

# On the Modeling and Parametric Identification of a Motor-Generator Set

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*Abstract*—This work deals with the modeling and parametric identification of a motor-generator set. Nonlinear and linearized dynamic models are derived. By utilizing the structure defined by the linear model and a predictor-error algorithm the parametric identification is accomplished. The methodology is performed on an experimental set-up. Several custom-made power, analog and digital electronics designs are employed for the acquisition of the transient experimental data.

## 1 Introduction

This paper presents the modeling and parametric identification of a motor-generator set. Our aim is to obtain a model including the corresponding parameters for experimental research in power systems. The long term objective would be to have a turbine-generator emulator as an experimental tool for studying the transient behavior of a power system when small disturbances appear. The electric machines involved in this turbine-generator emulator are a shunt connected DC motor and a 3-phase salient-pole synchronous generator.

One of the contributions of this work is that nonlinear and linearized dynamic models are derived. These models describe the steady state and transient behavior of the augmented (motor-generator) dynamic system. Once that these models are known, the structure and the order of the linearized dynamic model are used as a base for the predictor-error algorithm to identify the parameters. It is clear that our approach is based on a time-domain model.

Another contribution of this work is the design of several custom-made power, analog and digital electronics circuits that are employed for the acquisition of the transient experimental data. In other words, the methodology is tested in a experimental set up.

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Once that the experimental data is obtained a complex off-line processing is carried out. For example, a nonlinear mapping is employed to transform the original machine variables to variables in the rotor reference frame. This mapping is actually calculated by using the experimental dynamic values corresponding to the rotor position.

A brief review of the literature follows. The estimation of parameters of synchronous generators has been always an important topic since its relevance in steady state and dynamic studies. There are plenty of contributions regarding the modeling and parametric identification of these electric machines. For example, there are contributions that deal with different techniques to measure the experimental variables. There are also different identification schemes for a variety of models. Some online estimation techniques are presented in [1] [2] and [3]. Contribution in [4] provides an excellent review of parameters estimation and dynamic model identification of the synchronous generator. In [5], an identification method is presented; it is applied to a seventh order model that includes the effect of saturation. Methods based on state space models for identification of parameters have been also proposed. In [6], a continuous state space model is transformed to its discrete state space form to obtain a set of equations that allow the authors to calculate the parameters of the generator. An identification procedure for estimating the parameters of a synchronous machine in the time and frequency domains is presented in [7]. In [8], fundamental equations of the synchronous machine are used to obtain a state space model to analyze the interaction of different areas of generation in an interconnected system.

The above contributions are normally associated with a particular electric machine, i.e. the synchronous generator. In contrast, our modeling deals with the augmented (motor-generator) dynamic system. In addition, our work presents data that are obtained by using an experimental set-up that includes the effects related to the power electronics converter. This is important since the long term objective of this work is to have an emulator that will include a state feedback and an actuator as the one mentioned above.

## 2 Modeling

### 2.1 DC Motor

Consider the following voltage equations that describe the transient and steady state behavior of a shunt connected DC motor [9]

$$\begin{aligned} v_a &= r_a i_a + L_{aa} \dot{i}_a + L_{af} w_r i_f \\ v_a &= r_f i_f + L_{ff} \dot{i}_f \end{aligned} \quad (1)$$

the electromagnetic torque of the DC motor is given by

$$T_{e1} = L_{af} i_f i_a \quad (2)$$

rotor speed and torque are related by the following expression

$$T_{e1} = J_1 p w_r + T_L \quad (3)$$

Standard notation is employed in this part of our work [9].

