A Hybrid System Framework to Behavior Control of Nonholonomic AGV

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Abstract—In this paper, a new hybrid systems framework to behavior control of nonholonomic AGV is presented. Instead of using the traditional hierarchical structure or the hybrid automata separately, both were combined in this paper. This framework has the 3-layered hierarchical structure containing a hybrid automata of the motion control as the middle process. The hybrid automata has three states, stop, line path following and circle path following. Complex behavior of AGV is decomposed into discrete control sequences by discrete behavior planning from the top layer and implemented by the hybrid automata. The lower process is the AGV plant which is decoupled by input-output nonlinear feedback linearization. This framework not only avoids the state explosion problem of automation, but also reduces the control complexity and ensures the performance of the AGV behavior control system. The simulation results to the motion control, parking behavior and deadlock resolution of the AGV verify the validity of this framework.

Index Terms—Automated Guided Vehicle (AGV), Behavior control systems, hybrid systems, Nonholonomic constrains, Nonlinear control

I. INTRODUCTION

A
tomated guided vehicle (AGV) is a kind of simple mobile robots, which is powered by battery and could be controlled by programming. AGVs can be used widely for automated transportation in various industrial and service fields such as in factories, ports, banks, airport, post offices, etc. It occupies a center to material handing of the intelligent logistic systems due to its autonomous characteristics. AGVs not only reduce the labor costs, but also ensure the transport efficiency and security.

As the increasing complexity of AGV systems in commercial and industrial scenarios, control and coordination of the AGVs become much more difficult under uncertain dynamic environment. Along with higher requirements of complex task, the increased ability and responsiveness for AGV control systems have been received much consideration in the last decade. The multiagent-based or behaviors-based approaches have been the focus of the researchers. For examples, Choudhary et al. [1] proposed a multiagent-based framework representing zone controlled AGV environment incorporating various behaviors like path generation, collision and deadlock avoidance, etc. Christopher et al. [2] implemented behaviors-based intelligent distributed fuzzy logic control systems integrating the presentation of human knowledge to a meca- num wheeled AGV, the navigational and collision avoidance behaviors of AGV were controlled by using IF-THEN rules. The above mentioned approaches could achieve improved reliability and reduce complexity of AGV control systems to some extent. However, the AGV is a complex nonlinear system with nonholonomic constraint. It can not be controlled by smooth linear time invariant controls laws [3], which most of the assumptions made in the controls literature are not satisfied. It is especially hard to provide provable guarantees on safety and performance to its behavior control systems (BCS) which control the AGV to perform various tasks in uncertain dynamic environment.

One successful approach is to decompose it into hybrid systems that intermix discrete modes and continuous dynamics. Recently years, a rapid growth of interest in hybrid systems has developed efficient tools for synthesis and analysis of such complex systems. This hybrid, hierarchical approach to the design and control of BCS for AGV has proven to be very successful. For examples, Kress-Gazit et al. [4] applied linear-temporal logic speci- fications for generating robot behaviors. Hua et al. [5] applied motion description languages to solve the pose stabilization problem of nonholonomic wheeled mobile robots. Gayan et al. [3] explained a hybrid control strategy developed to coor- dinate multiple autonomous mobile robots with nonholo- nomic constraints in an obstacles populated environment. Wang et al. [6] adopted a hybrid input/output automata to decompose the behavior control systems of mobile robots and different languages were used to define the behavior speci- fications. Hua et al. [7] addressed a 3-layered hierarchical hybrid control structure to pose stabilization controller for wheeled mobile robots. Pu et al. [8] resolved the parking problem for wheeled mobile robot by using hybrid automata-based approach.

Traditional hierarchical structure [7, 9, 10] that each layer of controller will be designed and implemented separately can simplify the design process of complex systems, but due to the increasing complexity of controllers, these plans can not be applied to high-order systems.

