The Modification of EOQ Model under the Spare Parts Discrete Demand: A Case Study of Slow Moving Items

Sakon Wongmongkolrit, and Bordin Rassameethes

***Abstract*—Generally, EOQ inventory model is well known as an approach using for inventory control and spare parts stocking policy. However, EOQ model is basically thought on the basis of continuous demand. But for the discrete demand, EOQ model may not perform. This research study will be used for explanation about the constraints of EOQ model with discrete demand or slow moving items. According to this study, the modification of EOQ model is originally studied by based on spare parts discrete demand. This is the study of forming the extension of EOQ model conforming to discrete demand. In addition, the modification of EOQ model will be proved in according to test against real equipment. Hopefully, this study will be used for fulfillment a little niche of EOQ model. And it will be the new direction and useful for all manufacturers exploiting this model as an inventory management policy**.

***Index Terms*— discrete demand, spare parts, slow moving items, EOQ model, inventory cost**

I. Introduction

The order size that minimizes the total inventory cost is known as the Economic Order Quantity (EOQ). The classical inventory model assumes the idealized situation shown as Fig.1, where Q is an order size. Upon receipt of an order, the inventory level is Q units. Units are withdrawn from inventory at a constant demand rate, which is represented by the negative sloping lines. When the inventory reaches the reorder point (*ROP*), a new order is placed for Q units. After a fixed time period, the order is received all at once and placed into inventory. The vertical lines indicate the receipt of a lot into inventory. The new lot is received just as the inventory level reaches zero, so the average is $Q/2$ units [1].

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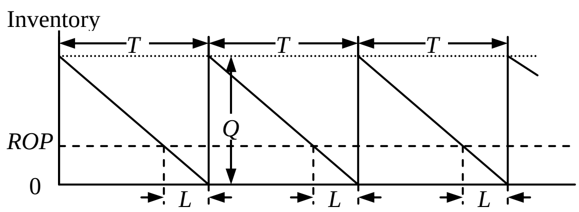


Fig. 1. Classical inventory model concept.
Source: adapted from [1].

According to Fig.1, this is the classical inventory model concept. The optimal lot size Q^* is concerned in order to minimize the total inventory cost. To obtain the minimum total cost, the lot size Q^* is equal to:

$$Q^* = \sqrt{\frac{2SD}{CI}} \tag{1}$$

Where: S is ordering cost per order.
D is annual demand in units.
C is part unit cost.
I is annual holding cost as a fraction of unit cost.

Indeed, the lot sizing techniques is developed for continuous and independent demand items such EOQ assume that demand occurs with certainty as a constant rate, whilst discrete demand occurs at discrete intervals or points in time rather than continuously over a time horizon. Demand requirements are usually time-phased in equal time increments over a finite time horizon. Uneven or lumpy demand requirements occurring over a finite time horizon complicate the lot sizing decision such as the utilization of spare parts consumptions or spare parts demand [1].

II. Relevant Literatures and Relate works

In area of inventory control study, several works and case studies in literatures on the inventory management decisions were applied EOQ model such as [2-6]. Almost of previous studies of EOQ model is possessed by continuous demand. Whilst, the studies of discrete demand are almost conformed to MRP approach such as Lot-for-Lot ordering [7], Wagner-Within algorithm [8], Least Period Cost model [9], Least Unit Cost model [10], Silver-Meal algorithm [11], etc.

In general, EOQ model is developed for continuous demand and it is never thought by based on entire discrete demand. However, a lot of previous studies of materials control for maintenance actions are still applied the EOQ model by the assumption that spare parts demand is continuous pattern such as [12-15]. Because of EOQ model is simple and easy to understand then it is still popular to use than the other approaches which are the complicated model with difficulty using or hard to implement. Actually, almost of spare parts consumptions look quite similar to discrete pattern rather than continuous pattern.

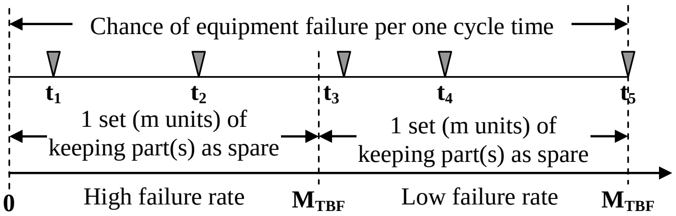
According to this study, the modification for EOQ model is originally thought by based on the entire discrete demand. A case study will be reflected to real equipment. The followings details are the explanation about these matters. Additionally, the independent demand items will be also considered in order to meet it variations. Then, this is our believing that is a cognitive thinking to establish the original context to be sustained the extension of EOQ model.

