Resistance to Clogging of Fluid Microfilters

Zbigniew Domański, Bożena Baran, and Mariusz Ciesielski

***Abstract*—The flow of particles suspended in fluids and transported through different geometries is a process with numerous industrial applications ranging from chemical processing to bio-analytic separation. A filtrate particle flowing through the channel may be trapped by the geometric constraint or other adhesive mechanisms. Realistic filters have randomly-interconnected channel space with a complex flow path. However, in micro-fluidic systems, channel space may resemble two-dimensional tessellation. Here we adopt the network flow concept to analyze two-dimensional micro-filters and study the filter efficiency (effective conductibility) and the clogging time i.e. the time until a filter becomes clogged due to the trapping of suspended particles**

***Index Terms*—clogging time, filter efficiency, microfilter.**

I. INTRODUCTION

The geometrical properties of networks have attracted much attention due to progress in the fields of computer science, mathematical biology, statistical physics and technology. Especially, the microfluidic systems are built with the use of methods borrowed from the semiconductor industry [1]. Such methods generally employ the fabrication of highly ordered microscale structures.

Molecular filtration using nanofilters is an important engineering problem, with very diverse applications ranging from chemical processing to biological applications. Biochemical analysis of aqueous solutions involves the flow of particles of different shapes suspended in fluids and transported through different geometries. A filtrate particle flowing through the pore space may be trapped by the geometric constraint or other adhesive mechanisms. Realistic filters have randomly-interconnected channel space with complex flow path. However, in microfluidic systems, channel space may resemble two-dimensional tessellation [1]-[5]. Here, the term "channel" refers to a conduit of any desirable shape through which liquids may be directed and the term "microfluidic" refers to the structure wherein one or more dimensions is less than 10^{-5} m.

The problem we consider is the clogging process of a hypothetical microfilter with the channel space built up according to a given two dimensional tessellation. The objective of our investigation is to determine the role played by the network geometry in this process provided that the flow of liquid and suspended molecules is laminar. We focus our analysis on two quantities: (i) the time until a filter becomes clogged by particles captured inside the network due to the size exclusion mechanism and (ii) the filter efficiency represented by a drop in filter permeability [6]-[10]. The results of our numerical computations give some insight into the question how resistant to clogging are the filters with two-dimensional channel spaces.

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II. MATHEMATICAL MODEL

A. Technological and Physical Aspects

Microfluidic devices are constructed in a planar fashion. Typically, they comprise at least two flat substrate layers that are mated together to define the channel networks. Channel intersections may exist in a number of formats, including cross intersections, "T" intersections, or other structures whereby two channels are in fluid communication [1], [4].

Due to the small dimension of channels the flow of the fluid through a microfluidic channel is characterized by the Reynolds number of the order less than 10. In this regime the flow is predominantly laminar and thus molecules can be transported in a relatively predictable manner through the microchannel.

B. Channel Space Geometry

In the context of artificial filter-channel space architectures the lattices of main interest have edges and vertices formed by a regular tiling of a plane, so that all corners are equivalent.

There exist exactly 11 lattices known in the literature as the Archimedean lattices [5]. Three of them: triangular, square and hexagonal are drawn in a plane such a way that all faces are the same whereas the remaining 8 lattices need more than one type of a face. The former lattices belong to the regular tessellations of the plane and the latter ones are called semiregular lattices. Another important group of lattices contains dual lattices of the Archimedean ones. The given lattice G can be mapped onto its dual lattice DG in such a way that the center of every face of G is a vertex in DG, and two vertices in DG are adjacent only if the corresponding faces in G share an edge. The square lattice is self-dual, and the triangular and hexagonal lattices are mutual duals. The dual lattices of the semiregular lattices form the family called Laves lattices [5]. Finally, there are 19 possible regular arrangements of fluid conduits.