As the complexity of AGV systems increasing, the existing methods for the control are not able to handle the problem. The conventional engineering methods are limited to the analysis functions, and the traditional control theory is powerless when handling the complex AGV models. The human's capability of intelligence and decision cannot be achieved in the traditional methods. Therefore, this paper presents a new hybrid systems framework to behavior control of AGV which can be applied to high-order systems.
to lacking of interaction between layers, it may have difficulty to control the complex BCS with various tasks. By using hybrid automata only [8, 11], the interaction between discrete and continuous parts of the system would be enhanced, however, with the growing number of complex tasks, the number of states would increase dramatically, this may cause a so called state explosion problem and make systems uncontrollable. Furthermore, the model in each state is usually rather arbitrary without exact specification, take [8] as an example, the motion trajectory of AGV is arbitrary without reasonable restrictions in each state, which may lead to some uncertain behaviors in some situations. To solve these problems, an innovative hybrid systems framework to BCS of Non-holonomic AGV is proposed in this paper. It has a unified 3-layered hierarchical structure which consists of discrete behavior planning (DBP) for the higher process, hybrid automata for the middle process and AGV plant for the lower process. In this framework, the complex behaviors (e.g., parking, deadlock resolution) control of AGV supervised by higher process will ultimately be decomposed into the path following to several straight lines and circular arcs. So, we get few continuous dynamics states for the hybrid automata in middle process, and each discrete state has a deterministic model, the motion trajectory of AGV is determined as straight line or circular arc. When adding new behaviors, we only need to add new discrete sequences in the layer of DBP, the number of states will not increase. In addition, this kind of slow switch systems has been proven to be stable as the subsystem of each model is stable [12]. The nonlinear plant with nonholonomic is decoupled by using input-output feedback linearization [13], and a set of switch time functions, \( \mathbf{v}(t) \in \mathbb{R}^{(n-m)} \), such that

\[
\dot{\mathbf{q}} = \mathbf{S} \mathbf{v}(t) \tag{4}
\]

Differentiating (4), then substituting it into (1) and premultiplying by \( \mathbf{S}^T \), we have

\[
\dot{\mathbf{y}} = (\mathbf{S}^T \mathbf{M})^{-1} \mathbf{S}^T (\mathbf{M} \mathbf{v} - \mathbf{V}_m + \mathbf{B} \mathbf{r}) \tag{5}
\]

Let

\[
\mathbf{x} = \begin{bmatrix} \mathbf{q}^T & \mathbf{v}^T \end{bmatrix}^T, \quad \text{from \ (4) and \ (5), we obtain the state space representation of dynamics model for nonholonomic AGV as}
\]

\[
\mathbf{x} = \begin{bmatrix} \mathbf{S} \mathbf{v} \\ \mathbf{f}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{H} \end{bmatrix} \mathbf{r} \tag{6}
\]

where

\[
\mathbf{f}_1 = (\mathbf{S}^T \mathbf{M})^{-1} \mathbf{S}^T (\mathbf{M} \mathbf{v} - \mathbf{V}_m), \quad \mathbf{H} = (\mathbf{S}^T \mathbf{M})^{-1} \mathbf{S}^T \mathbf{B}.
\]

Proper control laws can be developed by utilizing (6), which we discuss in next section.

A detailed analytical study of the structure of the kinematic and dynamic models of wheeled mobile robots can be found in [15]. Here, we are interested in the (2, 0) type as shown in Fig.1.it consists of a vehicle with two driving wheels mounted on the same axis and a front caster wheel. The two driving wheels are driven independently by DC motors to achieve the motion and orientation, they have the same wheel radius, \( r \), and are separated by \( 2b \). (\( X_i, Y_i \)) is a world coordinate system, (\( X_e, Y_e \)) is the local coordinate system fixed to the cart. The orientation of the AGV local coordinate system from the world coordinate system is denoted as \( \theta \). \( P \) is the center of mass (COM) of the cart. Point \( Q \) is the geometric center with coordinates and is located in the intersection of the axis of symmetry with the driving wheel axis. The distance between points \( P \) and \( Q \) is \( d \).