III. Occurrence of Spare part demand

Firstly, the cognitive thinking about spare parts demand should be considered in term of equipment failure rate and mean time between failures.

A. Failure occurrences

Equipment (or part, device) failure will be happen on anytime, and it is to be the probabilistic pattern under the exact standard deviation (σ) with average time of failure occurrence (or mean time between failure: M_{TBF}).



t_t is represented as probability of equipment failure

Fig. 2. Occurrences of equipment failure

According to Fig.2, equipment (part) may probably be failure on anytime. The failure pattern is probabilistic. So, time between failures at time t ($TBFt$) is equal to average time of failure occurrences plus failure variation as:

$$TBF_t = M_{TBF} \pm \varphi\sigma \qquad 2(a)$$

Where: φ is multiplication number of standard deviation.

If time between failures is falling into low failure rate period, then time between failures is:

$$TBFt = M_{TBF} + \varphi\sigma \qquad 2(b)$$

If time between failures is falling into high failure rate period, then time between failures is:

$$TBFt = M_{TBF} - \varphi\sigma \qquad 2(c)$$

If there are k identical parts in the system then mean time between failure of the system (θ) is become to be:

$$\theta = \frac{M_{TBF}}{k} \qquad (3)$$

And, time between failures of system (TBF_S) is:

$$TBF_S = \theta \pm \varphi\sigma_S \qquad (4)$$

Where: σ_S is failure standard deviation of the system.

B. Demand of spare parts

Spare parts demand (D) comes from equipment failure. It can be measured as frequent times of failure occurrences within annual that is called as failure rate (λ), and equal to:

$$D = \lambda = 1/M_{TBF} \qquad (5)$$

If there are m units of equipment (or parts) which are simultaneously changed within same time such as changing of 2 batteries, or double driving belt are simultaneously changed. So, demand of spare parts is:

$$D = m\cdot\lambda \qquad (6a)$$

If there are k identical equipment installed in the system then (6a) can be re-equated as:

$$D = m\cdot k\cdot\lambda = \frac{m}{\theta} \qquad (6c)$$

C. The discrete demand

Basically, slow moving item (or discrete demand) can be implied as non-continuous pattern. Or, parts (devices) are scarcely used. Indeed, the exact separation between discrete and continuous pattern (of demand) is not surely defined. But there are too many text books, articles, case studies which were given the definitions of discrete demand in many different types or several formats such as parts which are consumed less than one unit within an annual [16-17], items withdrawal less than one item in a quarter [18], part has been issued or sold at least one within last one year [19], an item where the mean time between demands is much longer than ten times of average lead time [20]. However, our point of view is to go along with the definition of [20]. Or, the equation can be shown as: M_{TBF}/k (or θ) > 10·L.

IV. Inventory Model

A. General EOQ model

According to Fig.1 and (1), purchasing quantities (Q^*) of EOQ model can be rewritten in term of failure rate as:

$$Q^* = \sqrt{\frac{2Smk\lambda}{CI}} \qquad (7)$$

Where: λ is annual failure rate.
m is equipment with simultaneous changed.
k is amount of identical equipment in a system.

The Re-Order Point (ROP) is ordered at equipment (or part/device) delivery lead time (L). And, total inventory cost composes of spare part ordering cost and holding cost:

$$TC_{EOQ} = \text{Ordering cost} + \text{Holding cost}$$

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$$TC_{EOQ} = S\left(\frac{mk\lambda}{Q^*}\right) + CI\left(\frac{Q^*}{2}\right) \tag{8}$$

B. The Extension of EOQ model

Basically, if spare parts demand is the discrete pattern, then general EOQ model may not perform. Supposedly, there are 4 sets of circuit breaker installed in the system with M_{TBF} of each breaker is 20 years. So, annual demand (or failure rate) is 0.2 sets. Whilst, unit price of circuit breaker is 30,000 Baht, ordering cost is 10,000 Baht, and fraction of holding cost is 4 percent. Thus, purchasing quantity is equal to 2 sets (if it is calculated by using general EOQ model). For this case, every ten years will purchase 2 sets of breaker (or one set of circuit breaker have been used for every five years). Recall to (8), total inventory cost is:

$$TC_{EOQ} = S\left(\frac{mk\lambda}{Q^*}\right) + CI\left(\frac{Q^*}{2}\right)$$

$$= 1{,}000 + 1{,}200 = 2{,}200 \text{ Baht per year}$$

Or: TC_{EOQ} = 44,000 Baht per 20 years (or M_{TBF})

