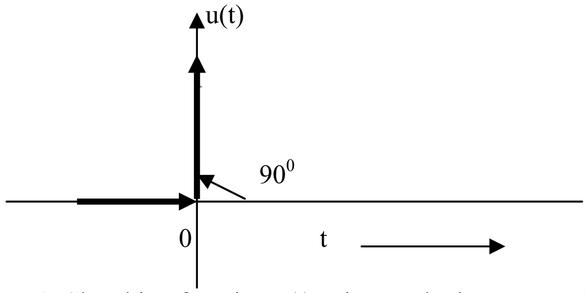

Figure 3. Sketching function u(t) using angle theory, step 2.

3. After attaining a unit height for this case again the shape takes a shift of 90 degree in clockwise direction and finally extends to infinity towards right. This is shown in figure 4.

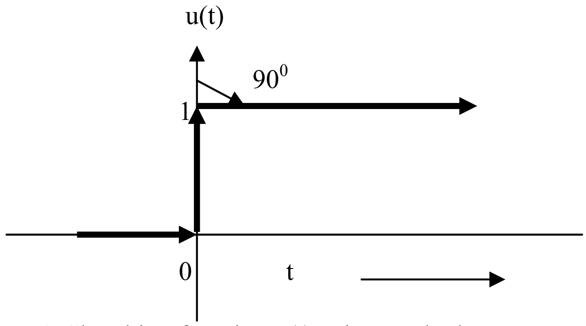

Figure 4. Sketching function u(t) using angle theory, step 3.

4. Thus u(t) signal takes two 90 degrees shifts – first shift in anticlockwise direction and second shift in clockwise direction with a unit height (amplitude) and finally extends towards right side (positive) infinity. Hence, plotting of u(t) by using angles can be shown as in figure 5.

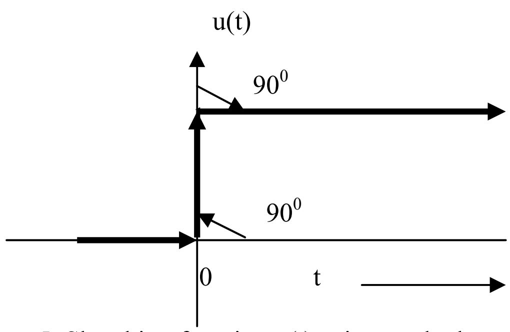

Figure 5. Sketching function u(t) using angle theory, step 4.

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

B. Drawing of u(-t)[4]

Mathematically u(-t) is represented[1], [2], [3] as –

$$u(-t) = \begin{cases} 1, \text{ when } t <= 0 \\ 0, \text{ otherwise (that is for } t > 0) \end{cases}$$

To understand this, let us understand the example of u(-t). It can be depicted as –

let x(t) = u(-t)

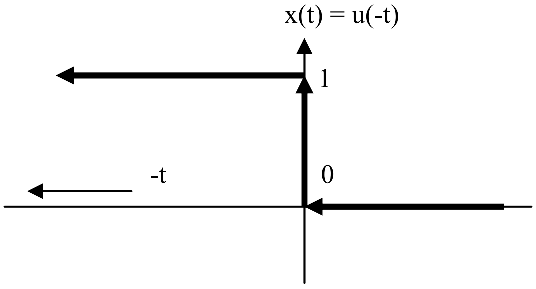

Figure 6. Function u(-t) based on existing theory.

Figure 6 shows that there are actually two 90 degrees shifts, it is explained with the help of first by taking a unit step function in negative direction i.e. u(-t) – For u(-t) case, u(-t) can be represented in terms of angles as shown below –

$$u(-t) = \begin{cases} 0, \text{ when } t > 0 \\ 1, 90^0 \text{ clockwise at } t = 0 \\ 1, 90^0 \text{ anticlockwise for } t < 0 \end{cases}$$

For u(-t) case –

7. First assume that, in idle case when no signal is there then the depiction line comes from positive infinity to origin. This situation is shown in figure 7.

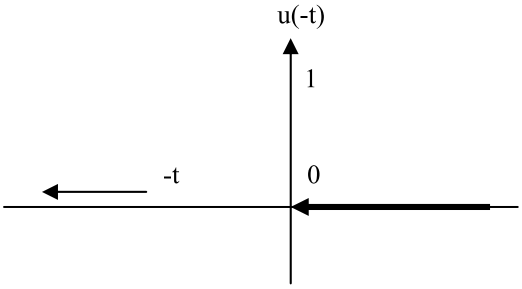

Figure 7. Sketching function u(-t) using angle theory, step 1.