The lattices are labeled according to the way they are drawn [5]. Starting from a given vertex, the consecutive faces are listed by the number of edges in the face, e.g. a square lattice is labeled as (4, 4, 4, 4) or equivalently as (4^4). Consequently, the triangular and hexagonal lattices are (3^6) and (6^3), respectively. Other, frequently encountered lattices are (3, 6, 3, 6) – called Kagomé lattice and its dual D(3, 6, 3, 6) - known as Necker Cube lattice. In some ways these 5 lattices serve as an ensemble representative to study

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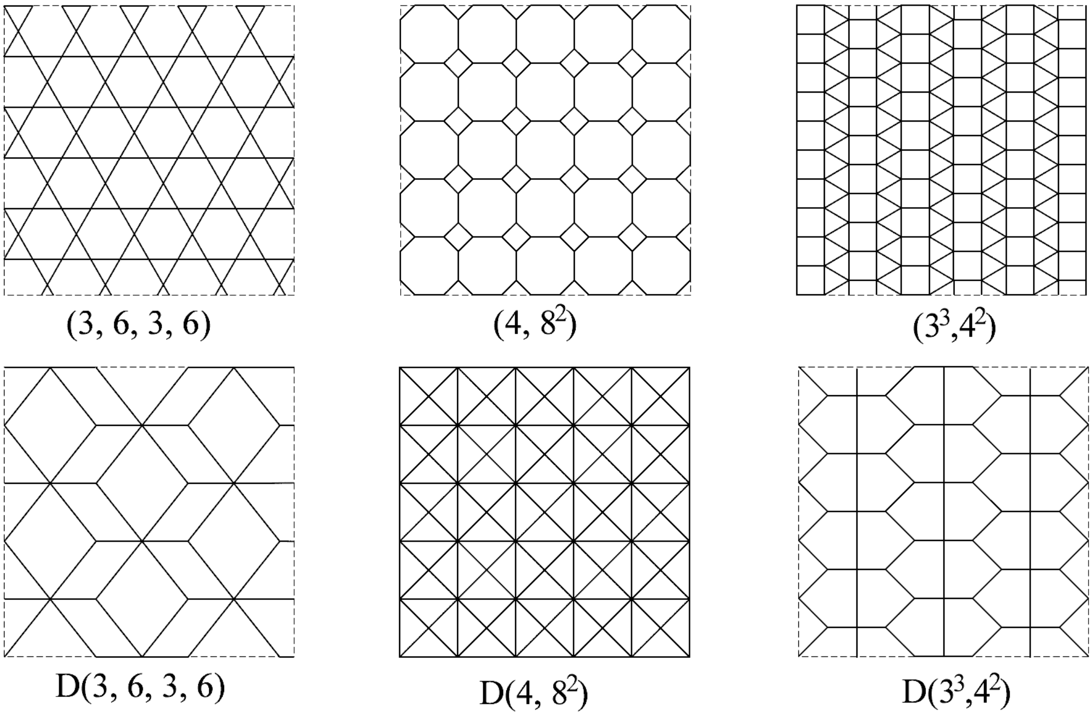

Fig. 1. Examples of two dimensional tessellations used in this work.

filtration problems in two dimension. They form pairs of mutually dual lattices and also share some local properties as e.g. the coordination number z being the number of edges with common vertex. One of the most interesting lattices in two dimension is the Kagomé lattice. Each its vertex touches a triangle, hexagon, triangle, and a hexagon. Moreover the vertices of this lattice correspond to the edges of the hexagonal lattice, which in turn is the dual of a triangular lattice. The Kagomé lattice is also related to the square lattice, they have the same value, $z = 4$, of the coordination number. Besides the above mentioned lattices, in this paper we have also analyzed other regular tiling, namely $(4, 8^2)$, $D(4, 8^2)$, $(3^3, 4^2)$, and $D(3^3, 4^2)$. Some of these lattices are presented in Fig. 1.

C. Filter Blockage

We consider a hypothetic flow of particles transported by fluid through the network of channels arranged according to the positions of the edges of the chosen lattice. All channels are characterized by their radii r which are quenched random variables governed by a given probability distribution. This distribution will be specified later.

