A Beamspace DOA Estimation Algorithm for Non-circular Signals

Wentao Shi, Jianguo Huang, Chengbing He and Yunshan Hou

Abstract—**Beamspace Root-MUSIC (BM-RTMUSIC) algorithm cannot resolve coherent sources, and the use of spatial smoothing pre-process leads to array aperture loss and reduction in detectable source numbers, a new beamspace conjugate Root-MUSIC (BMC-RTMUSIC) algorithm is proposed to estimate the Direction-Of-Arrival (DOA) for non-circular signals. The proposed method employs the properties of non-circular signal to construct a conjugate symmetric Toeplitz matrix from received data vectors, and forms the pseudo covariance matrix. It then estimates the source DOAs through the techniques similar to BM-RTMUSIC method. The proposed method does not require forward/backward spatial smoothing for the covariance matrix, hence decreasing the computational load and time. Simulation results show that the proposed method can resolve coherent and noncoherent sources using few snapshots and its performance is better than the BM-RTMUSIC method, especially in low SNR.**

Index Terms—**Beamspace Root-MUSIC (BM-RTMUSIC), Direction-Of-Arrival (DOA), non-circular signals, conjugate symmetric Toeplitz matrix, pseudo covariance matrix**

I. INTRODUCTION

In the last several decades, the high resolution direction of arrival (DOA) estimation methods based on beamspace has been a focus in radar, sonar and etc [1][2][3][4]. Compared with corresponding array-space DOA estimation methods, the beamspace methods have higher estimation precision, lower computational complexity, lower resolution threshold and more robustness performance, which leads to good application prospects in practice. However, the performance of these methods will degrade rapidly in the case of low SNR, few snapshots and too many incident signals.

Nowadays, lots of systems deal with non-circular incoming signals, as in telecommunication or satellite systems where amplitude modulation (AM) or binary phase shift keying (BPSK) modulated signals are often used. The non-circular incoming signals have been investigated in various literatures [5][6][7]. Many researcher have been exploiting to use the characteristics of non-circular signals to enhace the DOA estimation performance. Some high resolution DOA estimation methods like Conjugate MUSIC and C-SPRIT for non-circular were proposed in succession [8][9][10]. These methods can increase the number of the detectable targets and enhance the resolution and precision.

In this paper, we extend the conjugate Root MUSIC method proposed by Amjad Salameh[11] into beamspace bearing estimation, named BMC-RTMUSIC method. The proposed method employs the properties of non-circular signal to construct a conjugate symmetric Toeplitz matrix from received data vectors, and forms the pseudo covariance matrix. Then, it estimates the source DOAs through the techniques similar to BM-RTMUSIC method. It does not require forward/backward spatial smoothing for the covariance matrix, and estimates the DOAs of coherent sources using few snapshots. The maximum number of targets to be detected and located by the array can be increased to $M-1$ where M is the number of sensor. The proposed method can reduce the computational complexity while enjoying performance better than beamspace Root MUSIC (BM-RTMUSIC), especially in the low SNR.

The paper is organized as follows. In section II, the signal model is formulated. Section III proposes the novel method and describes the detail of BMC-RTMUSIC. Section VI presents simulation results and compare the performance of our proposed method with that of traditional beamspace Root MUSIC. The conclusion is given in Section V.

II. SIGNAL MODEL

Consider a uniform linear array(ULA) with M sensors. Assume that there are D point sources in the far field of this array with DOAs $\theta_d (d = 1, 2, \cdots, D)$. Then, the received signal vector can be expressed as

$$Y(t) = \sum_{d=1}^{D} a(\theta_d) s_d(t) + v(t) \tag{1}$$

where $s_d(t)$ is the signal from the d-th source with DOA equal to θ_d, $a(\theta_d)$ represents the array response vector, and $v(t)$ denotes the additional white Gaussian noise vector, which is zero mean with variance equal to σ^2.