2. As soon as the signal u(-t) comes/appears then the signal takes a 90 degree shift in clockwise direction and takes one straight line in signal u(-t) direction i.e. in positive y-axis direction line. This condition is shown in figure 8.

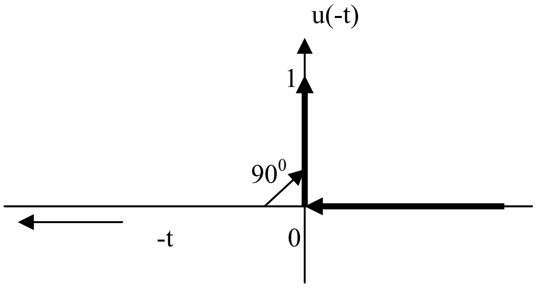

Figure 8. Sketching function u(-t) using angle theory, step 2.

3. After attaining a unit height again the shape takes a shift of 90 degree in anticlockwise direction and finally extends to infinity towards left. This is shown in figure 9.

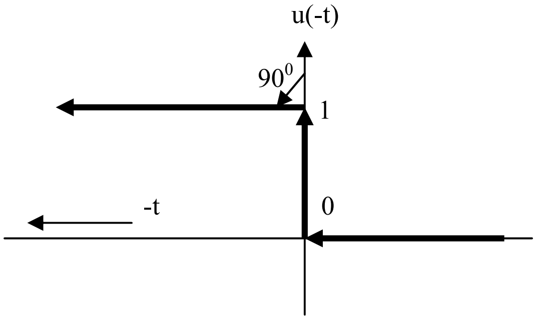

Figure 9. Sketching function u(-t) using angle theory, step 3.

4. Thus u(-t) signal takes two 90 degrees shifts – first shift in clockwise direction and second shift in anticlockwise direction with a unit height (amplitude) and finally extends towards left side (negative) infinity. The final figure is shown in figure 10.

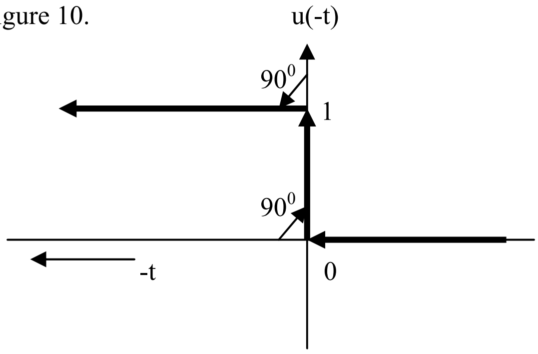

Figure 10. Sketching function u(-t) using angle theory, step 4.

C. Drawing of -u(t)[4]

Mathematically -u(t) is re presented as –

$$-u(t) = \begin{cases} -1, \text{ when } t >= 0 \\ 0, \text{ otherwise (that is for } t < 0) \end{cases}$$

To understand this, let us understand the example of -u(t). It can be depicted as –

if x(t) = -u(t)

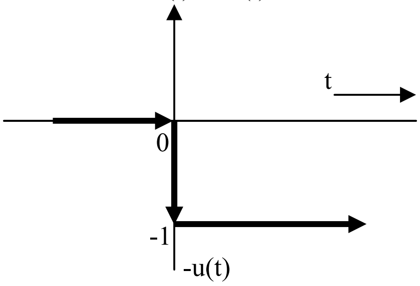

Figure 11. Function -u(t) based on existing theory.

Figure 11 shows that there are actually two 90 degrees shifts, it is explained with the help of first by taking a unitstep function in positive direction i.e. -u(t) –

For -u(t) case, -u(t) can be represented in terms of angles as shown below –

$$-u(t) = \begin{cases} 0, \text{ when } t < 0 \\ -1, 90^0 \text{ clockwise at } t = 0 \\ 1, 90^0 \text{ anticlockwise for } t > 0 \end{cases}$$

For -u(t) case -

1. First assume that, in idle case when no signal is there then the depiction line comes from negative infinity to origin. It is shown in figure 12.

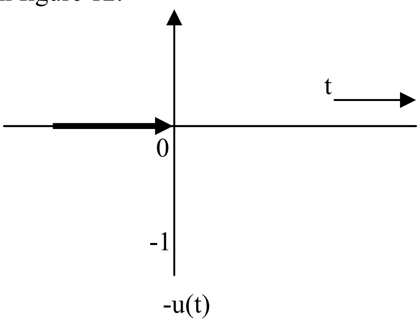

Figure 12. Function -u(t) using angle theory, step 1.

