An Improved Adaptive Time-Variant Model for Fuzzy-Time-Series Forecasting Enrollments based on Particle Swarm Optimization

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***Abstract*- In this paper an improved adaptive Time-Variant Model for fuzzy time series (ATVF) is proposed and this model try to predict the Alabama University enrollments well. This model acquires analysis window size (Time order) based on accuracy of forecasting in training phase and in testing phase heuristic rules help in the forecasting values and particle swarm optimization algorithm is uses for interval lengths improvement to acquire forecasting with better accuracy. The experiment results show that the proposed model achieves a significant improvement in forecasting accuracy as compared to other fuzzy-time-series models for forecasting enrollments of students of the University of Alabama.**

***Index Terms* - Fuzzy time series, Adaptive, forecasting, particle swarm optimization , fuzzy logical relationships.**

I.INTRODUCTION

Forecasting activities play an important role in our daily life. The forecasting problem of time series data, a series of data ordered in time sequence segmented by fixed time intervals [1], is an interesting and important research topic. In various disciplines it has been commonly tackled by using a variety of approaches such as statistics, artificial neural networks, etc. Traditional time series forecasting models are usually extensively dependent on historical data, which can be incomplete, imprecise and ambiguous. If these uncertainties were widespread in real-world data, they could hinder forecasting accuracy, thus limiting the applicability of forecasting models. As we know, to forecast these matters is generally believed to be a very difficult task. It looks like the performance of a random walk process on a different time. Obviously, we need to investigate some intelligent forecasting paradigm to solve the forecasting problems.

Zadeh proposed the fuzzy set theory first and then got fruitful achievements both in theory and applications [2]. In Li et al. Song and Chissom introduced a new forecast model based on the concept of fuzzy time series [3].

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They use the time variant fuzzy time series model and the time-invariant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama. Chen improved the fuzzy time series model by max–min composition operations [4]. Huarng presented a method to improve forecasting results in forecasting the enrollments of the University of Alabama and the Taiwan Futures Exchange (TAIFEX) [5], [6].

Chen and Chung used genetic algorithms to adjust each interval length of first-order and high-order forecasting models [7], [8]. Li et al. applied fuzzy c-means clustering to interval partitioning In [9]. Kuo et al. proposed an improved method of particle swarm optimization to find the proper content of the interval length [10]. Huarng et al. proposed a multivariate heuristic model by integrating a multivariate heuristic function and univariate fuzzy time-series models to form a multivariate model [11]. wong used adaptive model for fuzzy time series for forecasting [12].

We proposed combination of the adaptive time-variant model (ATVF) [12] with pso algorithm [19], [20] to improve Alabama University enrollments forecasting in this paper. ATVF model automatically adapts the analysis window size of fuzzy time series based on the predictive accuracy in the training phase and uses heuristic rules to determine forecasting values. in the testing phase the prediction accuracy is improved by using pso algorithm for improving interval length in testing phase. The proposed model is better than existing fuzzy-time series forecasting models for enrollments at the Alabama university. The rest of this paper is organized as follows. Section II summarizes the definitions of fuzzy time series from [13]. Section III explains the adaptive selection of analysis windows and heuristic rules in the testing phase (ATVF model). Section IV describes PSO Algorithm. Section V describe the ATVF-PSO proposed model. Section VI demonstrates the experimental results. Section VII is the conclusion.

II. FUZZY TIME SERIES

The concepts of fuzzy time series are described as follows [4], [12]. A fuzzy set A defined in the universe of discourse U = {u1, u2, . . . , un} can be represented as follows:

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdot \ \cdot \ \cdot + f_A(u_n)/u_n \qquad (1)$$

where f_A is the membership function of the fuzzy set A, $f_A : U \rightarrow [0, 1]$, $f_A(u_i)$ denotes the degree of membership of u_i belonging to the fuzzy set A, $f_A(u_i) \in [0, 1]$, and $1 \leq i \leq n$.