We rewrite (1) in matrix form as

$$Y(t) = A(\theta)S(t) + V(t) \tag{2}$$

where

$$A(\theta) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_D)] \tag{3}$$

The array response vector can be expressed as

$$a(\theta_d) = \left(1, \psi_d, \psi_d^2, \cdots, \psi_d^{M-1}\right)^{\mathrm{T}}, \quad d = 1, 2, \cdots, D \tag{4}$$

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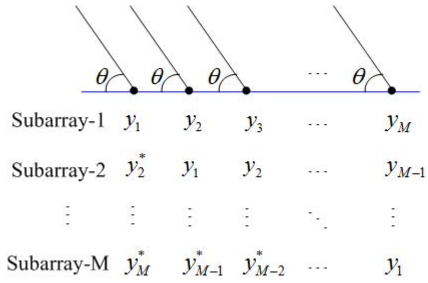

Fig. 1. The inputs to subarrays

where $(\cdot)^{\mathrm{T}}$ stands for the matrix transpose operator, and

$$\psi_d = \exp\left(\frac{-j2\pi\Delta\cos\theta_{\mathrm{d}}}{\lambda}\right) \tag{5}$$

Δ is the inter-element spacing of the ULA and λ is the wavelength of the signal. The covariance matrix of the received signal is defined as

$$R = E\left[Y(t)Y^{\mathrm{H}}(t)\right] \tag{6}$$

which can be estimated by

$$\hat{R} = \frac{1}{N}\sum_{n=1}^{N} Y(t)Y^{\mathrm{H}}(t) \tag{7}$$

where $(\cdot)^{\mathrm{H}}$ represents the matrix conjugate transpose operator, N is the number of the snapshots.

III. The proposed method

In the proposed method, the measurement is taken from all of the array elements and put into a column vector. Figure 1 shows the inputs to M subarrays. For instance, the puts to the subarray processing 1 are $(y_1, y_2, y_3, \cdots, y_M)$, the subarray processing 2 are $(y_2^*, y_1, y_2, \cdots, y_{M-1})$, and the subarray processing M are $(y_M^*, y_{M-1}^*, y_{M-2}^*, \cdots, y_1)$, $(\cdot)^*$ stands for the conjugate operater. In BMC-RTMUSIC, each subarray uses the maximum number of array elements equal to M.

The received vector of subarray-1 can be written as

$$Y_1(t) = [y_1(t), y_2(t), \cdots, y_M(t)]^{\mathrm{T}} \tag{8}$$

In this case, (8) can be written as

$$Y_1(t) = A(\theta)S(t) + V_1(t) \tag{9}$$

where

$$V_1(t) = [v_1(t), v_2(t), \cdots, v_M(t)]^{\mathrm{T}} \tag{10}$$

The definition of $V_1(t)$ is the same as $V(t)$ of Section II.

The received vector of subarray-2 can be written as

$$Y_2(t) = [y_2^*(t), y_1(t), \cdots, y_{M-1}(t)]^{\mathrm{T}} \tag{11}$$

For non circular signals, $s_d^*(t) = s_d(t)$, and (11) can be expressed as

$$Y_2(t) = A(\theta)\Psi^* S(t) + V_2(t) \tag{12}$$

where

$$\Psi^* = diag\left(\psi_1^*, \psi_2^*, \cdots, \psi_D^*\right) \tag{13}$$

$$V_2(t) = [v_2^*(t), v_1(t), \cdots, v_{M-1}(t)]^{\mathrm{T}} \tag{14}$$

Similarly, the received vector of subarray-3 can be written as

$$Y_3(t) = [y_3^*(t), y_2^*(t), \cdots, y_{M-2}(t)]^{\mathrm{T}} \tag{15}$$

and

$$Y_3(t) = A(\theta)(\Psi^*)^2 S(t) + V_3(t) \tag{16}$$

where

$$V_3(t) = [v_3^*(t), v_2^*(t), \cdots, v_{M-2}(t)]^{\mathrm{T}} \tag{17}$$

Finally, the received signal of subarray-M can be written as

$$Y_M = \left[y_M^*, y_{M-1}^*, \cdots, y_1\right]^{\mathrm{T}} \tag{18}$$

$$Y_M(t) = A(\theta)(\Psi^*)^{M-1} S(t) + V_M(t) \tag{19}$$

where

$$V_M(t) = [v_M^*(t), v_{M-1}(t)^*, \cdots, v_1(t)]^{\mathrm{T}} \tag{20}$$

Hence, the output data $Y_{out}(t)$ is

$$\begin{aligned} Y_{out}(t) &= [Y_1(t), Y_2(t), Y_3(t), \cdots, Y_M(t)] \\ &= \begin{bmatrix} y_1(t) & y_2^*(t) & y_3^*(t) & \cdots & y_M^*(t) \\ y_2(t) & y_1(t) & y_2^*(t) & \cdots & y_{M-1}^*(t) \\ y_3(t) & y_2(t) & y_1(t) & \cdots & y_{M-2}^*(t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_M(t) & y_{M-1}(t) & y_{M-2}(t) & \cdots & y_1(t) \end{bmatrix} \end{aligned} \tag{21}$$

Here $Y_{out}(t)$ is a $M \times M$ symmetric Toeplitz matrix, which enables the proposed method to detect $M-1$ sources even if the sources are coherent.