### II. HYBRID SYSTEM FRAMEWORK OF AGV

#### A. Modeling of Nonholonomic AGV

Without loss of generality, by using Lagrange formulation, the general dynamics model of AGV with \( n \)-dimensional generalized coordinates \( \mathbf{q} \) subjected to \( m \) nonholonomic independent constraints can be described as a generalized mechanical system that are in the form

\[
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}_m(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}(\mathbf{q}) + \mathbf{G}(\mathbf{q}) + \mathbf{\tau}_d = \mathbf{B}(\mathbf{q})\mathbf{r} - \mathbf{A}(\mathbf{q})\lambda \tag{1}
\]

\[
\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{2}
\]

where \( \mathbf{M}(\mathbf{q}) \in \mathbb{R}^{nxn} \) is a symmetric positive define inertial matrix of the system. \( \mathbf{V}_m(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{nxn} \) is a centripetal and coriolis forces matrix. \( \mathbf{F}(\mathbf{q}) \in \mathbb{R}^{nxn} \) is a surface friction matrix. \( \mathbf{G}(\mathbf{q}) \in \mathbb{R}^{nxn} \) is a gravity matrix. \( \mathbf{\tau}_d \in \mathbb{R}^{nx1} \) is the bounded unknown disturbances matrix. \( \mathbf{B}(\mathbf{q}) \in \mathbb{R}^{nx(n-m)} \)

### Concluding remarks

In conclusion, the development of a hybrid system framework for the control of the AGV is presented. The framework allows us to handle both discrete and continuous behaviors, providing a unified approach for the control of complex systems. The analysis and control design methods developed in this paper can be applied to various AGV systems, enabling more efficient and robust control strategies.
The configuration of the AGV plant can be described by five generalized coordinates as follows

\[ q = [x_p \ y_p \ \theta \ \varphi_A \ \varphi_B]^T \]  

where \( (x_p \ y_p) \) is the coordinates of the COM P, \( \varphi_A \) and \( \varphi_B \) are the angular displacements of driving wheels A and B respectively. For the pure rolling and no slipping nonholonomic condition states, the kinematic constraints equations of the AGV is given by

\[
\begin{align*}
-x_p \sin \theta + y_p \cos \theta - d \dot{\theta} &= 0 \\
\dot{x}_p \cos \theta + y_p \sin \theta - b \dot{\theta} &= r \dot{\varphi}_A \\
\dot{x}_p \cos \theta + y_p \sin \theta + b \dot{\theta} &= r \dot{\varphi}_B
\end{align*}
\]

From (8), (9), (10), the kinematic constraints matrix in (2) is given by

\[
A(q) = \begin{bmatrix}
-sin \theta & cos \theta & -d & 0 & 0 \\
-cos \theta & -sin \theta & b & r & 0 \\
-cos \theta & -sin \theta & -b & 0 & r
\end{bmatrix}
\]

then, it is easy to verify that the following matrix

\[
S(q) = \begin{bmatrix}
c(b \sin \theta + d \cos \theta) & c(b \sin \theta - d \cos \theta) & -c & 1 & 0 \\
c(b \sin \theta - d \cos \theta) & c(b \sin \theta + d \cos \theta) & c & 0 & 1
\end{bmatrix}^T
\]

satisfies (3), where the constant \( c = r / 2b \).

B. Hybrid Automata

The hybrid automata can be considered as a finite state machine, or finite automation, with a linear or nonlinear continuous dynamical system embedded in each discrete location, such systems evolve continuous evolution according to differential equations as well as instantaneous transitions triggered by external or deliberate events sensed. Hybrid automation model is useful in specification, analysis, verification or synthesis for complex hybrid system. The hybrid automata adopted in this paper is defined to be the tuple

\[ H = (Q, X, U, Y, init, f, h, I, E, G, R, \gamma) \]

in which \( Q = \{q_1, q_2, \cdots\} \) is a set of discrete states. \( X \) is the state space of the continuous variables. \( U \) is a finite collection of continuous input variables. \( Y \) is a set of continuous output variables. \( init \in Q \times X \) is a set of initial states. \( f = \{f_1, f_2, \cdots\} \) is a vector field. \( h = \{h_1, h_2, \cdots\} \) is an output map. \( I \) is an invariant set, the system may flow within \( q_i \) only if \( X \in I(q_i) \). \( E \in Q \times Q \) is set of discrete jumps. \( G \) is a guard condition, when \( G(e_i) \) is true, the system may instantaneously take a discrete transition from current state \( q_i \) to next state \( q_j \). \( R \) is a reset map, \( \gamma \) assigns to each state a set of admissible inputs.

