Dynamic Modeling and Current Mode Control of a Continuous Input Current Buck-Boost DC-DC Converter

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***Abstract*—This paper deals with the dynamic analysis and control of a continuous input current buck-boost DC-DC converter. Though this converter has been available in the literature for several years, its advantages have remained surprisingly hidden. This converter is able to provide the same voltage gain of a conventional buck-boost converter with a continuous input current by employing the same number of electronic components; these features are highly desirable for several applications. This paper is focused on issues as dynamic modeling, stability analysis and control of this converter. Both large signal and small signal dynamic models are provided. Experimental results of a current mode control based on Linux and an open-source real-time platform are presented.**

***Index Terms*—DC-DC converters, dynamic modeling of DC-DC converters, nonlinear control, current-mode control, real-time.**

I. Introduction

In the recent years new topologies of power converters are being constantly developed. These topologies are being designed to meet the requirements of several modern industrial applications.

During decades, the well-known topologies for DC-DC converters such as the buck, boost, buck-boost, sepic and cuk converters have been employed for a wide number of applications. In order to obtain improved features many other topologies can be proposed based on these basic topologies [1]-[2].

In the particular case of the Buck-Boost converter, it is a well-known topology that is able to provide higher or lower voltages at the output with respect to the input voltage. This feature is employed when for example; the input voltage is lower or higher than the desired output voltage. This converter is constructed with basic elements, a capacitor, an inductor, a transistor, a diode and a voltage source. However, the main disadvantage of this converter is that it provides a discontinuous input current.

In this paper, a continuous input current buck-boost converter is analyzed. This converter has been previously proposed and has been present in the literature for several years [3]-[4]. However, its main features have remained surprisingly hidden. The discussed converter is able to deal with the disadvantages of the conventional buck-boost converter without employing extra electronic components.

In the work presented in this paper, some issues regarding the dynamics of the discussed converter are analyzed. The large signal and small signal dynamic models of this converter are not currently available in the literature and the authors consider that they are important for the development of model-based control strategies as done for the well-known topologies of DC-DC converters [5]. For a detailed analysis of the operation and advantages from a designed-oriented point of view, the reference [6] is recommended for its review.

The body of this paper is constructed as follows. Section II shows a brief description of the converter. The corresponding steady state operation is analyzed by employing the averaging method. In section III, the large signal and average large signal sets of dynamic equations are derived for the presented converter. In addition, an average small signal model that is obtained via approximate linearization around an equilibrium point is obtained. Section IV presents a brief stability analysis by considering the capacitor voltage and the inductor current as the two possible outputs of the proposed dynamic system. Section V shows the synthesis of a current mode controller based on the input-output feedback linearization technique [7]. Finally, in section VI, experimental results of the implementation of the proposed controller are presented. The experimental setup is based on data acquisition hardware and RTAI-Lab, a Linux-based real-time platform [8].

This paper opens the door to the application of a wide number of control techniques and a diversity of model-based applications that has not been implemented for the continuous input current buck-boost converter.

II. Steady State Operation

Figures 1(a) and 1(b) show the topologies of the traditional buck-boost converter and the continuous input current buck-boost converter respectively.

It is well known that the input current of the conventional buck-boost converter is discontinuous. In other words, when the switch is opened (see Fig. 1(a)), the voltage source is disconnected, thus the current through it is interrupted. This feature represents several disadvantages such as EMI emission problems and low efficiency when batteries are

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used as power supply. Some converters that are able to provide higher or lower voltages at the output with respect to the input voltage have been developed such as the well-known Cuk and Sepic converters that are shown in figures 2 and 3.

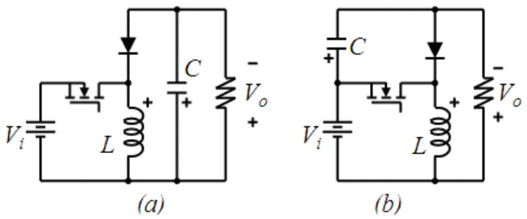

Fig. 1. (a) Traditional buck-boost converter, (b) Continuous input current buck boost converter.

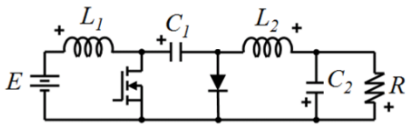

Fig. 2. DC-DC Cuk Converter.

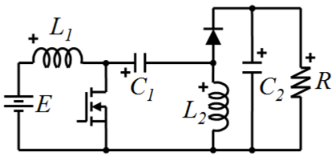

Fig. 3. DC-DC Sepic Converter.