Now, we can obtain the output data from element space to an M_b dimensional beamspace through the transformation.

$$Z(t) = W_M^{\mathrm{H}} Y_{out}(t) \tag{22}$$

where $Z(t)$ is the $M_b \times 1$ beamspace output vector, and W_M is the $M \times M_b$ beamforming matrix. In order to keep the noise of the output being white in beamspace, $W_M^{\mathrm{H}} W_M = I_{M_b}$, where I_{M_b} is the $M_b \times M_b$ identity matrix. Then, the beamforming matrix W_M has the form

$$\begin{aligned} W_M = \frac{1}{\sqrt{M}}\Bigg[a\left(\frac{2m}{M}\right), a\left(\frac{2(m+1)}{M}\right), \\ \cdots, a\left(\frac{2(m+M_b-1)}{M}\right)\Bigg] \end{aligned} \tag{23}$$

where m denotes the start location.

Finally, we can get the beamspace covariance matrix

$$\begin{aligned} R_Z &= E\left[Z(t)Z^{\mathrm{H}}(t)\right] \\ &= W_M^{\mathrm{H}} E\left[Y_{out}(t)Y_{out}^{\mathrm{H}}(t)\right] W_M \\ &= W_M^{\mathrm{H}} R W_M \end{aligned} \tag{24}$$

where $R = E\left[Y_{out}(t)Y_{out}^{\mathrm{H}}(t)\right]$ Then apply the eigen decomposition to R_Z of (24), we can obtain

$$R_Z = U_s \Lambda_s U_s^{\mathrm{H}} + U_n \Lambda_n U_n^{\mathrm{H}} \tag{25}$$

where Λ_s is the $D \times D$ diagonal matrix which contain D signal eigenvalues, and $U_s = [e_1, e_2, \cdots, e_D]$ which is the signal subspace spanned by the D signal eigenvalues. Λ_n is the $(M_b - D) \times (M_b - D)$ diagonal matrix which contain $M_b - D$ noise eigenvalues, and $U_n = [e_{D+1}, e_{D+2}, \cdots, e_{M_b}]$ which is the noise subspace spanned by the $M_b - D$ eigenvalues eigenvalues.

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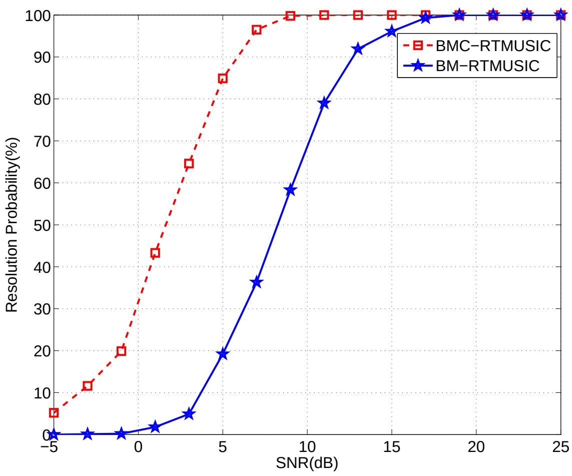

Fig. 2. Resolution probability for noncoherent sources

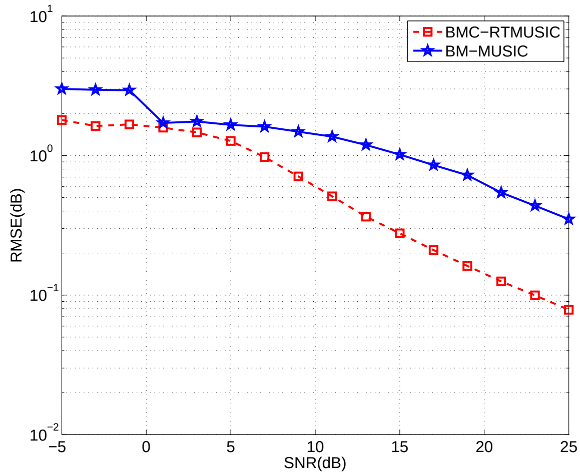

Fig. 3. Average RMSE for noncoherent sources

The proposed method employs the Beamspace Root MUSIC[12] to estimate the DOAs of the impinging signals. First, we define