In order to analyze the filter clogging process we employ a cellular automata model with the following rules [8]. Fluid and a particle of a radius R enter the filter and flow inside it due to an external pressure gradient. The particle can move through the channel without difficulty if $r > R$, otherwise it would be trapped inside a channel and this channel becomes inaccessible for other particles. At an end-node of the channel, the particle has to choose a channel out of the accessible channels for movement. If at this node there is no accessible channel to flow the particle is retained in the channel. Otherwise, if the radius of the chosen channel $r' > R$ the particle moves to the next node. The movement of the particle is continued until either the particle is captured or leaves the filter. Each channel blockage causes a small reduction in the filter permeability and eventually filter becomes clogged.

D. Effective Conductibility of the Filter

The next problem we consider is the conductibility of the filters with different channel-network geometries. We apply the network flow language. In this framework, all channels are characterized by their capacitances C. As previously, these capacitances are quenched random variables governed by a uniform probability distribution defined in the range [0, 1] to assure $C = 0$ for the clogged channel and $C = 1$ for the fully opened channel.

We define the filter's effective conductibility as follows

$$\phi(C_1, C_2, \ldots, C_n) = \frac{1}{\Phi_0} \Phi(C_1, C_2, \ldots, C_n) \qquad (1)$$

where $\Phi(C_1, C_2, \ldots, C_n)$ is the flux transmitted by the filter whose channels have restricted possibilities to maintain the flow and $\Phi_0 = \Phi(C_1 = 1, C_2 = 1, \ldots, C_n = 1)$. Equation (1) permits to compare performance of different lattice geometries in their job as a potential filtering network.

III. Numerical Modeling and Results

The cellular automata approach constitutes the effective tool for numerical computations of particles transfer. For the filter blockage investigation a minimalist description requires two assumptions: (i) injected particles are identical spheres with the radius R and (ii) the channel radius is drawn from a discrete two-point probability distribution function, whereas $P(r > R) = p$ is the only model parameter. Thus, the channel space is represented by a network of interconnected, wide (W) and narrow (N), cylindrical pipes (Fig. 2). Fluid containing

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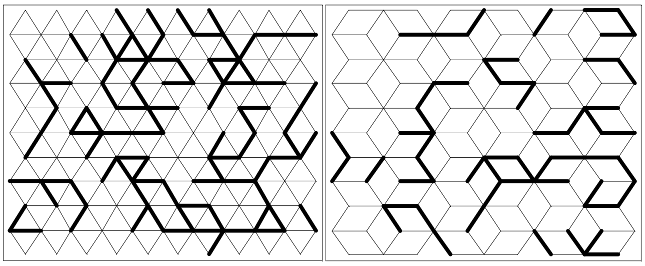

Fig. 2. Examples of two-dimensional model filters: N channels – thin lines, W channels – thick lines. Fluid with suspended particles is injected on the left side of the filter and exits the right side.

suspended particles flows through the filter according to the previously stated rules (see Section II.C).

We present the results of the numerical simulations of the above specified filter. Every time step particles enter the filter - one particle per each accessible entry channel and we count the time t required for the filter to clog. For each analyzed geometry and for several values of p from the range $[0.05, p_c]$ we performed 10^3 simulations and then we have built empirical distributions of the clogging time t. Here p_c is the fraction of W channel for which the network lost its filtering capability. It is because of sufficiently high p values that there exist statistically significant number of trajectories formed only by W channels and spanned between input and output of the filter.

Our simulations yield a common observation: the average time required for the filter to clog can be nicely fitted as

$$\bar{t} \approx \tan[\pi p/(2 p_c)] \tag{2}$$

where the values of p_c are in excellent agreement with the bond percolation thresholds of the analyzed networks (see Table I). Fig. 3 shows $\bar{t}$ as a function of p for selected lattices, 3 lattices out of 9 lattices we have analyzed.