C. Hierarchical Hybrid Control Architecture of AGV

In the face of dynamic changes in the environment and description requires to complex task, the whole AGV system includes both continuous activities and discrete-event features and the developed continuous dynamic subsystems must be switch between. So a hybrid system is developed, which can greatly simplify the behavior planning and control by generating plans at the level of the discrete modes.

The hybrid systems control architecture considered here is a 3-layered hierarchical structure as depicted in Fig.2.

**The plant layer.** The AGV system to be controlled belongs to a special class of Euclidean SE(2) dynamic systems. Since it is nonholonomic and couldn’t be asymptotically stabilized to a single equilibrium point by any continuous feedback control. So, the input-output nonlinear feedback linearization and input-output decoupling are applied to make system controllable in this layer.

**The hybrid automata layer.** In this layer, the AGV controller is modeled as a hybrid automata to reduce the difficulties of motion control with nonholonomic constraints and allows for the application in various complex task domains. In order to simplify control mode sequence generation and avoid the state explosion problem, it is more desirable to reduce the number of control modes of the hybrid automata. Moreover, based on the fact that the shortest path for AGV are composed of circular arcs and straight lines, complex path for AGV can be decomposed into piecewise continuous path with discontinuities in curvature [13]. Thus, we model the hybrid automata with three specific states: \( q_1 \): stop, \( q_2 \): path following to a straight line and \( q_3 \): path following to a circle. By using path following, the starting posture of AGV need to be as close to the reference path as possible, however, it could have more stable speed than by using trajectory tracking, so it is more suitable to practical applications. This layer serves as the communication between the AGV plant and the behavior planning layer. The hybrid automata would evolve according to the discrete sequence of operations from higher process and give the control input to the plant.

**The behavior planning layer.** This layer is the top layer, which responses to the environment, communicates with the hybrid automata layer. It is concerned with the planning of the AGV behavior and supervises the execution of a mission plan. In this layer, the behavior specification of AGV can be described by many methods and languages, such as automation, linear-temporal logic method, pseudo code, C language, etc. Here, the pseudo code is used, in which the specification described consists of behavior decomposition strategy, path planning, control parameters, transition rules and trigger sequences.

III. IMPLEMENTATIONS

A. Control Laws of Plant

The AGV system modeled as continuous-time system described by (6) is a multi-input-multi-output (MIMO) nonlinear system, it may be input-output linearizable if a set of
output equation is chosen appropriately. Utilizing the nonlinear feedback control, let
\[
\tau = H^T (U - f_v)
\]  
(13)
where \( \tau = [\tau_d \, \tau_B]^T \) is the control torque to two actuators of AGV, \( H^T \) is the pseudo inverse matrix of \( H \), \( U \in \mathbb{R}^{2 \times 1} \) is a new auxiliary input vector associated with the output equation.

Substituting (13) into (6), the state equation simplifies to the form of
\[
\dot{x} = k(x) + g(x)U
\]
(14)

\[\begin{bmatrix}
S_v \\
0
\end{bmatrix}
\]
where \( k(x) = \begin{bmatrix}
S_v \\
0
\end{bmatrix} \), \( g(x) = \begin{bmatrix}
0 \\
1
\end{bmatrix} \)

Let the output equation be denoted by
\[
y = h(q) = [h_1(q) \, \ h_2(q)]^T
\]  
(15)
and the full rank decoupling matrix \( P(x) \in \mathbb{R}^{2 \times 2} \) for the system be denoted by
\[
P(q) = \frac{\partial h}{\partial q} S(q)
\]  
(16)
then defining a new auxiliary input vector \( u \in \mathbb{R}^{2 \times 1} \) and utilizing the following state feedback
\[
U = P^{-1} (u - P_v)
\]  
(17)
we achieve input-output linearization, \( u \) can be designed by the linear controller, we use PD controller in this paper. Let error \( e = h_{\text{desired}} - h \), the PD controller is given as
\[
u = \dot{y}_{\text{desired}} + k_p e + k_d \dot{e}
\]  
(18)
where \( k_p \in \mathbb{R}^{2 \times 2} \) and \( k_d \in \mathbb{R}^{2 \times 2} \) are constant gains to ensure the convergence of the control errors. The schematic block diagram of the plant is shown in Fig.3.