The latter converters allow continuous input current, however inductors and capacitors are added in both cases. On the other hand, the converter in Figure 1(b) provides a continuous input current by making a straightforward arrangement of the components to the conventional buck-boost converter.

Figure 4 shows the equivalent circuits of the continuous input current buck-boost converter depending on the position of the switch. It is clear that the input current is now continuous when the switch is opened, because the input voltage along the capacitor feed the load directly. It is clear that no extra components are required as well.

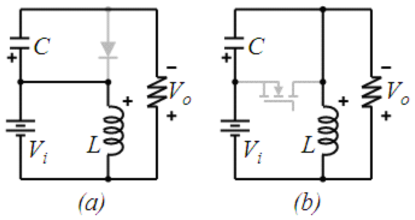

Fig. 4. Equivalent circuits for the switching states in CCM when; (a) the switch is closed; (b) the switch is opened.

In this paper, a particular description of the dynamics and control of this converter is presented. The purpose of this work is to obtain the dynamic equations that describe the behavior of the converter. The traditional method that considers the position of the switch is considered [5].

Let us analyze first the steady state operation of the converter by employing the averaging method, in steady state operation,

$$DV_i + (1 - D)(V_i - V_c) = 0 \tag{1}$$

Where D is the duty cycle; V_i is the input voltage and V_c is the capacitor voltage. Equation (1) can be rewritten as

$$V_c = \frac{V_i}{1 - D} \tag{2}$$

The steady state capacitor voltage V_c provides the same equilibrium that the correspondent in the traditional boost converter. In other words, this converter may operate as a boost or a buck-boost converter. Note that the presented topology in Figure 4 considers that the load is connected not only to the capacitor but also to the input voltage, thus the output voltage gain in steady state operation is given by

$$V_o = V_c - V_i = \frac{V_i}{1 - D} - V_i = V_i \frac{D}{1 - D} \tag{3}$$

Equation (3) shows that the output voltage V_o provides the typical steady state voltage gain of a buck-boost converter.

This paper is mainly focused in several issues as the dynamic modeling, stability and control of the continuous input current buck-boost converter.

III. Dynamic Modeling

In this section, a large signal model for the continuous input current buck-boost converter is presented. The nature of this model is nonlinear from the state space approach. In addition, a small signal model is obtained by employing approximate linearization around an equilibrium point (see [5]).

The provided models are not currently available in the literature and are essential for the optimal control of the converter.

Other issues as the stability and current mode control of this converter will be presented in the following sections.

A. *Large Signal Modeling*

Let us consider the equivalent circuits shown in Fig. 4 and define the position of the switch as $u = 1$ and $u = 0$ when it is closed and opened respectively.

When the switch is closed as illustrated in Fig. 4(a), the following set of dynamic equations are obtained

$$\begin{cases} L\dfrac{d}{dt}i = E \\[2ex] C\dfrac{d}{dt}v = \dfrac{(V_i - v)}{R} \end{cases} \tag{4}$$

Where i is the inductor current; v the capacitor voltage; C the capacitance; L the inductance; R the resistive load; and

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V_i the input voltage. Note that the variables represent time-dependent instantaneous values.

On the other hand, when the switch is opened, this is $u = 0$, the following set of dynamic equations are obtained

$$\begin{cases} L\dfrac{d}{dt}i = -v + V_i \\ C\dfrac{d}{dt}v = i + \dfrac{(V_i - v)}{R} \end{cases} \tag{5}$$

A set of dynamic equations that is valid for both switching states is obtained by including the input u, this is

$$\begin{cases} L\dfrac{d}{dt}i = -(1-u)v + V_i \\ C\dfrac{d}{dt}v = (1-u)i + \dfrac{(V_i - v)}{R} \end{cases} \tag{6}$$

In some cases, average dynamic models are employed for dynamic analysis and control of power electronics devices. The average model is obtained by substituting the binary input $u = \{0,1\}$ by the duty cycle $u_{av} = [0,1]$ of the converter, this is

$$\begin{cases} L\dfrac{d}{dt}i_{av} = -(1-u_{av})v_{av} + V_i \\ C\dfrac{d}{dt}v_{av} = (1-u_{av})i_{av} + \dfrac{(V_i - v_{av})}{R} \end{cases} \tag{7}$$

Where i_{av} and v_{av} represent the average time-dependent variables of the inductor current and capacitor voltage.

