Theoretical Foundations of Fuzzy Bi-criterial Approach to Project Cost and Schedule Buffers Sizing

Pawel Błaszczyk, Tomasz Błaszczyk, Maria B. Kania,

Abstract—**The aim of this research was the trial of modelling and optimizing the time-cost trade-offs in project planning problem with taking into account the behavioral impact of performers' (or subcontractors') estimations of basic activity parameters. However, such a model must include quantitative measurements of budget and duration, so we proposed to quantify and minimize the apprehension of their underestimations. The base of the problem description contains both safe and reasonable amounts of work estimations and the influence factors matrix. We assumed also the pricing opportunity of performance improving. Finally we introduce fuzzy measurements for work amount.**

Index Terms—**critical chain, buffer sizing, fuzzy numbers**

I. Introduction

The time-cost trade-off analysis, allowing for establishment of such a project plan which satisfies the decision-maker's expectations for the soonest completion date with as low budget as possible, is one of the basic multicriterial problems in project planning. The first researches in this subject, conducted by Fulkerson [7] and Kelley [12], have been publicated in 1960's. Precise reviews of temporary results were widely described by several authors, for instance by Brucker et al. [4]. The aim of the follow-ing research is to consider the critical chain approach described by Goldratt [8] in multiple-criteria environment. The primal description of the method was based on verbal language, rather than formal. The chain and time buffers quantification methods were the results of successive authors. One of the detailed approaches was formally described by Tukel et al. [18]. The issues of buffering some project characteristics, other than duration, were considered by Leach [13], Gonzalez et al. [9], Błaszczyk and Nowak [1]. The general critical chain approach, widely discussed by various authors (compare Herroelen and Leus [10], Rogalska et al. [16], Van de Vonder et al. [19]), is not drawback-free. So that, the range of its practical implementation is not as wide as the regular CPM and PERT methods. However, the critical chain has an important advantage because of the behavioral aspects inclusion, what can make it more useful in the real-life planning problem descriptions. Thus, by including the impact of the human factor on measurable project features, we are capable of using it to improve these features in return for financial equivalent. An example of such a solution, with using the extraordinary premium fund, was described by Błaszczyk and Nowak in [1]. The following part of the paper is the consequence of continuing this research on buffering different project features. Here we took into consideration the duration and budget expectations by modelling the project with time and cost buffers. Apart from temporary results in the described procedure, we introduced buffers on overestimated amounts of labor which are given by employees or subcontractors. In this paper we introduce fuzzy measure of amount of labor to represent the uncertainty of afford estimations. Fuzzy approach to critical chain modelling has been considered by CHen et al. [6], Long and Ohsato [14], Shi and Gong [17]. The model we are proposing assumes the opportunity to motivate them to participate in the risk of delays and budget overrunning in return for probable profits, in case of faster and cheaper realization.

Pawel Błaszczyk is with the Institute of Mathematics, University of Silesia, Katowice, Poland, email: pblaszcz@math.us.edu.pl
Tomasz Błaszczyk is with Department of Operations Research, University of Economics in Katowice, Katowice, Poland, email: blaszczyk@ue.katowice.pl
Maria B. Kania is with the Institute of Mathematics, University of Silesia, Katowice, Poland, email: mkania@math.us.edu.pl

II. First Mathematical model: cost and time buffers

We consider project which consist $x_1, \ldots, x_n$ activities characterized by cost and time criteria. We assume that only q factors has any influence on the cost and the time of the project. Let us consider the following matrix X:

$$X = \begin{bmatrix} x_{11} & \dots & x_{1q} \\ \vdots & \ddots & \vdots \\ x_{n1} & \dots & x_{nq} \end{bmatrix} \tag{1}$$

Elements of the matrix X equals 0 or 1. If x_{ij} equals 1 it means that factor j has influence on the completion of activity x_i. In the other case there is no influence of factor j on activity x_i. The matrix X we will call the *factor's matrix*. Let

$$K = [k_{ij}]_{i=1,\ldots,n; j=1,\ldots,q} \tag{2}$$

to be the matrix of cost's ratios of all q factors for all activities and

$$W^m = [w_1^m, \ldots, w_n^m] \tag{3}$$

to be the vector of minimal amounts of work for the activities $x_1, \ldots, x_n$. On the basis of matrix X and vector W^m for activity x_i we can calculate the total amount of work w_i by:

$$w_i = f_{w_i}(x_{i1}, \ldots, x_{iq}, w_i^m) \tag{4}$$

where f_{w_i} is a work assigning function. Moreover we assume that there is vector

$$R = [r_1, \ldots, r_q] \tag{5}$$

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

describing the restrictions of accessibility of factors for whole project. Let

$$T = [t_{ij}]_{i=1,\ldots,n;j=1,\ldots,q} \tag{6}$$

be the matrix of amounts of work for each factor in each activity. On the basis of the matrix X, T and K we calculate the cost and the duration of each activity by:

$$k_i = f_{i_k}(x_{i1}, \ldots, x_{iq}, t_{i1}, \ldots, t_{iq}, k_{i1}, \ldots, k_{iq}) \tag{7}$$

and

$$t_i = f_{i_t}(x_{i1}, \ldots, x_{iq}, t_{i1}, \ldots, t_{iq}) \tag{8}$$

where f_{i_k} and f_{i_t} are some functions. We called this functions the *cost* and the *time* functions, respectively. Thus the total cost and the total duration of the project are given by

$$K_c = \sum_{i=1}^{n} k_i \tag{9}$$

and

$$T_c = \max_{i=1,\ldots,n} (ES_i + t_i) \tag{10}$$

where ES_i is the earliest start of activity x_i. Under the following assumptions we minimize total cost of the project. If the functions f_{i_k} and f_{i_t} are linear functions than this optimization problem can be solved by Linear Programming (LP). In typical case the linear programming model is given by

$$c \cdot x \longrightarrow \min \tag{11}$$
$$A \cdot x \leq b \tag{12}$$
$$x \geq 0 \tag{13}$$

where c, x, A, b are coefficient vector of object function, coefficient vector of decision variables, matrix of coefficient of restriction and vetor of absolute terms respectively. In our case we have the following linear programming model

$$\sum_{i=1}^{n} f_{i_k}(x_{i1}, \ldots, x_{iq}, t_{i1}, \ldots, t_{iq}, k_{i1}, \ldots, k_{iq})$$
$$= \sum_{i=1}^{n} k_i \longrightarrow \min \tag{14}$$
$$X^{'} \cdot T \leq R \tag{15}$$
$$X \cdot T^{'} = W \tag{16}$$
$$t_i \geq 0. \tag{17}$$

It leads to find the optimal work assignments for every factor in each activities. From the set of alternate optimal solutions we choose this one, for which the total duration of project is minimal. In this way we obtain the optimal solution in safe case. According to the contractors' safe estimations the amount of work could be overestimated. It leads up to overestimations of the activities' cost and duration expected values and afterwards the total cost and the total duration of the whole project. That means

$$k_i = k_i^e + k_i^B \tag{18}$$

and

$$t_i = t_i^e + t_i^B \tag{19}$$

where k_i^e, t_i^e are the reasonable cost and reasonable duration for activity x_i and k_i^B, t_i^B are the buffers of budget and time for activity x_i, respectively. Therefore we can write the total cost and total duration of project by

$$K_c = K^e + K^B \tag{20}$$

and

$$T_c = T^e + T^B \tag{21}$$

where K^e, T^e are the reasonable cost and reasonable duration of the project and K^B, T^B are the buffers of budget and time, respectively. To set the buffers K^B, T^B up we must estimate the most probable amounts of work. We do that by changing appropriate elements x_{ij} in matrix X from 1 to 0 or vice versa. It means that some factors which had influence on activity x_i in safe estimation case does not have it in real estimation case and vice versa. Than we using the function w_i for each activity x_i. In this way we get the new factor's matrix $X^\star$ and the new vector of amounts of work $W^\star$. Then we execute the same procedure for the most probable amount of work but under additional condition $t_{ij} \geq t_{ij}^\star$ for $i = 1, \ldots, n; j = 1, \ldots, q$, where $T^\star = \left[t_{ij}^\star\right]$ is the matrix of amounts of work for each factor in each activity calculated for the new data. Since that is unlikelihood that all factors will occur, we can reduce the buffers for project by:

$$K_r^B = \alpha K^B \tag{22}$$

and

$$T_r^B = \beta T^B \tag{23}$$

where $\alpha, \beta \in [0, 1]$ are the ratios revising amount of buffers.