\[
\begin{bmatrix}
A \\
B \\
0 \\
0
\end{bmatrix} \sqrt{A^2 + B^2}
\]
\[
\begin{bmatrix}
x_{\text{desired}} - x \\
y_{\text{desired}} - y
\end{bmatrix}
\]
\[
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
where
\[
J_e = \begin{bmatrix}
x_{\text{desired}} - x \\
y_{\text{desired}} - y
\end{bmatrix} \sqrt{A^2 + B^2}
\]
\[
J_\varphi = \begin{bmatrix}
r & r \\
r & r
\end{bmatrix}
\]

According to sub-control systems described above, the hybrid automata for corresponding control mode is obtained as shown in Fig.4.

**B. Switching Models of Hybrid Automata**

The AGV controller shaded as oval in Fig.3 is modeled by a hybrid automata with three states. We now describe the continuous dynamics within each discrete state. Considering the problem of path following, by following the conventional wisdom in which one drives a car. Two requirements should be considered, the car should pursue the path as closely as possible and travel the path with a given forward velocity. Let us define \( h_1 \) be the shortest distance from the point P to the reference trajectory and \( h_2 \) be the AGV forward velocity of point P along the Xr-axis.

For the path following of a straight line described by
\[
A x + B y = C
\]
and \( \theta \), the output equation is
\[
h_{\text{L1}}(x, y, \theta) = \frac{A x_{\text{desired}} + B y_{\text{desired}} + C}{\sqrt{A^2 + B^2}}
\]
(19)
\[
h_{\text{L2}}(v) = \dot{x} \cos \theta + \dot{y} \sin \theta - \frac{r}{2} (\dot{\phi}_A + \dot{\phi}_B)
\]  
(20)

Likewise, if the path is a circle with center \( (x_{\text{desired}}, y_{\text{desired}}) \) and radius \( R \), the output equation is
\[
h_{\text{C1}}(x, y, \theta) = \sqrt{(x_{\text{desired}} - x)^2 + (y_{\text{desired}} - y)^2 - R}
\]
(21)
\[
h_{\text{C2}}(v) = h_{\text{C1}}(v)
\]  
(22)

then, utilizing (16), we can obtain the decoupling matrix for straight line and circle respectively, they are
\[
P_L(q) = \begin{bmatrix}
J_1 S(q) \\
J_\varphi S(q)
\end{bmatrix}
\]  
(23)
\[
P_C(q) = \begin{bmatrix}
J_1 S(q) \\
J_\varphi S(q)
\end{bmatrix}
\]  
(24)

According to sub-control systems described above, the hybrid automata for corresponding control mode is obtained as shown in Fig.4.
In Fig.4, the transaction conditions turn = i(i = 0, 1, 2) is determined by the arrival of switching points which satisfy the restrictions, turn = 0 may also be true for emergency. For presence of position errors, the transaction condition is true when the distance from point P to switching point is within a tolerance. Depending on a set of discrete sequences of turn and switching points from layer of behavior planning, the switching is triggered between the states.

C. Behavior Supervise and Control of AGV

This section we will consider the applications of the hybrid system framework described above in motion control, parking behavior and deadlock resolution for AGV.

(a) Motion control of AGV

The motion control is the foundation for all other activities of AGV, the complex behaviors of AGV are ultimately implemented by motion control. For the nonholonomic AGV, here, its desired motion manner including moving (backward, forward), turning (left, right), and stationary are chosen by the switching point, each control specifications of motion manner is described as follows.

Motion Control Specifications
IF: Forward or backward;
SET: Turn=1;
THEN: Given the line path parameters A, B and C. For Forward, the velocity is positive. For backward, the velocity is negative.
IF: Left turn or right turn;
SET: Turn=2;
THEN: For left turn, given the circle path parameters center \( O_i \) and radius \( R_i \). For right turn, given the circle path parameters center \( O_i \) and radius \( R_i \). And for forward, the velocity is positive. For backward, the velocity is negative.
IF: Stationary;
SET: Turn=0;
THEN: Given all the path parameters to zero. The velocity is zero.
DO: Path following
The center \( O_i \) and \( O_j \) are given by
\[
\begin{align*}
    x_c &= x_s + R \cos(\pi / 2 + \theta) \\
    y_c &= y_s + R \sin(\pi / 2 + \theta)
\end{align*}
\]
where \((x_c, y_c)\) is the coordinates of circle center, \( R \) is the circle radius, \((x_s, y_s)\) is the coordinates of switch point.