Since the input u (or the duty cycle u_{av}) multiplies the state variables in both dynamic equations, the large signal dynamic model of the converter is clearly nonlinear.

On the other hand, small signal models that are valid around of a specific operation range (or equilibrium) are frequently employed for this type of converters.

B. Small Signal Modeling

In this paper, a small signal model of the continuous input current buck-boost converter is obtained by employing approximate linearization around an equilibrium point (see [5]).

The equilibrium of the dynamic system is obtained from (7), this is

$$\begin{cases} \bar{v}_{av} = \dfrac{V_i}{(1-\bar{u}_{av})} \\ \bar{\imath}_{av} = \dfrac{(\bar{v}_{av} - V_i)}{R(1-\bar{u}_{av})} \end{cases} \tag{8}$$

Let us define the following state space representation for the discussed converter

$$\begin{cases} F_1(x) = \dfrac{d}{dt}i_{av} = -\dfrac{(1-u_{av})}{L}v_{av} + \dfrac{V_i}{L} \\ F_2(x) = \dfrac{d}{dt}v_{av} = \dfrac{(1-u_{av})}{C}i_{av} + \dfrac{(V_i - v_{av})}{RC} \end{cases} \tag{9}$$

Now, let us employ the following linear approximation for (9) around the equilibrium in (8), this is

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{av} \\ \Delta v_{av}\end{bmatrix} = \begin{bmatrix} \left.\dfrac{\partial F_1}{\partial i_{av}}\right|_{p=\bar{p}} & \left.\dfrac{\partial F_1}{\partial v_{av}}\right|_{p=\bar{p}} \\ \left.\dfrac{\partial F_2}{\partial i_{av}}\right|_{p=\bar{p}} & \left.\dfrac{\partial F_2}{\partial v_{av}}\right|_{p=\bar{p}} \end{bmatrix} + \begin{bmatrix} \left.\dfrac{\partial F_1}{\partial u_{av}}\right|_{p=\bar{p}} \\ \left.\dfrac{\partial F_2}{\partial u_{av}}\right|_{p=\bar{p}} \end{bmatrix}\Delta u_{av} \tag{10}$$

Where $\Delta i_{av} = (i_{av} - \bar{\imath}_{av})$; $\Delta v_{av} = (v_{av} - \bar{v}_{av})$; are linear incremental variables; $\Delta u_{av} = (u_{av} - \bar{u}_{av})$ is an incremental input and $\bar{p}$ is a set of the values of the system at the equilibrium that are shown in equation (8). Equation (10) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix}\Delta i_{av} \\ \Delta v_{av}\end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{(1-\bar{u}_{av})}{L} \\ \dfrac{(1-\bar{u}_{av})}{C} & -\dfrac{1}{RC} \end{bmatrix} + \begin{bmatrix} \dfrac{\bar{v}_{av}}{L} \\ -\dfrac{\bar{\imath}_{av}}{C} \end{bmatrix}\Delta u_{av} \tag{11}$$

Equation (11) is a small signal state space representation of the continuous input current buck-boost converter.

IV. Stability Analysis of the Large Signal Model

One of the most important issues regarding the dynamics of this converter is the stability. It is well-known that the most of the nonlinear models of power electronics converters have non-minimum phase variables (see. [5]. In this section, the stability of the system is analyzed by choosing the capacitor voltage as the output y of the system at first. Let us consider the following dynamic system

$$\begin{cases} \dfrac{d}{dt}i_{av} = -\dfrac{(1-u_{av})}{L}v_{av} + \dfrac{V_i}{L} \\ \dfrac{d}{dt}v_{av} = \dfrac{(1-u_{av})}{C}i_{av} + \dfrac{(V_i - v_{av})}{RC} \\ \qquad y = v_{av} \end{cases} \tag{12}$$

Let us consider the zero dynamics of the system [7]. An input u_{av} and an initial condition for the output $v_{av}(0)$ are chosen such that $v_{av}\forall\, t$.

This is,

$$\begin{cases} u_{av} = 1 + \dfrac{(V_i - v_{av})}{R i_{av}} \\ \\ v_{av}(0) = 0 \end{cases} \tag{13}$$

By considering conditions in (13) for the dynamic system (12), the following equations are obtained

$$\begin{cases} \dfrac{d}{dt} i_{av} = \dfrac{V_i}{L} \\ \\ \dfrac{d}{dt} v_{av} = 0; \ \text{(with } v_{av} = 0\ \forall\ t\text{)} \\ \\ y = v_{av} \end{cases} \tag{14}$$

Since parameters in the state equation of the inductor current are strict positive, the resulting system is clearly unstable. In other words, this converter has the same feature that the traditional buck-boost converter and the nonlinear converters (boost, Cuk, Sepic, and so forth) which have a non-minimum phase capacitor (or output) voltage.