### 2.2 Synchronous Generator

In order to define an adequate dynamic model for the synchronous generator a change of coordinates is necessary. The well known reference frame theory provides the basis necessary to obtain a dynamic model that does not include any time-varying coefficient [9]. Since the circuits of the rotor are normally unsymmetrical we will be using the rotor reference frame [9]. Variables associated with the stator of the generator can be transformed from the original coordinates (machine variables) to variables in the rotor arbitrary reference frame, i.e.

$$\begin{bmatrix} F_{qs} \\ F_{ds} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta_r & \cos \left( \theta_r - \frac{2\pi}{3} \right) & \cos \left( \theta_r + \frac{2\pi}{3} \right) \\ \sin \theta_r & \sin \left( \theta_r - \frac{2\pi}{3} \right) & \sin \left( \theta_r + \frac{2\pi}{3} \right) \end{bmatrix} \begin{bmatrix} F_{as} \\ F_{bs} \\ F_{cs} \end{bmatrix} \quad (4)$$

where  $F$  may represent voltage, current or flux linkages. We employ the standard notation presented in [9] excepting that we have omitted the superindex  $r$  for simplifying purposes. That superindex is normally associated with the variables in the rotor reference frame. The mechanical rotor position is denoted by  $\theta_r$ . Let us consider the voltage equations in the rotor reference frame for the stator of a pole-salient synchronous generator [9]

$$\begin{aligned} v_{qs} &= -r_s i_{qs} + w_r \lambda_{ds} + p \lambda_{qs} \\ v_{ds} &= -r_s i_{ds} - w_r \lambda_{qs} + p \lambda_{ds} \end{aligned} \quad (5)$$

If the electric load at the generator terminals is a 3-phase symmetrical resistive circuit, then expression (5) now becomes

$$\begin{aligned} r_L i_{qs} &= -r_s i_{qs} + w_r \lambda_{ds} + p \lambda_{qs} \\ r_L i_{ds} &= -r_s i_{ds} - w_r \lambda_{qs} + p \lambda_{ds} \\ v_{fd} &= r_{fd} i_{fd} + p \lambda_{fd} \end{aligned} \quad (6)$$

where the expression that represents the behavior of the field winding has been included;  $r_L$  denotes the load resistance. The flux linkages may be written as [9]

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{fd} \end{bmatrix} = \begin{bmatrix} -i_{qs}(L_{ls} + L_{mq}) \\ -i_{ds}(L_{ls} + L_{md}) + i_{fd} L_{md} \\ -i_{ds} L_{md} + i_{fd}(L_{lfd} + L_{md}) \end{bmatrix} \quad (7)$$

It is easy to show that the currents can be expressed as a function of the flux linkages, this is

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{fd} \end{bmatrix} = \begin{bmatrix} -\lambda_{qs}/(L_{ls} + L_{mq}) \\ -\lambda_{ds}(L_{lfd} + L_{md})/\Lambda + \lambda_{fd} L_{md}/\Lambda \\ -\lambda_{ds} L_{md}/\Lambda + \lambda_{fd}(L_{ls} + L_{md})/\Lambda \end{bmatrix} \quad (8)$$

where

$$\Lambda = L_{ls} L_{lfd} + L_{ls} L_{md} + L_{md} L_{lfd} \quad (9)$$

the electromagnetic torque of the synchronous generator in terms of the currents in the rotor reference frame is

$$T_{e2} = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \left[ L_{md} (-i_{ds} + i_{fd}) i_{qs} + L_{mq} i_{qs} i_{ds} \right] \quad (10)$$

using (8) it is possible to obtain a new expression for the electromagnetic torque of the synchronous generator

$$T_{e2} = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \left[ \lambda_{qs} \lambda_{ds} \left( \frac{L_{lfd} + L_{md}}{\Lambda} + \frac{1}{L_{ls} + L_{mq}} \right) - \lambda_{qs} \lambda_{fd} \frac{L_{md}}{\Lambda} \right] \quad (11)$$