$$K^P = K^e + K_r^B \tag{24}$$

and

$$T^P = T^e + T_r^B \tag{25}$$

Part of saved money can generate bonus pool B and be divided between the factors. Let us introduce the weight of importance of activities

$$S = [s_i]_{i=1,\ldots,n}, \tag{26}$$

where $s_i \in [0, 1]$. To share the bonus pool we define function which depends on saved amount of work, importance of activity x_i and if the activity is critical or not and on the reduced buffers of cost and time. In the general case that factor i can receive the amount of money b_i

$$b_i = f_{b_i}(s_i, D_i^W, c, D_B^K, D_B^T) \tag{27}$$

where s_i is the importance of activity x_i, D_i^W is the saved amount of work for activity x_i, $c = 1$ if the activity is on critical path or $c = 0$ if is not on the critical path, D_B^K is the amount of saved cost, D_B^T is the amount of saved time and f_{b_i} is some function. For example we can used the following function

$$b_i = \begin{cases} \frac{s_j}{s^1} \frac{D_j^W}{D^1} \gamma_1 B & \text{if } x_i \text{ is on critical path} \\ \\ \frac{s_j}{s^2} \frac{D_j^W}{D^2} \gamma_2 B & \text{else} \end{cases} \tag{28}$$

where B is the bonus pool $\gamma_2 < \gamma_1$, $\gamma_1 + \gamma_2 = 1$, s^1 is the sum of importances of activities which is on critical path,

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

s^2 is the sum of importances of activities which is not on critical path, D_j^W is the sum of saved amounts of work for activity x_i, D_i^1 is the saved amounts of work for activities which are critical and D_i^2 is the sum saved amounts of work for activities which are beside any critical path.

III. Second Mathematical model: Work amount buffer

In this section we discuss another mathematical model for the project considered above, which was introduced in [3]. Like in the first model we introduce factor's matrix X, matrix of cost's ratios K, vector of minimal amounts of work W^m, vector R describing the restrictions of accessibility of factors and the matrix of amounts of work for each factor in each task T (see (1)-(3), (5),(6), respectively). On the basis of the matrix X, T, K we calculate the cost and the duration of each activity using formulas (7) and (8) and then using formulas (9) and (10) the total cost and total duration of the project. Like in previous model we minimize the total cost of the project. If the functions f_{i_k} and f_{i_t} are linear than this optimization problem can be solved by LP. From the set of alternate optimal solution we choose this one, for which the total duration of project is minimal.

For task x_i the amount of work could be written as

$$w_i = f_{w_i}(x_{i1}, \ldots, x_{iq}, w_i^m) = w_i^e + w_i^B. \tag{29}$$

Therefore we can write the total amount of work of project by

$$W_c = W^e + W^B, \tag{30}$$

where W^e is the reasonable amount of work of the project and W^B is the buffer of amount of work. To set the buffer W^B up we must estimate the most probable amounts of work. We do that by changing appropriate elements in matrix X and using the function w_i for each task x_i. In this way we get the new factor's matrix X^* and the new vector of amounts of work W^*. Since that is unlikelihood that all factors will occur, we can reduce the buffer for project by:

$$W_r^B = [\alpha_1, \ldots, \alpha_n] W^B, \tag{31}$$

where $\alpha \in [0,1]$ for $i \in \{1, \ldots, n\}$ are the ratios revising amount of work for tasks $x_1, \ldots, x_n$. So the total project amount of work is given by

$$W^P = W^e + W_r^B. \tag{32}$$

The overestimation of amount of work leads up to overestimations of the tasks' cost and duration expected values and afterwards the total cost and the total duration of the whole project. Because the amount of work changed the duration and cost of project also changed. Therefore we can write the total cost and total duration of project as in (20) and (21), respectively. Like in previous model part of saved money can generate bonus pool B and be divided between the factors. The weight of importance of tasks S is given by formula (26). The bonus pool for the factor i can be shared by using function (27). Like above we can you the function (28).

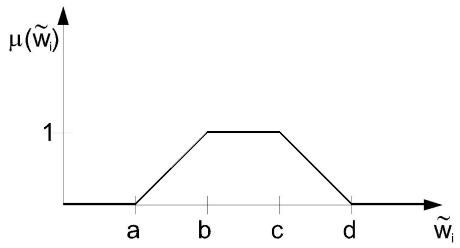

Fig. 1. An example of trapezoid fuzzy number (TrFN)

IV. Fuzzy approach

Usually deterministic values in Classic Linear Programming model does not correspond with real and uncertain conditions expected during project execution. To deal with this problem we propose extention of the models above using fuzzy approach. The proposed method use trapezoidal fuzzy numbers (TrFN). First, let us introduce some basic facts, which we use in our fuzzy model extension.