This time, let us consider the inductor current as the output of the dynamic system, this is

$$\begin{cases} \dfrac{d}{dt} i_{av} = -\dfrac{(1-u_{av})}{L} v_{av} + \dfrac{V_i}{L} \\ \\ \dfrac{d}{dt} v_{av} = \dfrac{(1-u_{av})}{C} i_{av} + \dfrac{(V_i - v_{av})}{RC} \\ \\ y = i_{av} \end{cases} \tag{15}$$

Let us consider the zero dynamics of the system, but this time an input u_{av} and an initial condition $i_{av}(0)$ is chosen such that $i_{av} \forall\ t$. This is

$$\begin{cases} u_{av} = 1 - \dfrac{V_i}{v_{av}} \\ \\ i_{av}(0) = 0 \end{cases} \tag{16}$$

By considering conditions in (13) in the dynamic system (12), the following equations are obtained

$$\begin{cases} \dfrac{d}{dt} i_{av} = 0; \ \text{(with } i_{av} = 0\ \forall\ t\text{)} \\ \\ \dfrac{d}{dt} v_{av} = -\dfrac{v_{av}}{RC} + \dfrac{V_i}{RC} \\ \\ y = i_{av} \end{cases} \tag{17}$$

Since parameters in (17) are strict positive, it is clear that the zero dynamics of the system is stable. In other words, the inductor current is a minimum phase variable. A current control mode is allowed by employing the presented large signal dynamic model. On the other hand, an indirect output voltage control can be implemented as it has been employed for the traditional power electronics converters (see. [5]).

V. Current Mode Control

Let us consider the dynamic system in (15) in which the average inductor current is the variable to be controlled. The input-output feedback linearization technique [7] is employed for this case.

Let us rewrite equations in (15). An input affine dynamic system is obtained, this is

$$\begin{cases} \dfrac{d}{dt} x = f(x) + g(x) u_{av} \\ \\ y = h(x) \end{cases} \tag{18}$$

where

$$f(x) = \begin{bmatrix} -\dfrac{v_{av}}{L} + \dfrac{V_i}{L} \\ \dfrac{i_{av}}{C} + \dfrac{(V_i - v_{av})}{RC} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \dfrac{v_{av}}{L} \\ -\dfrac{i_{av}}{C} \end{bmatrix}$$

$$y = h(x) = i_{av}; \ x = [i_{av} \quad v_{av}]^T$$

In order to obtain a subsystem that represents the input-output dynamics of the system the output is derived, this is

$$\frac{d}{dt} y = \frac{\partial h(x)}{\partial x}[f(x) + g(x)u_{av}] \stackrel{\text{def}}{=} L_f h(x) + L_g h(x) u_{av} \tag{19}$$

Where $L_f h(x)$ and $L_g h(x)$ are known as the Lie derivatives of $h(x)$ along the vectors f and g [7]. Using the definition in (18) and employing equations in (15), the following expressions can be obtained

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = [1 \quad 0] f(x) = -\frac{v_{av}}{L} + \frac{V_i}{L}$$

$$L_g h(x) = \frac{\partial h(x)}{\partial x} g(x) = [1 \quad 0] g(x) = \frac{v_{av}}{L}$$

therefore

$$\frac{d}{dt} y = -\frac{v_{av}}{L} + \frac{V_i}{L} + \frac{v_{av}}{L} u_{av} \tag{20}$$

This state equation clearly corresponds to the inductor current state equation because $y = h(x) = i_{av}$. Since the input u_{av} is directly involved in the output derivative, the following expression for the input can be employed

$$u_{av} = \frac{1}{L_g h(x)}\left[-L_f h(x) + u_L\right]$$

In this case,

$$u_{av} = 1 + \frac{V_i}{v_{av}} + \frac{L u_L}{v_{av}} \tag{21}$$

By substituting (21) into (20), equation (20) can be rewritten as

$$\frac{d}{dt} y = u_L \tag{22}$$

In order to address system uncertainties, the following integrator is employed

$$\frac{d}{dt} = x_I = i_{av} - i_{ref} \tag{23}$$

Where i_{ref} is the reference or the desired inductor current. By employing equation (22) and (23), the following linear subsystem is obtained

$$\frac{d}{dt}\begin{bmatrix} x_I \\ i_{av} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_I \\ i_{av} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_L - \begin{bmatrix} 1 \\ 0 \end{bmatrix} i_{ref}$$
$$y = i_{av} \tag{24}$$