the torque-rotor speed relationship is defined by the following expression

$$T_{e2} = -J_2 \left( \frac{2}{P} \right) p w_r + T_L \quad (12)$$

### 2.3 Nonlinear model of the mechanical coupled system

In order to establish the mathematical model of the motor-generator set, an equation that defines the interaction between both subsystems is required. Since both electrical machines rotate at the same speed only one state equation for the rotor speed is necessary. It is clear that the load torque of the DC motor is the input (load) torque of the synchronous generator. Using (3) and (12) we obtain

$$T_{e1} - J_1 p w_r = T_{e2} + \frac{2}{P} J_2 p w_r \quad (13)$$

Substituting (8) into (6) and using (1), (6) and (13) the following nonlinear dynamic model can be obtained

$$\begin{cases} p\lambda_{qs} = (r_s + r_L)\lambda_{qs}/(L_{ls} + L_{mq}) - w_r\lambda_{ds} \\ p\lambda_{ds} = w_r\lambda_{qs} + (r_s + r_L)[-(L_{lfd} + L_{md})\lambda_{ds}/\Lambda + L_{md}\lambda_{fd}/\Lambda] \\ p\lambda_{fd} = r_{fd}L_{md}\lambda_{ds}/\Lambda - r_{fd}(L_{ls} + L_{md})\lambda_{fd}/\Lambda + v_{fd} \\ p i_a = -r_a i_a/L_{aa} - L_{af}w_r i_f/L_{aa} + v_a/L_{aa} \\ p i_f = -r_f i_f/L_{ff} + v_a/L_{ff} \\ p w_r = P(T_{e1} - T_{e2})/(2J_2 + PJ_1) \end{cases} \quad (14)$$

$$\begin{cases} v_{qs} = -r_L\lambda_{qs}/(L_{ls} + L_{mq}) \\ v_{ds} = -r_L(L_{lfd} + L_{mq})\lambda_{ds}/\Lambda + r_L L_{md}\lambda_{fd}/\Lambda \\ w_r \end{cases} \quad (15)$$

where  $T_{e1}$ ,  $\Lambda$  and  $T_{e2}$  are defined in (2), (9) and (11) respectively. It is clear that expression (14) is the state equation and (15) is the output equation. The following remarks are in order:

**Remarks**

- Some of the experimental variables to be measured are the 3-phase instantaneous voltages (original variables) at the terminals of the generator, that is  $[v_{as} \ v_{bs} \ v_{cs}]^T$ . Then the electrical outputs ( $v_{qs}$  and  $v_{ds}$  in (15)) defined in the rotor reference frame can be calculated by using a particular case of expression (4).
- The experimental synchronous generator that is employed in this work does not have any damper winding. This is the reason for not including any state equation associated with those windings.
- Since the 3-phase voltages at the terminals of the generator are balanced the 0s voltage ( $v_{0s}$ ) is zero  $\forall t$ . This is the reason for not including any 0s equation and/or variable.
- The external resistive circuit that is connected at the terminals of the generator is considered to have parameters that are known. Additionally, it is clear that a resistive circuit does not increase the order of the system to be tested.

**2.4 Linearized system**

The dynamic model presented in (14) and (15) can be linearized by using the Taylor's expansion about an equilibrium point. It is important to note that during steady state conditions the machine variables (original state variables) of the DC motor are normally constants. In contrast, during steady state conditions the stator machine variables (original stator state variables) of the synchronous generator consist of a balanced 3-phase sinusoidal set. Transforming the machine variables of the

generator to the rotor reference frame allows us to obtain an augmented model (motor-generator set) that has an equilibrium point instead of having a state vector containing periodic solutions during steady state conditions. The existence of the equilibrium point is a basic requirement for our linearization process. The linearized dynamic system follows.