Definition 1: Let A be a subset in some space X. A fuzzy set A in X is a set of ordered pair

$$(x; \mu_A(x)) \colon x \in X \tag{33}$$

where

$$\mu_A \colon X \to \mathbb{R} \tag{34}$$

is membership function of set A.
For each $x \in A$, $\mu(x)$ is called the grade of membership of x in (A, μ). To define fuzzy number, first we must introduce some basic facts.

Definition 2: The set A is called normal if

$$h(A) = \sup_{x \in X} \mu_A(x) = 1. \tag{35}$$

Definition 3: The set

$$supp(A) = \{x \in A : \mu(x) > 0\} \tag{36}$$

is called the support of (A, m).

Definition 4: Let $\alpha \in [0,1]$. The set

$$A_\alpha = \{x \in X : A(x) \geq \alpha\} \tag{37}$$

is called alpha cut.

Definition 5: Let $X = \mathbb{R}$. A fuzzy number is such fuzzy set $A \in F(\mathbb{R})$ which satisfy following conditions:

1) A is normal set,
2) A^α is closed for each $\alpha \in [0,1]$,
3) $supp(A)$ is bounded,

Definition 6: The trapezoidal fuzzy numbers $TrFN(a,b,c,d)$ (see fig. 1) is a fuzzy number for which the membership function is given by the following formula

$$\mu(x) = \begin{cases} (x-a)/(b-a) & \text{for} \quad x \in [a,b] \\ 1 & \text{for} \quad x \in [b,c] \\ (d-x)/(d-c) & \text{for} \quad x \in [c,d] \\ 0 & \text{for} \quad x \notin [a,d] \end{cases} \tag{38}$$

The membership function μ depends on expert's judgment about availability of factors, workers, materials etc.

ISBN: 978-988-19251-7-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Definition 7: Let $x \in \mathbb{R}$ and $\epsilon \in [0,1]$ be sufficient small. The trapezoid fuzzy number $\widetilde{x}$ is called the fuzzy number close to real number x when is given by:

$$\widetilde{x} = (x-\epsilon, x, x, x+\epsilon) \tag{39}$$

In the hereinafter of this article we denote the fuzzy number close to real number x by $\widehat{x}$ We write that trapezoid fuzzy number $A(a,b,c,d) \geq \delta$, where δ is some real number, if $a \geq \delta$, $A > \delta$, $A \leq \delta$ for $d \leq \delta$ and $A(a,b,c,d) < \delta$ id $d < \delta$. If A, B are two fuzzy subset of set a space X, than $A \leq B$ mean that $A(x) \leq B(x)$ for all $x \in X$, or A is a subset of B, $A < B$ holds when $A(x) < B(x)$ for all x. There is a potential problem with the symbol $\leq$. In this article $A \leq B$ for fuzzy numbers A, B means that A is less than or equal to B.

Definition 8: For two fuzzy numbers the basic four arithmetic operation are given by the following formulas

$$\mu_{B=A_1\oplus A_2}(y) = \sup_{x_1,x_2\in X, y=x_1+x_2} \min\{\mu_A(x_1), \mu_a(x_2)\} \tag{40}$$

$$\mu_{B=A_1\ominus A_2}((y) = \sup_{x_1,x_2\in X, y=x_1-x_2} \min\{\mu_A(x_1), \mu_a(x_2)\} \tag{41}$$

$$\mu_{B=A_1\odot A_2}((y) = \sup_{x_1,x_2\in X, y=x_1\cdot x_2} \min\{\mu_A(x_1), \mu_a(x_2)\} \tag{42}$$

$$\mu_{B=A_1\oslash A_2}((y) = \sup_{x_1,x_2\in X, y=x_1/x_2} \min\{\mu_A(x_1), \mu_a(x_2)\} \tag{43}$$

In all above cases the result is also a fuzzy number, but not necessary trapezoid fuzzy number. In the case when objective functions and and restrictions are given by fuzzy numbers the Fuzzy Linear Programming (FLP) model is given by the following formula

$$\widetilde{c} \cdot x \longrightarrow \min \tag{44}$$

$$\widetilde{A} \cdot x \leq \widetilde{b} \tag{45}$$

$$x \geq 0 \tag{46}$$

where $\widetilde{c}, \widetilde{A}, \widetilde{b}$ are fuzzy coefficient vector of object function, matrix of fuzzy coefficient of restriction and vector of fuzzy numbers respectively.