2. As soon as the signal –u(t) comes/appears then the signal takes a 90 degree shift in clockwise direction and takes one straight line in -u(t) direction i.e. in negative y-axis direction line. This situation is shown in figure 13.

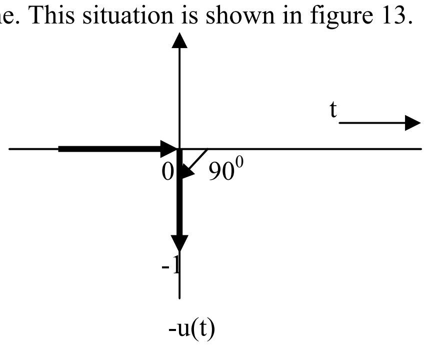

Figure 13. Function -u(t) using angle theory, step 2.

3. And after attaining a negative unit height again the shape takes a shift of 90 degree in anticlockwise direction and finally extends to infinity towards right. This is shown in figure 14.

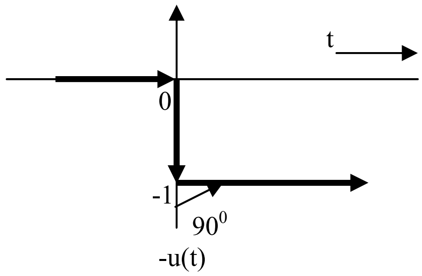

Figure 14. Function -u(t) using angle theory, step 3.

4. Thus -u(t) signal takes two 90 degrees shifts – first shift in clockwise direction and second shift in anticlockwise direction with a unit height (amplitude) in –y axis and finally extends towards right side (positive) infinity. The final figure can be shown as –

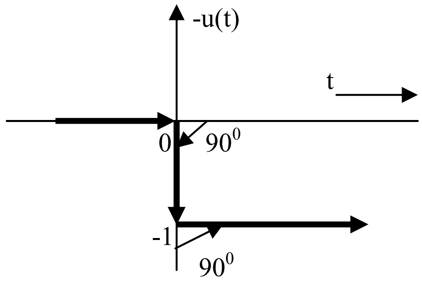

Figure 15. Function -u(t) using angle theory, step 4.

Likewise, one can plot the signals –u(-t) unit step function. After learning this, it is the time to move to have some practical examples.

V. Examples Based on Theory Developed

Example 1. – Drawing of u(t-2)

Solution : u(t-2) function can be shown as –

$$u(t-2) = \begin{cases} 1, \text{ when } t >= 2 \\ 0, \text{ otherwise} \end{cases}$$

It can be depicted as –

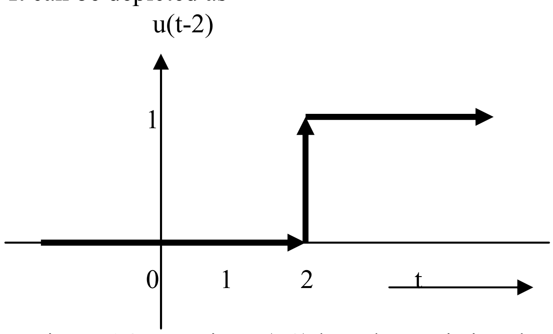

Figure 16. Function u(t-2) based on existing theory.

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Let us see the case step by step, which is more convincing –

1. First assume that, in idle case when no signal is there then the depiction line comes from negative infinity to t = 2 as shown in figure 17.

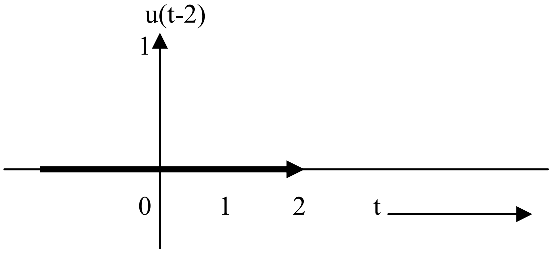

Figure 17. Function u(t-2) using angle theory, step 1.

2. As soon as the signal u(t-2) comes/appears then the signal takes a 90 degree shift at t = 2 in anticlockwise direction and takes one straight line parallel to x(t) i.e. parallel to y-axis line. This situation is shown in figure 18.