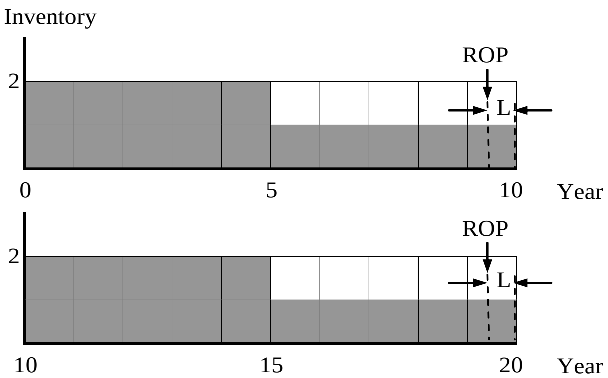

Fig. 3. Consideration for Holding cost

According to Fig.3, each gray block is represented by parts stocking (or holding cost) which is equal to material or spare part unit cost (C) multiplying by holding cost fraction (I). Thus, each block is equal to 30,000*0.04 = 1,200 Baht. There are totally 30 blocks, so total holding cost is 36,000 Baht per 20 years (M_{TBF}). During 20 years, circuit breakers are equally purchased by 2 times (year 0, and the end of year 10). Then, ordering cost is equal to 2*10,000 = 20,000 Baht (for 20 years). Consequently, total inventory cost is 56,000 Baht. This value is not as same as the previous.

If n is defined as the optimal purchasing quantity. So, the extension EOQ model can be shown as the followings:

Total Cost = Ordering Cost + Holding Cost

$$TC = S\frac{k}{n} + CI \cdot m \cdot \left(\sum_{i=1}^{n} i\right) \cdot \left(\frac{M_{TBF}}{n}\right)$$

$$= S\frac{k}{n} + CI \cdot m\left[\left(\frac{n}{2}\right)\cdot(n+1)\right]\cdot\left(\frac{M_{TBF}}{n}\right)$$

$$= S\frac{k}{n} + CI \cdot m \cdot \left(\frac{n+1}{2}\right)\cdot M_{TBF} \tag{9}$$

DWT TC to n: $\frac{dTC}{dn}$ and set it as zero, then:

$$\frac{dTC}{dn} = -\frac{Sk}{n^2} + CI \cdot m\left(\frac{M_{TBF}}{2}\right) = 0$$

Thus: $$n = \sqrt{\frac{2Sk}{C\,Im\,M_{TBF}}} = \sqrt{\frac{2S}{C\,Im\,\theta}} \tag{10}$$

And: $$p = j*m \tag{11}$$

Where: p is actual purchasing quantity.
j is the rounding number of n,
Or: j is actual purchasing lot-size (of m units)

According to (9), ordering cost is 20,000 Baht, holding cost is 36,000 Baht, and total inventory cost (to have circuit breaker as spare part) is 56,000 Baht.

C. The modification for the extension of EOQ model

Practically, the exact time of equipment failure cannot be surely known. Then, new part(s) will be replaced when the existing part(s) is breakdown. And the average purchasing time is still equal to mean time between failures.

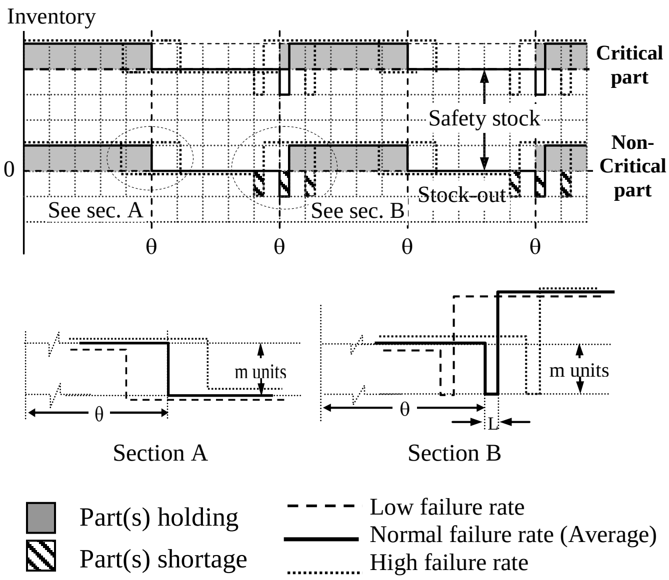

Fig. 4. Consideration for Equipment failure

Regarding to EOQ model, spare parts can be separated into 2 groups as critical part and non-critical part. The critical part is an important part in system, and should have the safety stock for ensuring that part(s) can be in hand at all. Otherwise, non-critical part is an unimportant part in system, and parts can allow being shortage (no safety stock).