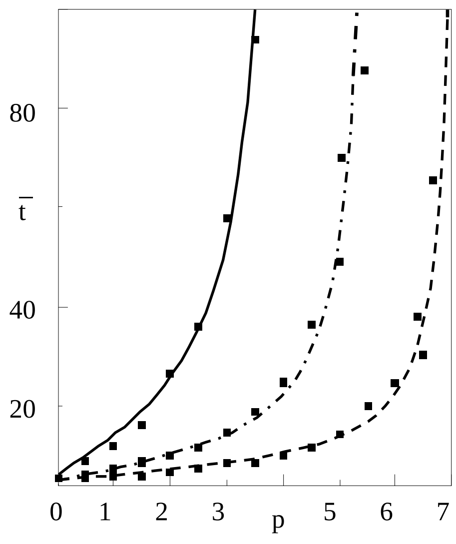

Fig. 3. Average clogging time for regular lattices: solid line, triangular lattice; dash-dotted line, square lattice; dashed line, hexagonal lattice. The lines are drawn using (2) and they are only visual guides.

Table I. Bond percolation thresholds and coordination numbers for 9 networks analyzed in this work.

Lattice	Bond percolation threshold p_c
(3^6) triangular	0.3473
(4^4) square	0.5000
(6^3) hexagonal	0.6527
(3, 6, 3, 6)	0.5244
D(3, 6, 3, 6)	0.4756
$(4, 8^2)$	0.6768
D$(4, 8^2)$	0.2322
$(3^3, 4^2)$	0.4196
D$(3^3, 4^2)$	0.5831

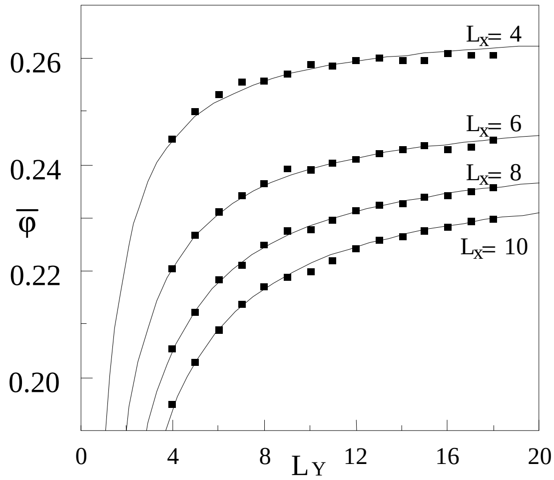

Fig. 4. Average filter's effective conductibility, defined by (1) computed for different values of length (L_X) and width (L_Y) of the filter for the hexagonal lattice. The lines are drawn using (3) and they are only visual guides.

As was pointed out in Section II.D we are also interested in the filter's effective conductibility defined by (1). For our 9 lattices we have computed the average values of $\bar{\phi}$ for an ample set of values of length (L_X) and width (L_Y) of the filter. As an example, in Fig. 4 we present $\bar{\phi}$ for the hexagonal lattice. We have found that for all lattices $\bar{\phi}$ has the following form:

$$\bar{\phi}(L_X, L_Y) = \left(a_1 + a_2 / L_X^{\delta}\right)\tan^{-1}\left[\psi(L_X)\cdot L_Y\right] \tag{3}$$

where: a_1, a_2, δ are the parameters and $\psi(L_X)$ is the function, all dependent on the lattice symmetry.

IV. Conclusion

In this paper we have studied a minimalist model for particle flow through the microfluidic filter with an artificial geometry of the channel space. We have exploited two extreme pictures: a cellular automata microscopic-like picture and a completely statistical approach to an operating filter considered it as the network supporting the flow trough a collection of randomly conducting channels. Even though the cellular automat rules for the movement of particles are too

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simple to capture the detailed interactions of real particles this approach enable us to see how the system becomes clogged. Also the network flow concept is useful to study the interplay between geometry and transport properties of ordered lattices. Its main advantage relays on a very simple representation of the inner filter's structure yet keeping a bridge between the conductibility, the geometry (lattice's symmetry, coordination number) and the statistical global property (bond percolation threshold).

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