$$\begin{aligned} p\Delta x &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + E\Delta u \end{aligned} \quad (16)$$

where  $\Delta x = [\Delta\lambda_{qs} \ \Delta\lambda_{ds} \ \Delta\lambda_{fd} \ \Delta i_a \ \Delta i_f \ \Delta w_r]^T$  is state vector,  $\Delta y = [\Delta v_{qs} \ \Delta v_{ds} \ \Delta w_r]^T$  is the measurable output,  $\Delta u = [\Delta v_a \ \Delta v_{fd}]^T$  is the input and

$$A = [A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6]$$

$$A_1 = \begin{bmatrix} (r_s + r_L)/(L_{ls} + L_{mq}) \\ w_{r0} \\ 0 \\ 0 \\ 0 \\ k_1 k_2 (-k_3 \lambda_{ds0} + L_{md} \lambda_{fd0}/\Lambda) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -w_{r0} \\ -(r_s + r_L)(L_{lfd} + L_{md})/\Lambda \\ r_{fd} L_{md}/\Lambda \\ 0 \\ 0 \\ -k_1 k_2 k_3 \lambda_{qs0} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 \\ (r_s + r_L)L_{md}/\Lambda \\ -r_{fd}(L_{ls} + L_{md})/\Lambda \\ 0 \\ 0 \\ k_1 k_2 L_{md} \lambda_{qs0}/\Lambda \end{bmatrix}; \quad A_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -r_a/L_{aa} \\ 0 \\ k_1 L_{af} i_{f0} \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -L_{af} w_{r0}/L_{aa} \\ -r_f/L_{ff} \\ k_1 L_{af} i_{a0} \end{bmatrix}; \quad A_6 = \begin{bmatrix} -\lambda_{ds0} \\ \lambda_{qs0} \\ 0 \\ -L_{af} i_{f0}/L_{aa} \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1/L_{aa} & 0 \\ 1/L_{ff} & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = [C_1 \ C_2]$$

$$C_1 = \begin{bmatrix} -r_L/(L_{ls} + L_{mq}) & 0 \\ 0 & -r_L(L_{lfd} + L_{mq})/\Lambda \\ 0 & 0 \end{bmatrix}$$

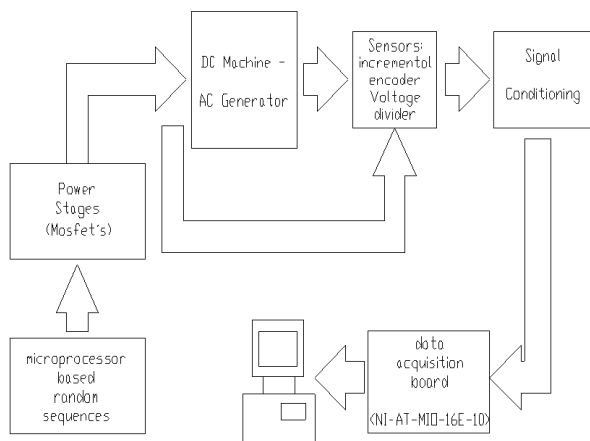


Figure 1: Block diagram of the experimental setup

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ r_L L_{md}/\Lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = 0$$

where

$$k_1 = P/(J_1 P + 2J_2) \quad ; \quad k_2 = 3P/4$$

$$k_3 = (L_{fd} + L_{md})/\Lambda + 1/(L_{ls} + L_{mq})$$

and  $x_0 = [\lambda_{qs0} \lambda_{ds0} \lambda_{fd0} i_{a0} i_{f0} w_{r0}]^T$  is the equilibrium point where the linearization was performed.

### 3 Experimental Setup

Important components of the experimental system are a personal computer, a National Instruments AT-MIO-16E-10 data acquisition card, NI Labview software, two custom made PWM-based power electronics converters, an incremental encoder [10], a custom made microcontroller based design for defining several random sequences and signal conditioning circuits for measuring several variables. A block diagram of the experimental setup is shown in Fig. 1. To identify parameters in (16) an experimental test is carried out. Basically, the experimental test consists of applying (at the same time) two different random sequences to the incremental inputs of the system  $\Delta u$ . During this test, we restrict those inputs such that the dynamics of the augmented system is located in a small neighborhood of the equilibrium point. Microcontroller in Fig. 2.A contains the assembler source program for defining the random sequence. The PWM signal is generated by using the design presented in Fig. 2.B. The schematic diagram of the power stage and the DC motor is depicted in Fig. 2.C. A design similar to the one that is presented in Fig. 2 is used to apply the random sequence to the field winding of the synchronous generator. The only difference is that both designs contain different random sequences. In addition, a circuit is