According to Fig.4, new order (replenishment) can be delayed until spare part(s) is empty (for non-critical part) and new order will be purchased at period after spare part(s) stocked-out. Or, new order will be purchased when spare parts are touched safety stock (for critical part) or period after safety stock is begun to use. By this way, total holding cost will be reduced at all.

Recall to (9), if it is considered by applying Fig.(4), so this equation can be rewritten as:

$$TC_{EOQ} = S\left(\frac{k}{n}\right) + C\,Im \cdot \left(\frac{n-1}{2}\right) \cdot M_{TBF} \quad (12)$$

Note: The explanations and example case study for (10) and (12) are shown in appendix 1.

DWT TC_{EOQ} to n and set it as zero.

Then: $$\frac{dTC_{EOQ}}{dn} = -S\frac{k}{n^2} + \frac{CI}{2} m M_{TBF} = 0$$

$$n = \sqrt{\frac{2Sk}{C\,Im\,M_{TBF}}} = \sqrt{\frac{2S}{C\,Im\,\theta}}$$

Refer to (12), this equation will be true when spare part lead time is very short (or assumed to be zero) if compare with system mean time between failure, (Or $L << \theta$). And, purchasing lots (or n) must be over than one ($n > 1$). So, lot size of purchasing is same as previous but total cost is not same. Particularly, term of holding cost is always reduced, because part can be allowed being shortage (for non-critical part), or safety stock is firstly used (for critical part).

The purchasing quantity is referred as (10). Safety stock is issued by based on failure variation, and it must conform to discrete pattern then Poisson distribution is applied. Hence, the variation of equipment failure on lead time is:

$$P_x(t = L) = \frac{(\lambda_S \cdot L)^x}{x!} e^{-\lambda_S \cdot L} \quad (13)$$

For this case, safety stock (SS) is always based on failure rate during lead time which is equal to parts usage during lead time plus its variation that is equal to:

$$SS = m \cdot (1 + Px(t = L)) \quad (14)$$

Where: SS is safety stock.
Px is probability of occurrences.
λ_S is system failure rate.
x is probability of occurrences.
L is equipment delivery lead time.

Note: The example equipment and numerical result is shown in next section.

Whilst, Re-Order Point (ROP) for this case is equal to safety stock, because this concept is thought by based on parts borrowing from safety stock. And after replenishment, parts will be refilled back to the safety stock.

V. Application

Indeed, the most important useful of this study is "How can this concept be applied to spare parts inventory management?", and this concept can be used for reducing the complicated thinking about previous studies. Therefore, the contributions to knowledge of this study are:

1. The Re-Order Point and Purchasing Quantity
2. The Safety Stock concept.

These two contributions to knowledge can be illustrated as the followings:

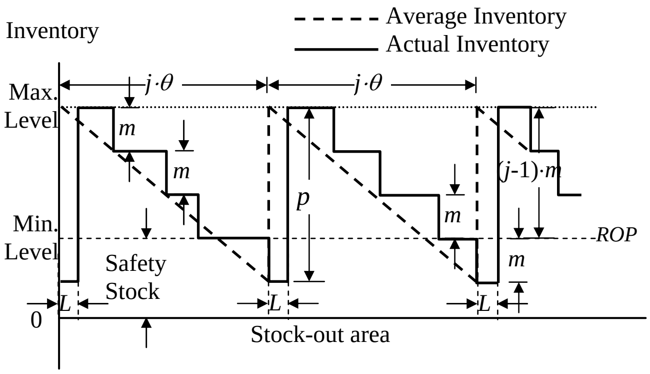

Fig. 5. Extension of EOQ model for critical part

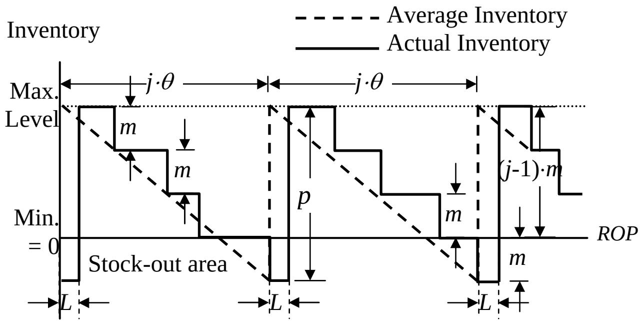

Fig. 6. Extension EOQ model for Non-Critical part

Regarding to Fig.5 and Fig.6, there are two cases for considerations. The first case is inventory management for critical part and the second is inventory management for non-critical part. For the critical part, safety stock is applied as it buffer, while non-critical part is not (or parts can be shortage). Both of them are still applied with concept of Extension EOQ model (which is shown in previous section).

VI. Numerical Result

The case study is spare parts for automatic baggage sorting machine (which is called as Tilt Tray Sorter) of the Baggage Handling System in Suvarnabhumi International Airport, Bangkok, Thailand. Three equipment are the examples as Auxiliary Switch 1no/1nc (for circuit breaker), Battery (for PLC), and under voltage coil (230VAC). The automatic baggage sorting machine composes of 4 closed loops of Tilt Tray Sorter. Whole system composes of 12 pieces of auxiliary switch, 4 sets of PLC (each PLC needs 8 cells of battery for backup), and 8 sets of under voltage coil. The equipment details with variables are shown in table 1.

TABLE I
Equipment details and Variables

Variable	Unit	Equipment		
		Auxiliary contact	*Battery (for PLC)*	*Under voltage*
Unit price (C)	Baht	975	750	3,250
Amount of parts in the system	unit	12 eachs	32 cells	8 sets
Number of identical parts (k)	unit	4	4	4
Usage per each changing (m)	unit	3	8	2
Lead time (L)	day	20	3	28
Mean time between failure (M_{TBF})	year	10	5	10
System mean time between failure (θ)	days	304	57	456
Ordering Cost (S)	Baht	1,500	250	2,000
Fraction of Holding Cost (I)	% of unit cost	3%	15%	10%

Note: Currency is 30 Thai Baht per 1 US$ (on June, 2011).

TABLE II
The safety stock for each equipment

Equipment	System failure rate	No.of Failure occurences during lead time	Probability of occurences	Cumulative or Service level	Safety stock (ea)
Auxiliary contact 1no+1nc	0.400	0	0.9780	0.9780	3
		1	0.0217	0.9998	6
		2	0.0002	1.0000	9
		3	0.0000	1.0000	12
		4	0.0000	1.0000	15
Battery (for PLC)	0.800	0	0.9934	0.9934	8
		1	0.0066	1.0000	16
		2	0.0000	1.0000	24
		3	0.0000	1.0000	32
		4	0.0000	1.0000	40
Under voltage coil (230 VAC)	0.400	0	0.9694	0.9694	2
		1	0.0302	0.9995	4
		2	0.0005	1.0000	6
		3	0.0000	1.0000	8
		4	0.0000	1.0000	10

Regarding to table 2, safety stock is calculated by using (14). For this case, if the required service level is supposedly equal to 100 percent then the gray stripe is represented the safety stock for each equipment.

TABLE III
Summary of Inventory control

Result / Equipment	Optimal Lot Size (*n*)	Actual Lot Size (*j*)	Actual Purchasing (*p*)	Safety Stock (*SS*)	Re-Order Point (*ROP*)
Auxiliary contact 1no+1nc	3.698	4	12	9	when parts thouch *SS*
Battery (for PLC)	0.667	1	8	16	when parts thouch *SS*
Under voltage coil	1.569	2	4	6	when parts thouch *SS*

Table 3 is represented the inventory management for each equipment. All parts are supposedly to be the critical part then safety stock is issued in order to meet failure variation during lead time (see table 2).