$$F = QU_nU_n^{\mathrm{H}}Q^{\mathrm{H}} \tag{26}$$

where

$$Q = DW_MG \tag{27}$$

$$D(i,k) = \exp\left[-j\pi\frac{M_B-1}{M}\left(i-\frac{M_B+1}{2}\right)\right]\delta_{ik} \tag{28}$$

$$\begin{aligned} G(i,k) = {} & \frac{1}{M_B}(-2)^{M_B-1}(-1)^{k-i}\exp\left[-j(i-1)\frac{\pi}{M_B}\right] \\ & \left\{\prod_{n=0,n\neq k-1}^{M_B-1}\sin\left[\frac{1}{2}\pi\left(\frac{M_B-1}{M}+(i-1)\frac{2}{M_B}\right.\right.\right. \\ & \left.\left.\left.-(n+m)\frac{2}{M}\right)\right]\right\} \end{aligned} \tag{29}$$

for $i = 1, 2, \cdots, M_B$ and $k = 1, 2, \cdots, M_B$, where δ_{ik} denotes the Kronecker delta. Then the polynomial

$$\begin{aligned} D(z) = {} & b_0 + b_1z + \cdots + b_{M_B-1}z^{M_B-1} \\ & + b_1^*z^{2M_B-3} + b_0^*z^{2M_B-2} \end{aligned} \tag{30}$$

where

$$b_j = \sum_{i=0}^{j} F(M_B-j+i, i+1) \quad j = 0, 1, \cdots, M_B-1 \tag{31}$$

and

$$z = \exp(-j2\pi\Delta\cos\theta/\lambda) \tag{32}$$

Finally, the roots of the polynomial $D(z)$ can be used to estimate the DOAs of the incident signals. The estimated DOAs can be given as

$$\theta_d = -\cos^{-1}\left(\frac{\lambda}{2\pi\Delta}\arg(z_d)\right) \tag{33}$$

where z_d is the d-th roots of the polynomial.

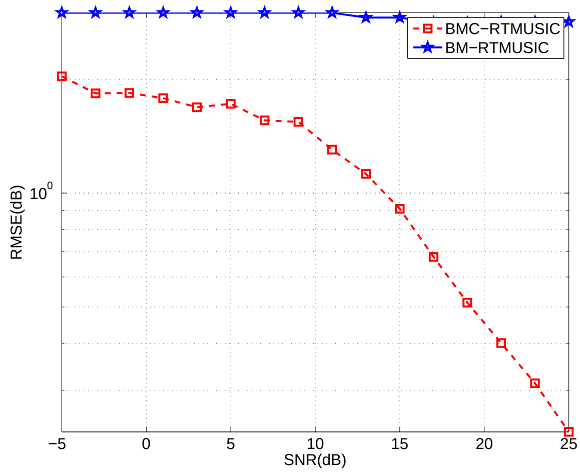

Fig. 4. Average RMSE for coherent sources

IV. Performance Analysis

In this section, we conduct simulations and evaluate the performances of BMC-RTMUSIC and BM-RTMUSIC algorithms.

Consider a uniform linear array (ULA) of 8 monostatic sensors and the sensors are spaced half a wavelength apart. Assume that $D = 2$ and 7 BPSK sources, 20 snapshots per trial, and 500 Monte Carlo trials for simulations. The sampling frequency is $30kHz$ and the signal frequency is $7kHz$.

In figures 2 and 3, there are two noncoherent sources located at $-3°$ and $3°$. Figures 2 and 3 show the resolution probability and the root mean-square error (RMSE) of these algorithms. It is very clear that the performance of BMC-RTMUSIC is better than that of BM-RTMUSIC, especially in the low SNR.

In figures 4, we give the RMSE of the two algorithms under the same scenario except for coherent sources. It is clearly seen that the BM-RTMUSIC fails to resolve the incident sources. In contrast, the proposed algorithm gives very well performance without forward/backward smoothing.