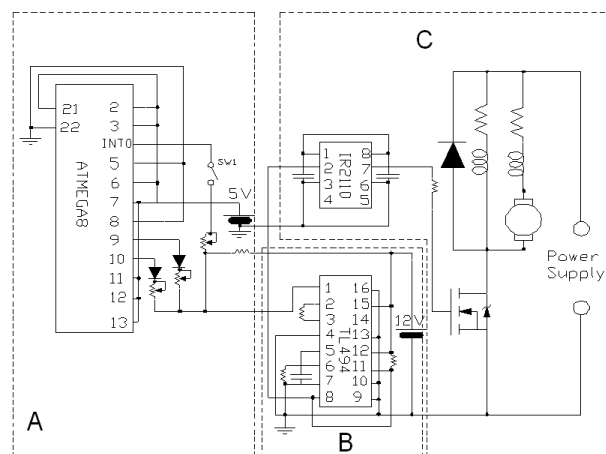


Figure 2: Circuit for applying a random sequence to the DC motor

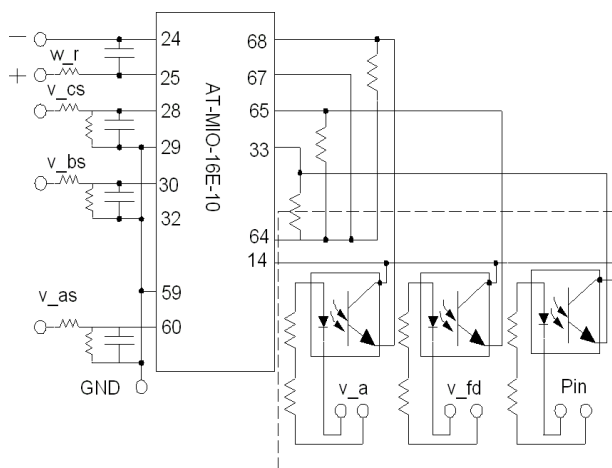


Figure 3: Signal conditioning circuit

designed for conditioning the signals to be measured, see Fig. 3. Once the variables have been isolated and have adequate amplitudes, they are connected to the analog inputs of the data acquisition board, see Fig. 3.

Additional diagrams of the experimental setup for the parametric identification can be found in [11].

### 4 Results

The measured trace of the PWM field winding voltage of the synchronous generator ( $v_{fd}$ ) is depicted in Fig. 4. After some off-line filtering and subtracting the input corresponding to the equilibrium point ( $v_{fd0}$ ) the PWM voltage in Fig. 4 becomes the incremental voltage ( $\Delta v_{fd}$ ) depicted in Fig. 5. It is evident that the measured PWM field winding voltage corresponds to an incremental voltage having a time-varying (random) period. Fig. 5 also shows the ideal (not measured) trace of the correspond-

ing random sequence. Similar traces were obtained for the other incremental input, i.e. for the measured voltage of the DC motor ( $\Delta v_a$ ).