APPENDIX

Firstly, the thinking about the lowest total inventory cost for general EOQ model and extension EOQ model will be issued in term of the comparison between both models. The example equipment is the under voltage coil which can be shown as:

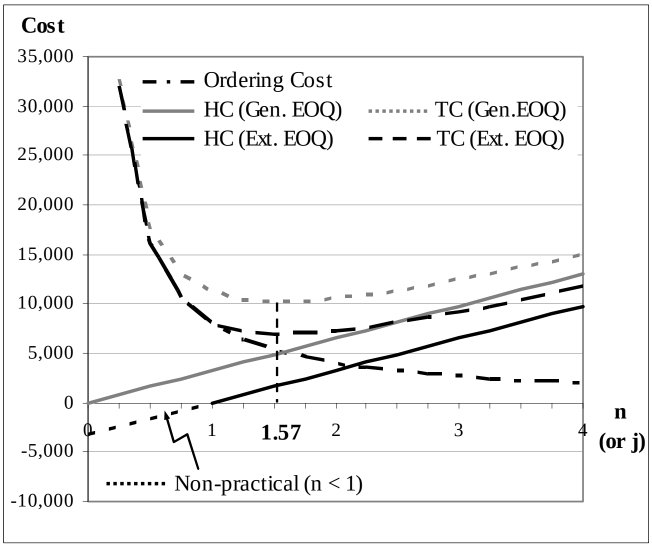

Fig.7. Plotting of Ordering cost, Holding cost, and Total inventory cost (for under voltage coil equipment) against n lots for EOQ model.

According to Fig.7, this is the plotting of under voltage coil. The ordering cost, holding cost, and total cost are plotted against *n* lots (or *j*). Full gray line is represented the plotting of holding cost, and dot gray line is represented the plotting of total cost for general EOQ model. Whilst, full black line is represented the plotting of holding cost, and long dot black line is represented the plotting of total cost for extension EOQ model. And short dot black line is represented the plotting of ordering cost (It is the same value for both of general EOQ model and extension EOQ model). The lowest total cost of general EOQ model is same point as extension EOQ model. At the lowest total cost for

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extension EOQ model, the holding cost is not equal to ordering cost.

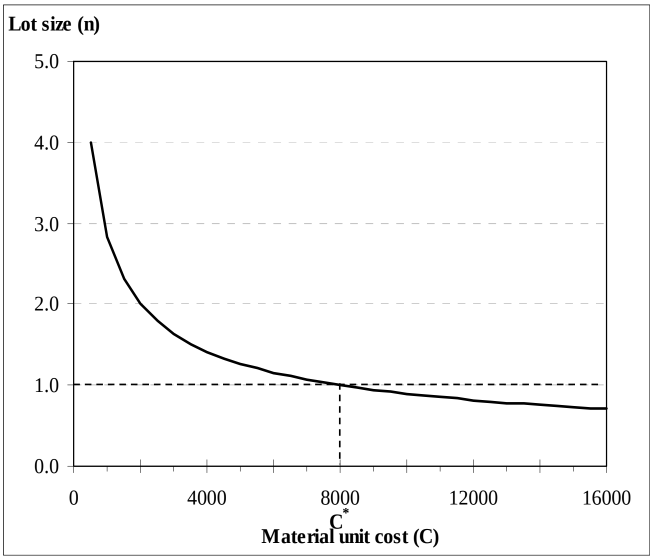

Fig. 8. Simulation of Lot sizing against Material unit cost.

Fig.8 is used for confirmation about the material unit cost which is more than the break-even point, thus it will let the optimal lot size (*n*) to be less than one.

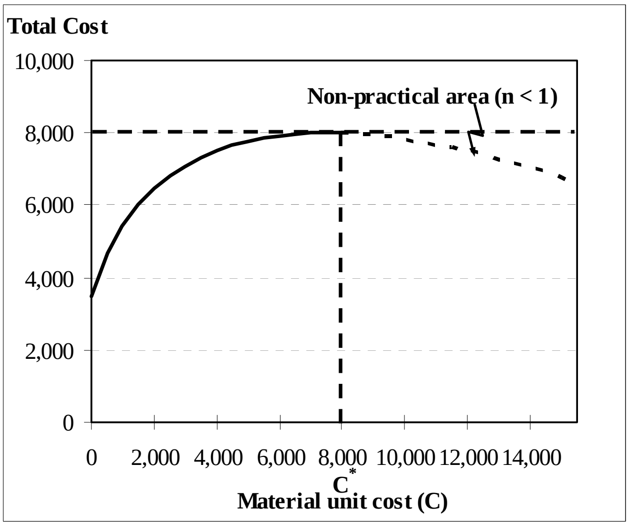

Fig.9. Simulation of Total cost against Material unit cost.