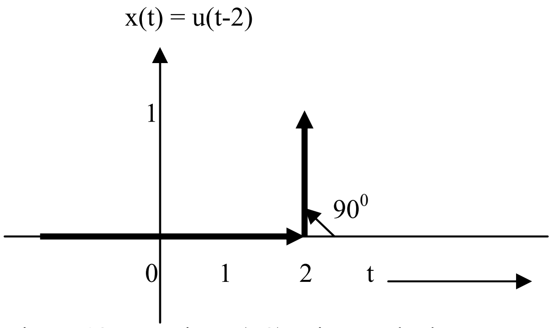

Figure 18. Function u(t-2) using angle theory, step 2.

3. After attaining a unit height again the shape takes a shift of 90 degree in clockwise direction and finally extends to infinity towards right. This is shown in figure 19.

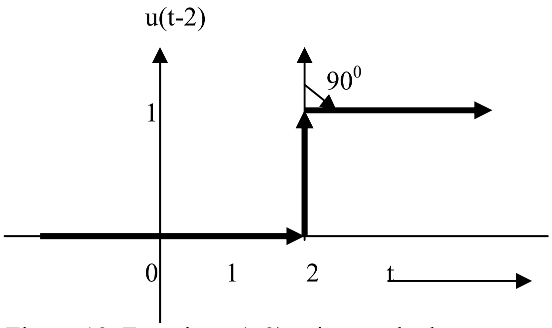

Figure 19. Function u(t-2) using angle theory, step 3.

4. The whole diagram can be shown as below for two 90^0 shifts.

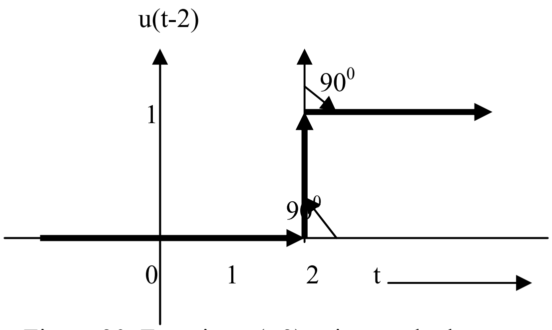

Figure 20. Function u(t-2) using angle theory, step 4.

Example 2. – Drawing of -u(t-2)

Solution : -u(t-2) function can be shown as –

$$-u(t-2) = \begin{cases} -1, & \text{when } t >= 2 \\ 0, & \text{otherwise} \end{cases}$$

It can be depicted as –

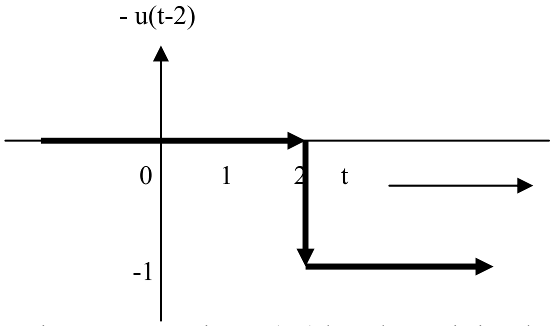

Figure 21. Function –u(t-2) based on existing theory.

Let us see the case step by step, which is more convincing –

1. First assume that, in idle case when no signal is there then the depiction line comes from negative infinity to t = 2 as shown in figure 22.

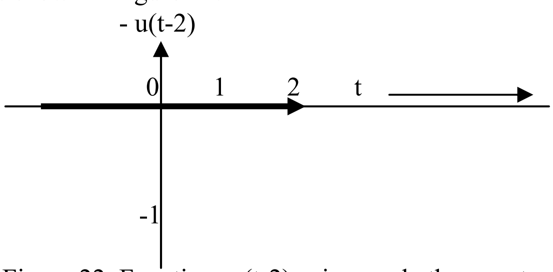

Figure 22. Function -u(t-2) using angle theory, step 1.

2. As soon as the signal -u(t-2) comes/appears then the signal takes a 90 degree shift at t = 2 in clockwise direction and takes one straight line parallel to x(t) i.e. parallel to y-axis line. This situation is shown in figure 23.

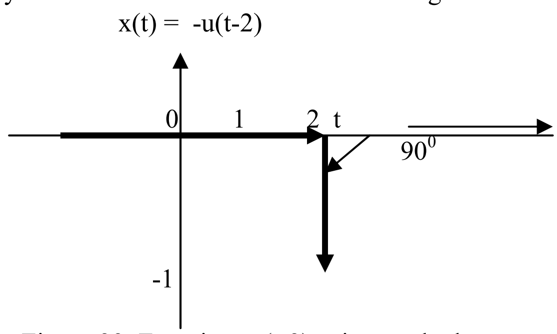

Figure 23. Function -u(t-2) using angle theory, step 2.