(b) Parking behavior

Parking behavior is usually attributed to posture stabilizaion problem which has been regarded as a very difficult problem. Here, we attempt to solve the parking problem using our hybrid system framework. Figure 6 shows the parking behavior of AGV from point S to origin K, its specifications can be described as follows.

Parking Behavior Specifications
INITIAL: The path parameters of lines SD, EF and JK. The path parameters of circles O1, O2 and O3. The position coordinates of points D, E, F, G, H, I and J. The forward velocity;
START: Forward;
IF: Reach D THEN: Forward right turn;
IF: Reach E THEN: Forward;
IF: Reach F THEN: Forward left turn;
IF: Reach H THEN: Forward right turn;
IF: Reach J THEN: Forward;
IF: Reach K THEN: Stationary.

All the circles have the same radius \( R \). The distance from point D to Xi-axis is \( 5R \), from points F, H and J to Yt-axis is \( R \).

(c) Deadlock resolution

A traffic control scheme for multiple AGVs aims for a collision free motion plan, the collision should be predict and resolved well in advance, however, it is more flexible that the collision could be resolved dynamically. Figure 7 shows the dynamic deadlock resolution for head on collision of two AGVs under our hybrid system framework, its specifications is given as follows.

Deadlock Resolution Specifications
IF: The distance of AGV P1 and P2 is equal to DG=\( 2\sqrt{3}R \);
INITIAL: The path parameters of circles O1, O2, O3, O4 and O5. The position coordinates of points D, E, F, G, H and I. The forward velocity;
AGV P1: Forward right turn
IF: Reach I THEN: Forward left turn;
IF: Reach H THEN: Forward right turn;
IF: Reach G THEN: Forward;
AGV P2: Forward right turn
IF: Reach F THEN: Forward left turn;
IF: Reach E THEN: Forward right turn;  
IF: Reach D THEN: Forward.

All the circles have the same radius $R$ which must be greater than the swing radius of AGV. The points E, F, H, I are the common tangent points of two circles.

IV. SIMULATIONS

The validity of our approach is verified by simulation, the simulation codes are written in Matlab M file. The fifth order Runge-Kuttta integration routine is used with the integration step setting to 0.001s. The AGV starts with a forward velocity of 1.414m/s.

Figure 8 shows how the AGV follows a line path, then a circular path when it starts with a posture $(0, 4.1, -\pi/4)$ under the control of hybrid automata. We can see that the output velocities of left wheel and right wheel are very smooth.

Figure 9 and figure 10 show how our hybrid system framework to accomplish the parking behaviors and deadlock resolution for AGV. Related parameters are given as follows.

For parking behaviors, the starting posture of AGV is $S(6.1, 10, -2\pi/5)$.  
$R = 1.5m$, $O(4.5, 7.5)$, $O2(1.5, 4.5)$, $O3(1.5, 1.5)$.

For deadlock resolution, the starting posture of AGV is $P1(-2, 4.01, 0)$, $P2(13, 4.01, \pi)$.

$R = 2m$, $D(5 - 2\sqrt{3}, 4)$, $G(5 + 2\sqrt{3}, 4)$.

V. CONCLUSION

This paper introduces a new hybrid systems framework for the behavior control of AGV with nonholonomic constraints. It has the 3-layered structure including a hybrid automaton of motion control with three states: stop, line path following, circle path following. The complex behavior of AGV is decomposed into discrete control sequences by the top layer of DBP and implemented by the hybrid automata in the middle layer. The input-output nonlinear feedback linearization is applied to make the AGV plant controllable in the lower layer. This approach not only reduces control complexity of the BCS for AGV, but also avoids the state explosion problem. The simulation results of the motion control, parking behavior and deadlock resolution of the AGV demonstrated the feasibility of this framework. In the future work, we will implement this framework to pioneer P3 robots.

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