Fig.9 is used for explanation about the curve of total cost is turning back (or diminishing return curve) after passed the break-even point material (spare part) unit cost is more than C^* (for this case is equal to 8,000 Baht).

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REFERENCES

[1] R.J. Tersine, *Principle of Inventory and Materials Management*, 4th ed., PTR Prentice Hall, Englewood Cliff, NJ, Ch.4, pp.177-203, 1994.

[2] A.A. Kranenburg. 2006. *Inventory control under system availability constraints*. A Ph.D dissertation, Technische Universiteit, Eindhoven, Netherland.

[3] Z. Wen-Yong, X. Ying, and S. Bing, “Study on pare parts inventory control by quantitative analysis in the environment of erp system,” *Proc. of the 2011 Int. Conf. of Business Management and Electronic Information (BMEI)*, pp. 259 - 263, 2011.

[4] A. K. Pal and B. Mandal, “An EOQ model for deteriorating inventory with alternating demand rates,” *Journal of Applied Mathematics and Computing*, Vol. 4(2), pp.397-407, 1997.

[5] R. Begum, “An EOQ Model for Deteriorating items with Weibull distribution deterioration, unit production cost with quadratic demand,” *J. Applied Mathematical Sci.*, Vol.4(6), pp.271–288, 2010

[6] E.Porras, and R. Dekker, “An inventory control system for spare parts at a refinery: An empirical comparison of different re-order point methods,” *Eur. J. Oper. Res., doi:10.1016/j.ejor.2006.11.008.*, 2007.

[7] M. Omar, and S.S. Supadi, “A Lot-for-Lot Model with Multiple Instalments for a Production System under Time-Varying Demand Process,” *Journal Matematika*, 2003, Jilid 19, Bil. 2, pp. 101–106.

[8] S.J. Sadjadi, M.B.Gh. Aryanezhad, and H.A. Sadeghi, “An Improved WAGNER-WHITIN Algorithm,” *International Journal of Industrial Engineering & Production Research*, pp. 117-123, 2009.

[9] J.C. Ho, Y.L. Chang, and A.O. Solis, “Two modifications of the least cost per period heuristic for dynamic lot sizing,” *Jounal of Operation Research Society*, vol. 57(8), pp. 1005-1013, 2006.

[10] D.Louit, R. Pascual, D.Banjevic, and A.K.S. Jardine, “Dynamic lot sizing with product returns and remanufacturing,” *Int. J. of Production Research*, Vol.44(20), pp. 4377-4400(24), 2006.

[11] M. Omar, and M. M. Deris, “The Silver-Meal Heuristic Method for Deterministic Time-Varying Demand,” *Journal of Matematika*, Vol.17(1), pp. 7–14, 2001.

[12] W. Cheangkul, K. Damrongrat, and D. Muangman, *Reliability based maintenance management*. SE-Education PCL, Bangkok, Thailand, ch.9, pp. 233 – 271, 2010.

[13] S. Suppamongkol, *Spare parts optimization*. SE-Education Public Company Limited, Thailand, ch.10, pp.111-132, 2004.

[14] W. Cheangkul, and K. Damrongrat, *Maintenance the Profit Maker*. SE-Education PCL, Bsngkon, Thailand, ch.5, pp.82-90, 2003.

[15] M. Sakaguchi, and Ma. Kodama, “Sensitivity analysis of an economic order quantity for dynamic inventory models with discrete demand,” *Int. J. of Manufacturing Technology and Management*, Vol. 18(4), pp. 438–452, 2009.

[16] S. Siasiriwattana, W. Cheangkul, and K. Damrongrat, *Efficiency of Maintenance*. SE-Education PCL,Thailand, ch.16, pp. 313–350, 2006.

[17] L.Pintelon, L.Gelders, and F.V.Puyvelde. *Maintenance Management*. Acco Leuven/Amersfoort Belgium, ch.8, pp.143-167, 1997.

[18] P. Gopalakrishnan, *Handbook of Materials Management*. Prentice-Hall of India Private Limited,New Delhi, Ch.29, pp.164-169, 2005.

[19] P. Gopalakrishnan, and A.K. Banerji, *Maintenanc and Spare parts management*. Prentice-Hall of India Private Limited, New Delhi, Ch.30, pp. 295-301, 2006.

[20] N.A.J Hastings, *Physical Asset Management*. Springer-Verlag London Limited. New York, Ch.22, pp.295-310, 2009.