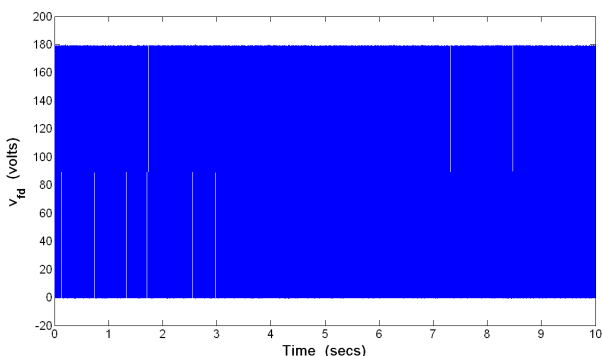


Figure 4: Measured PWM field winding voltage of the synchronous machine

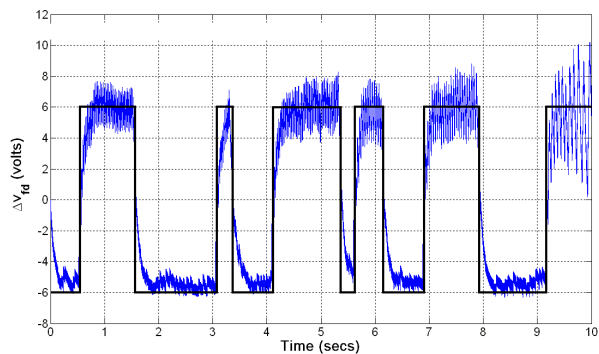


Figure 5: Incremental field winding voltage of the synchronous machine

Incremental inputs ( $\Delta v_a$  and  $\Delta v_{fd}$ ) having time-varying (random) periods clearly affect the measured voltages in original coordinates at the generator terminals, see the voltage  $v_{as}$  shown in Fig. 6. As it was explained earlier, the experimental voltages in the rotor reference frame ( $\Delta v_{qs}$  and  $\Delta v_{ds}$ ) can be obtained by performing an off-line calculation, i.e. transforming the incremental version of the measured 3-phase voltages at the terminals of the generator ( $[v_{as} \ v_{bs} \ v_{cs}]^T$ ). As an example, voltage denoted by  $\Delta v_{ds}$  (expressed in the rotor reference frame) is shown in Fig. 7. It is important to note that the unsymmetrical nature of the voltage in Fig. 7 is related to the initial condition of the angle employed in the mapping (4).

Once we obtained the transient experimental data corresponding to the output ( $\Delta y = [\Delta v_{qs} \ \Delta v_{ds} \ \Delta w_r]^T$ ) and the input ( $\Delta u = [\Delta v_a \ \Delta v_{fd}]^T$ ), we utilized Matlab for the off-line processing of the data. In particular, prediction-error technique is used to calculate the parameters in (16). A key assumption here is that the model

order and the structure of the vector/matrices used as a base for the prediction-error algorithm is the same as the one defined by model in (16). The resulting vectors/matrices are

$$A_1 = \begin{bmatrix} 201.1 \\ 134.57 \\ 0 \\ 0 \\ 0 \\ -89708 \end{bmatrix}; A_2 = \begin{bmatrix} -134.57 \\ -4678.2 \\ 820.93 \\ 0 \\ 0 \\ 1920.4 \end{bmatrix}; A_3 = \begin{bmatrix} 0 \\ 16183 \\ -2897.8 \\ 0 \\ 0 \\ -6539.3 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -130.7 \\ 0 \\ 44.234 \end{bmatrix}; A_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3689.9 \\ -73.486 \\ 1944.6 \end{bmatrix}; A_6 = \begin{bmatrix} -140.86 \\ -104.78 \times 10^3 \\ 0 \\ -30.377 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0.045635 & 0 \\ 0.11965 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -152.93 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.6195 & 3.8727 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The initial condition of the incremental state vector is  $\Delta x(0) = \mathbf{0}$ .