Figure 5 and 6 show the histograms of the DOA es-

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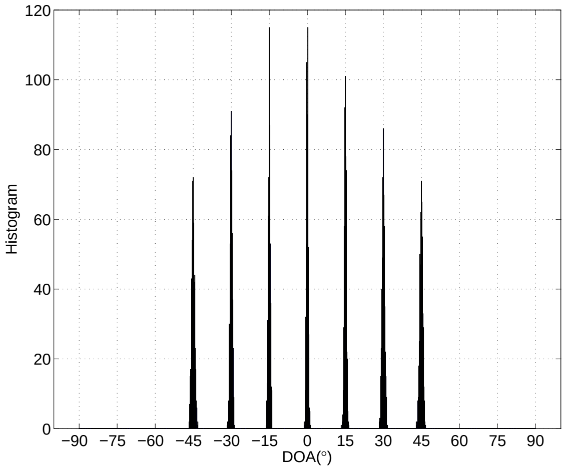

Fig. 5. Histogram of DOA estimation for noncoherent sources, BMC-RTMUSIC

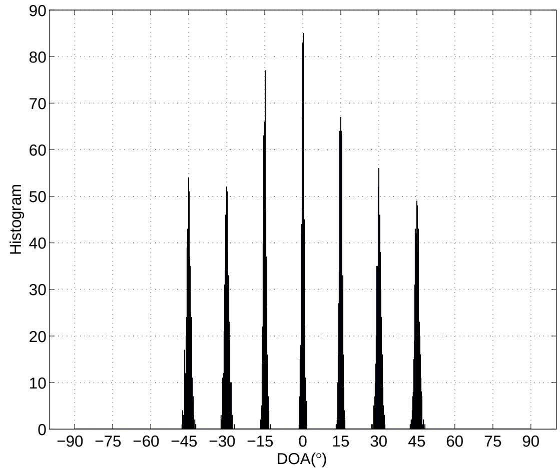

Fig. 7. Histogram of DOA estimation for coherent sources, BMC-RTMUSIC

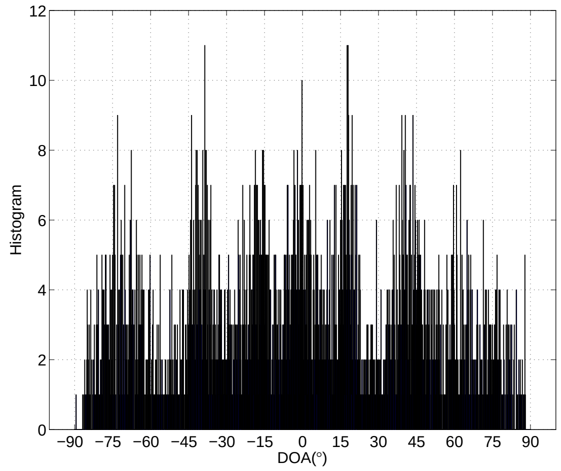

Fig. 6. Histogram of DOA estimation for noncoherent sources, BM-RTMUSIC

timation for $D = 7$ noncoherent sources of DOAs at $[-45°, -30°, -15°, 0°, 15°, 30°, 45°]$, and SNR=$10dB$. It becomes clear in figure 5 and 6 that our proposed algorithm gives the accurate DOAs estimation, and we can observe all seven peaks. However, the BM-RTMUSIC fails to give accurate DOA estimation and clear peaks around the true DOAs.

Then we present the simulation results for the case of $D = 7$ coherent sources. The signals are located at $[-45°, -30°, -15°, 0°, 15°, 30°, 45°]$ and SNR=$10dB$. Figure 7 shows the sharp histograms for the azimuth DOA estimations using our proposed BMC-RTMUSIC.

V. Conclusion

In this paper, we apply the conjugate Root MUSIC method to beamspace bearing estimation, and present a novel beamspace conjugate MUSIC (named MC-RTMUSIC) method to find the DOA for noncoherent/coherent and non-circular signals such as BPSK. The proposed method employs the properties of non-circular signal to construct a conjugate symmetric Toeplitz matrix from received data vectors, and forms the pseudo covariance matrix. The proposed method can correctly estimate the DOAs of noncoherent/coherent sources with a few snapshots and resolve $M-1$ sources where M is the number of sensors. Simulation are shown to support our statements and shown the exemplary performance of the proposed algorithm in low SNR. These advantages make our proposed method more appropriate for real-time implementation.

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