Theorem 1: Let $\tilde{c_j}, \tilde{a_{ij}}$ be a fuzzy quantities. Than the fuzzy set $\tilde{c_1}x_1 + \ldots + \tilde{c_n}x_n$ and $\tilde{a_1}x_1 + \ldots + \tilde{a_n}x_n$ defined by the extension principle is again fuzzy quantity.
Detailed information about solving fuzzy linear programming can be found in [5], citeJamison, [15].

V. Third Mathematical model: Fuzzy Work amount and fuzzy buffers

In this section we discuss third mathematical model for the project considered above. Like in the first two models we introduce factor's matrix X, matrix of cost's ratios K, vector of minimal amounts of work W^m, vector R describing the restrictions of accessibility of factors and the matrix of amounts of work for each factor in each task T (see (1)-(3), (5), (6), respectively). On the basis of the matrix X, T, K we calculate the cost and the duration of each activity using formulas (7) and (8) and then using formulas (9) and (10) the total cost and total duration of the project. Like in previous model we minimize the total cost of the project. If the functions f_{i_k} and f_{i_t} are linear than this optimization problem can be solved by LP. From the set of alternate optimal solution we choose this one, for which the total duration of project is minimal. Like in the second model, for task x_i, the amount of work could be written using the formula (29). Therefore the total amount of work of project is give by formula (30). To set the buffer W^B up we must estimate the most probable amounts of work. In some cases it could be hard to estimate the amount of work for task x_i and therefore, for some tasks, it could be impossible to set up deterministic value of amount of work. To solve this problem we can used the trapezoid fuzzy numbers described in (38). For the safe estimation the amount of work for task x_i is given by real number. before we estimate the real amount of work we must rewrite this real numbers as a fuzzy number close to real number using definition 7 and the formula (39). Now the amount of work can be write using the following formula

$$\hat{w}_i = \widetilde{w_i}^e + \widetilde{w_i}^B. \tag{47}$$

where $\widehat{w_i}$ is fuzzy number close to real number w_i, $\widetilde{w_i}^e$ fuzzy number describing to real estimation for work amount of the task x_i and $\widetilde{w_i}^B$ is the buffer for the work amount for the task x_i. Therefore we can write the total amount of work of project by

$$\widetilde{W_c} = \widetilde{W^e} + \widetilde{W^B}, \tag{48}$$

where $\widetilde{W^e}$ is the reasonable amount of work of the project and $\widetilde{W^B}$ is the buffer of amount of work. The buffer $\widetilde{W^B}$ is setting by expert's judgment about availability of factors in the matrix X. Under the following assumptions we minimize total cost of the project. If the functions f_{i_k} and f_{i_t} are linear than this optimization problem can be solved by FLP. From the set of alternate optimal solution we choose this one, for which the total duration of project is minimal. Since that is unlikelihood that all factors will occur, we can reduce the buffer for project by:

$$\widetilde{W_r^B} = [\alpha_1, \ldots, \alpha_n]\widetilde{W^B}, \tag{49}$$

where $\alpha \in [0,1]$ for $i \in \{1, \ldots, n\}$ are the ratios revising amount of work for tasks $x_1, \ldots, x_n$. So the total project amount of work is given by

$$\widetilde{W^P} = \widetilde{W^e} + \widetilde{W_r^B}. \tag{50}$$

The overestimation of amount of work leads up to overestimations of the tasks' cost and duration expected values and afterwards the total cost and the total duration of the whole project. Because the amount of work changed the duration and cost of project also changed. Therefore we can write the total cost and total duration of project as

$$K_c = K^e + K^B \tag{51}$$

$$\widetilde{T}_c = \widetilde{T^e} + \widetilde{T^B} \tag{52}$$

where $K^e, \widetilde{T^e}$ are the reasonable cost and reasonable duration of the project and $K^B, \widetilde{T^B}$ are the buffers of budget and time, respectively. The $\widetilde{T^e}$ and $\widetilde{T^B}$ are trapezoid fuzzy number.

Like in previous models part of saved money can generate bonus pool B and be divided between the factors. The weight of importance of tasks S is given by formula (26). The bonus pool for the factor i can be shared by using function (27). Like above we can use the function (28).