Where the input u_L can be defined by a linear control law based on the pole placement technique (see [9]), this is

$$u_L = -k_I x_I - k_1 i_{av} \tag{25}$$

The stability of the linear subsystem (24) is given by the selection of gains k_I and k_1. In other words, the input-output subsystem is stable if the well-known closed loop matrix $(A - BK)$ is a Hurwitz matrix. On the other hand, the stability of the full-order system can be demonstrated by analyzing the zero dynamics of the closed loop system. Note that this proof is given by setting $u_L = 0$ and considering the same conditions in (16), thus the zero dynamics are given by (17) for this case as well. Therefore we conclude that the internal dynamics of the closed-loop system is stable.

VI. Experimental Results of The Controller

For the experimental setup, the control law given in equation (21) is implemented. Figures 5 and 6 show the experimental results of the closed-loop system. The experimental setup was carried out by employing RTAI-Lab [8] which is a Linux-based real-time platform. In addition, a data acquisition card PCI-6024E is used as interface between the real-time software platform and the converter. The set-point for the inductor current is changed several times during the experiment by employing the real-time user interface shown in Fig. 7. The parameters of the converter involved in the experiment are $C = 222.2\mu F$, $L = 550\mu H$, $R = 100\Omega$, $V_i = 30V$. The PWM frequency is set at $20KHz$.

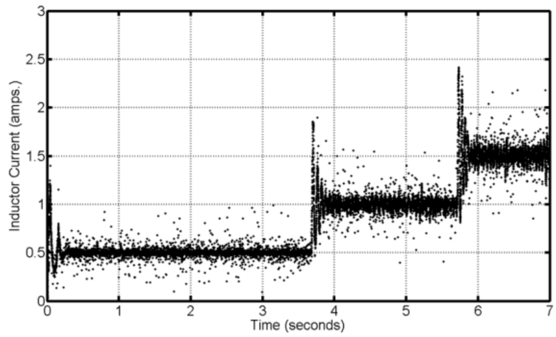

Fig. 5. Experimental results of the closed-loop inductor current i_{av} with different set-points (i_{ref} = {0.5,1,1.5})

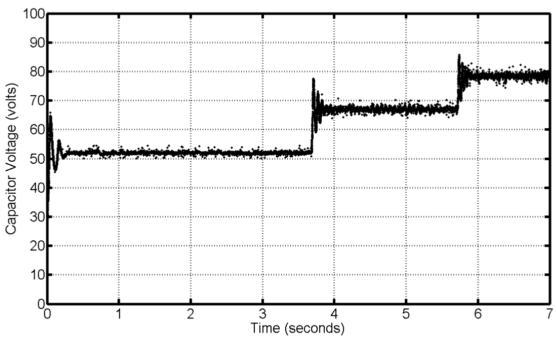

Fig. 6. Experimental results of the closed-loop capacitor voltage v_{av} with different set-points for the inductor current (i_{ref} = {0.5,1,1.5}).

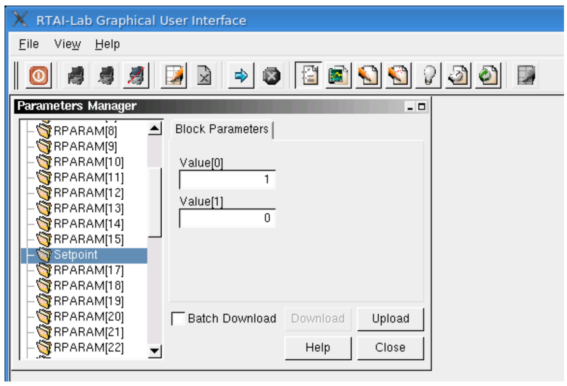

Fig. 7. Set-point selection in the RTAI-Lab Graphical User Interface.

Note that the output voltage is given by $V_o = (V_i - v_{av})$ which would be negative with respect to the input voltage as the conventional boost converter.

VII. Conclusion

In this paper, a brief analysis of the basic issues regarding the dynamics of the continuous input current buck-boost converter was presented. The usefulness of the proposed average large signal dynamic model is shown with the

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experimental results of a current mode control. In future works, other model-based nonlinear control techniques based on the presented theory can be employed. Indirect output voltage control can be implemented as well for this converter.

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