3. After attaining a unit height in negative direction, again the shape takes a shift of 90 degree in anticlockwise direction and finally extends to infinity towards left. This is shown in figure 24.

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

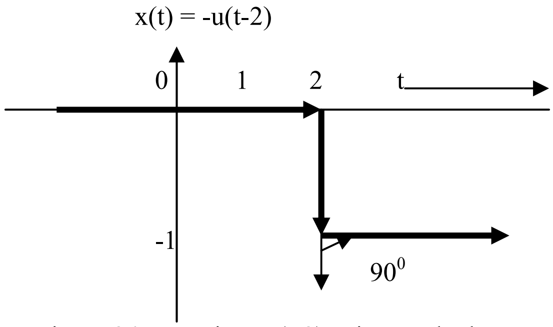

Figure 24. Function -u(t-2) using angle theory, step 3.

4. The whole diagram can be shown as below for two 90^0 shifts.

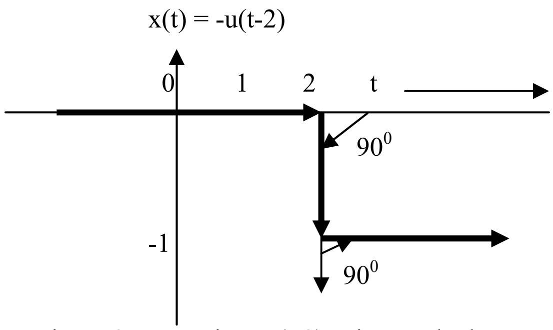

Figure 25. Function -u(t-2) using angle theory, step 4.

Let us take one complex example without explaining steps.

Example 3. Sketch the shape of the given equation –

x(t) = u(t-2)-2u(t-5)+3u(t-7)-2u(t-9)

Solution : The solution is given below by keeping the given theory in the previous section.

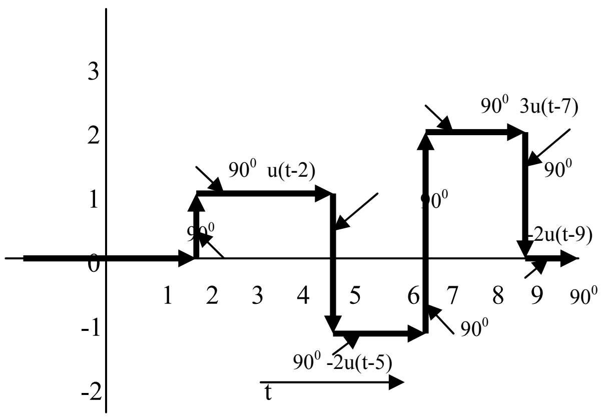

Figure 26 Function -u(t-2) using angle theory.

Now it might be cleared that each and every unit function is having two 90^0 angles hence the theory developed fits best in the numerical examples.

Likewise some other complex examples can be considered for better understanding of the theory developed.

This is what I call concept of 90 degree related to unit step function u(t). This concept plays a vital role while solving the related numerical.

VI. Conclusion

When I applied this theory to the B.Tech. (Subject : Signals and Systems) students, then I found that students not only grasped this theory but also solved a number of problems based on this.

This paper is an outcome from the teaching experience where the students faced a lot of problems to understand the ramp function numerical problems. This work is an attempt to teach the students step by step construction procedure of ramp signal functions and this work has an attempt to explore the new and easy theory specially written for ramp related to the basic signal functions. No doubt the future studies will further explore my work in deep.

On the basis of this theory, some other signals could be developed that will go long to the scientists and students. As no matter is available on the internet, hence *I claim that this theory is purely based on my research work*/affords and has a bright chance to explore new theory and ideas on this. .

Acknowledgment

I would like to thank to my B.Tech. pursuing students who posed a lot of questions in the form of doubts and inclined me to think more and more to clarify their complex doubts. What I feel in this context that it is only the students for a teacher who can make a teacher gold from silver. Hence, I acknowledge my students and again I pay special thanks to my students who made me to reach at this stage where I could produce this paper.

References

[1] **A.V. Oppenheim, A.S. Willsky** with **S. Hamid Nawab**, Signals and Systems – by – Pearson Education, Second edition, 2002.

[2] **Simon Haykin, Barry Van Veen,** Signals and systems – by – John Wiley & Sons (Asia) Pte. Ltd., Second Edition, 2004 .

[3] **P. Ramakrishna Rao,** Signals and Systems – by – Tata McGraw Hill, First edition, 2008.

[4] **Satyapal Singh** own concept, based on teaching experience.

ISBN: 978-988-17012-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)