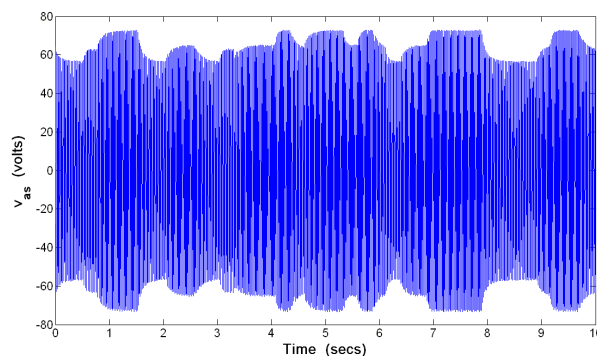


Figure 6: Measured phase a voltage at the terminals of the synchronous machine

It is important to note that the filtered versions of the inputs (as the one presented in Fig. 5) were employed just to tune the incremental values. The prediction-error algorithm is actually performed by using the PWM version of the incremental inputs.

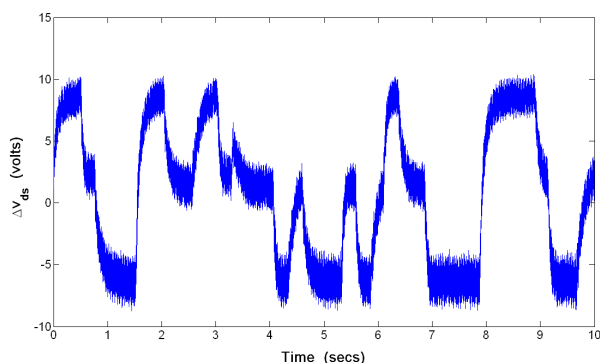


Figure 7: Incremental  $ds$  voltage of the synchronous machine

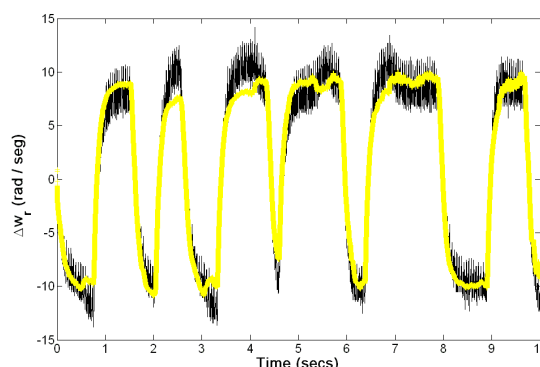


Figure 10: Comparison between the measured and simulated output ( $\Delta w_r$ )

A good agreement is observed when the measured and simulated outputs are compared, see Fig. 8 - Fig. 10. It is clear that the simulated response is obtained by solving the linear system that results of substituting the calculated parameters.

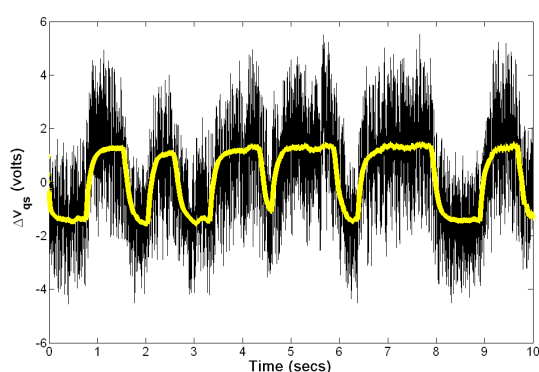


Figure 8: Comparison between the measured and simulated output ( $\Delta v_{qs}$ )

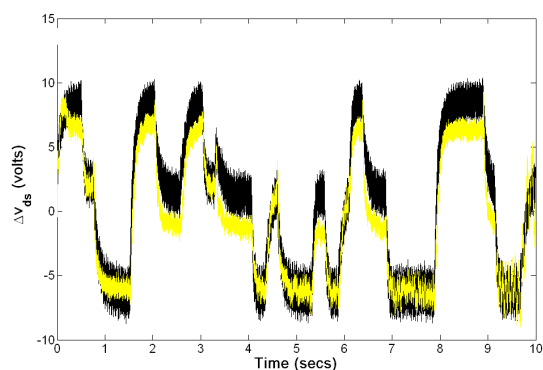


Figure 9: Comparison between the measured and simulated output ( $\Delta v_{ds}$ )

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