VI. Conclusion

According to the authors of the paper, in the project planning issues it is possible to extract the safety buffers hidden in schedule estimations. The results of prior researches indicate that this mechanism is useful in project budgeting processes. The main thesis of our study, stating the existence of required labor overestimations, seems to be justified as well. The theoretical consideration is compliant with the project cost buffering approach, described in [1] in terms of the procedure of buffers sizing and profits distributing. Another extension of the prior approach was done with including the influence matrix describing the hypothetical dependence of resources on several time and cost drivers. The introduction of fuzzy measures allowed us to improve the representation of estimations on the required amounts of labor. It has to be highlighted, though, that proving its efficiency requires further empirical study in real-life conditions. It will be a subject of the next stage of this research.

References

[1] T. Błaszczyk and B. Nowak, "Project costs estimation on the basis of critical chain approach," (in Polish), T. Trzaskalik (ed.): Modelowanie Preferencji a Ryzyko '08, Akademia Ekonomiczna w Katowicach, Katowice, 2008.

[2] P. Błaszczyk, T. Błaszczyk and M. B. Kania, "The bi-criterial approach to project cost and schedule buffers sizing," *Lecture notes in mathematics and economy*, in press.

[3] P. Błaszczyk, T. Błaszczyk and M. B. Kania, "Task duration buffers or work amount buffers?," *The first Earned Value Analysis Conference for the Continental Europe (proceedings)*, vol. 1, pp. 345-375, 2009.

[4] P. Brucker, A. Drexl, R. Möhring, K. Neumann and E. Pesch, "Resource-constrained project scheduling: Notation, classification, models and methods," *European Journal of Operational Research*, vol. 112, pp. 3–41, 1999.

[5] J. J. Buckley, E. Eslami and T. Feuring, "Fuzzy Mathematics in Economy and Engineering," Springer 2002.

[6] L. Chen, F. Liang, S. Xiaoran, Y. Deng and H. Wang, "Fuzzy-Safety-Buffer approach for project buffer sizing considering the requirements from project managers and customers," *Information Management and Engineering (ICIME), 2010 The 2nd IEEE International Conference on*, pp. 482–486, June 2010.

[7] D. R. Fulkerson, "A network flow computation for project cost curves," *Management Science*, vol. 7, pp. 167–178, 1961.

[8] E. Goldratt, "Critical Chain," North River Press, 1997.

[9] V. Gonzalez, L. F. Alarcon and K. Molenaar, "Multiobjective design of Work-In-Process buffer for scheduling repetitive projects," *Automation in Construction*, vol. 18, pp. 95–108, 2009.

[10] W. Herroelen and R. Leus, "On the merits and pitfalls of critical chain scheduling," *Journal of Operations Management*, vol. 19, pp. 559–577, 2009.

[11] K. D. Jamison and W. A. Lodwick, "Fuzzy linear programming using a penalty method," *Fuzzy Sets and Systems* vol. 119pp.97–110, 2001

[12] J. E. Kelley, "Critical-path planning and scheduling: Mathematical basis," *Operations Research*, vol. 9, pp. 296–320, 1961.

[13] L. Leach, "Schedule and cost buffer sizing: how account for the bias between project performance and your model," *Project Management Journal*, vol. 34, pp. 34–47, 2003.

[14] L. D. Long and A. Ohsato, "Fuzzy critical method for project schedulling under resource constraints and uncertainty," *International Journal of Project Management*, vol. 26, pp. 688-698, 2008.

[15] J. Ramik, "Duality in fuzzy linear programming with possibility and necessity relations," *Fuzzy Sets and Systems* 157 pp.1283-1302, 2006.

[16] M. Rogalska, W. Bozejko and Z. Hejducki, "Time/cost optimization using hybrid evolutionary algorithm in construction project scheduling," *Automation in Construction*, vol. 18, pp. 24–31, 2008.

[17] Q. Shi and T. Gong, "An Improved Project Buffer Sizing Approach to Critical Chain Management Under Resources Constraints and Fuzzy Uncertainty," *Artificial Intelligence and Computational Intelligence*, 2009. AICI '09. International Conference on, pp. 486–490, January 2010.

[18] O. I. Tukel, W. O. Rom and S. D. Eksioglu, "An investigation of buffer sizing techniques in critical chain scheduling," *European Journal of Operational Research*, vol. 172, pp. 401–416, 2006.

[19] S. Van de Vonder, E. Demeulemeester, W. Herroelen and R. Leus, "The use of buffers in project management: The trade-off between stability and makespan," *International Journal of Production Economics*, vol. 97, pp. 227–240, 2005.