Definition 1: Let Y (t)(t = . . . , 0, 1, 2, . . .) be the universe of discourse and also a subset of R. It is assumed that $f_i(t)$

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(i = 1, 2, . . .) is defined on Y (t), and F(t) is the collection of $f_i(t)$. Therefore, F(t) is called a fuzzy time series on Y (t).

Definition 2: It is assumed that F(t) is a fuzzy time series and

$$F(t) = F(t-1) \times R(t, t-1) \tag{2}$$

where R(t, t − 1) is a fuzzy relation, and × is an operator that is caused by F(t − 1). The relationship between F(t) and F(t − 1) can be denoted by F(t − 1) → F(t) when F(t) = F(t − 1) × R(t, t − 1) is the first-order fuzzy-time-series model of F(t).

Definition 3: Let F(t) be a fuzzy time series. For any t, F(t) = F(t − 1) and F(t) have only finite elements, and therefore, F(t) is a time-invariant fuzzy time series; otherwise, it is a time-variant fuzzy time series.

Definition 4: If F(t) is caused by F(t − 1), F(t − 2), . . . , F(t − n), the fuzzy relationship is represented by

$$F(t-1), F(t-2), \ldots, F(t-n) \rightarrow F(t) \tag{3}$$

It is the nth-order fuzzy-time-series model.

Definition 5: It is supposed that F(t − 1) = A_{i1}, F(t − 2) = A_{i2}, . . . , F(t − n) = A_{in} and F(t) = A_j . The relationship among n + 1 consecutive data can be denoted by

$$A_{i1}, A_{i2}, \ldots, A_{in} \rightarrow A_j \tag{4}$$

where $A_{i1}, A_{i2}, \ldots, A_{in}$ is the left-hand side, and A_j is the right-hand side.

Definition 6: It is supposed that F(t) is simultaneously caused by F(t − 1), F(t − 2), . . . , F (t − m)(m > 0), and the relations are time variant. The F(t) is a time-variant fuzzy time series, and the relation can be expressed as

$$F(t) = F(t-1) \times R^w(t, t-1) \tag{5}$$

where w > 1 is a time parameter affecting the forecast F(t), which is the analysis window of time-variant models.

III. ATVF MODEL

Step 1) Define the universe of discourse U and the intervals.

For U = [$D_{min} - D_1, D_{max} + D_2$] ≡ [L,R], D_1 and D_2 are two proper positive numbers, and for the intervals u_i = [L + (i − 1)l,L + il), i = 1, 2, . . . , t, l is the interval length, l is an even number, and t = (R − L)/l. The midpoints of these intervals are mi, i = 1, 2, . . . , t. According to Table 1 for enrollments forecasting at the University of Alabama, it is obvious that D_{min} = 13055 and D_{max} = 19337. For convenience of illustrating the forecasting example here, we set U_{min} = 55 and U_{max} = 663, and get the universe of discourse on Y(t) =[13000,20000].

Step 2) Define the fuzzy sets and fuzzify the data. Each fuzzy set A_i is assigned a linguistic term and can be defined by the intervals $u_1, u_2, \ldots, u_t$, i.e.,

$$A = fA_i(u_1)/u_1 + fA_i(u_2)/u_2 + \cdot\ \cdot\ \cdot + fA_i(u_n)/u_n$$

In other words, Ai denotes a fuzzy set = {$I_1, I_2, I_3, I_4, I_5, I_6, I_7$} with different membership degree = {$u_1,u_2,u_3,u_4,u_5,u_6,u_7$}. The detailed definitions of all fuzzy sets are described in the following equation:

$$\begin{aligned}
A_1 &= 1/u_1+0.5/u_2+0/u_3+0/u_4+0/u_5+0/u_6+0/u_7\\
A_2 &= 0.5/u_1+1/u_2+0.5/u_3+0/u_4+0/u_5+0/u_6+0/u_7\\
A_3 &= 0/u_1+0.5/u_2+1/u_3+0.5/u_4+0/u_5+0/u_6+0/u_7\\
A_4 &= 0/u_1+0/u_2+0.5/u_3+1/u_4+0.5/u_5+0/u_6+0/u_7\\
A_5 &= 0/u_1+0/u_2+0/u_3+0.5/u_4+1/u_5+0.5/u_6+0/u_7\\
A_6 &= 0/u_1+0/u_2+0/u_3+0/u_4+0.5/u_5+1/u_6+0.5/u_7\\
A_7 &= 0/u_1+0/u_2+0/u_3+0/u_4+0/u_5+0.5/u_6+1/u_7
\end{aligned} \tag{6}$$

linguistic values is shown in Table 2, for example, A_1 means the linguistic value ‘‘not many” and denotes a fuzzy set = {I1, I2, I3, I4, I5, I6, I7} consists of seven members with different membership degree = {1, 0.5, 0, 0, 0, 0, 0} . The descriptions with respect to the remaining fuzzy sets are similar to A1 as mentioned in Table 1 for all of actual data.

Step 3) Establish the fuzzy relationships of time t and t + 1. In the training phase, the fuzzy relationship is supposed to be Ai → Aj. F(1971) → F(1972) is a relationship; and a fuzzy relationship A1→A1 is obtained by replacing F(1971) and F(1972) to A1 and A1, respectively. In the testing phase, the fuzzy relationship is supposed to be Ai → #, fuzzy relationship A6 → # is got as F(1992) → F(1993).

Step 4) Calculate the forecasted values. In the training phase, each round calculates values **For1** and **For2** and compares the two values to the actual value **Act** with the better one as the forecasting value, i.e., Forecast =

$$\begin{cases} \text{For1}, & \text{if } |\text{For1} - \text{Act}| \le |\text{For2} - \text{Act}| \\ \text{For2}, & \text{if } |\text{For1} - \text{Act}| > |\text{For2} - \text{Act}| \end{cases} \tag{7}$$

The analysis window is determined by Algorithm 1.

The computations of For1 and For2 are carried out by Algorithm 2.

[1973] Select analysis window sizes 1 and 2 as initial values and flag n = 1. Fuzzy relationship of 1972 and 1973: $A_1 \rightarrow A_1$. According to Algorithm 2, For1 = m1 = 13500, and For2 = 13492. Because PA1 > PA2 (PAi denotes the predictive accuracy of the analysis window size i), the forecasting value of 1973 is **13500**. Window sizes 1 and 0 are selected for forecasting the enrollment in 1974, and flag n = 0. According to Algorithm 1, window sizes 1 and 2 are selected to forecast 1974. Update flag n = 1.

In the testing phase, each forecast calculates values **For3** and **For4**. The analysis window size is determined by rules 1–3. The computations of For3 and For4 are carried out by Algorithm 2 or 3. The forecasted value is determined by rules 4–6.

[1990] Fuzzy relationship A6 → #. Because the flag of 1989 is 1, according to Rule 1, the analysis window sizes are 1 and 2 to forecast 1990. For3 = m6 = 18500, and For4 = 18577. Because both the flags of 1988 and 1989 are 1 and E1989 > E1988, according to Rule 4, Forecast = max (For3, For4) = **18577**.

Algorithm 1

1) Select two initial window sizes 1 and 2 and flag n = 1. Initialize i = 1.

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2) Predict the next data point with window sizes 1 and 2 using the ATVF model and measure their predictive accuracy **PA** by the difference between forecasting values and actual values (ie. eq (6)). If no difference is found, PA_1 (the predictive accuracy of the small window size) is the higher value.

3) Select another two window sizes n and n − i or n + i and n + 2i based on which window size has higher accuracy obtained in step 2. If window size n has higher prediction accuracy, n and n − i are selected and flag n = n − i or n + i and n + 2i are selected and flag n = n + i. If n = 0, go back to step 1.

4) Slide the analysis window and include the next time series observation. Use the two selected window sizes in step 3 to predict future data with the ATVF model and measure their prediction accuracy again. If no difference in accuracy is found, PAn (i.e., the predictive accuracy of the small window size) is the higher value.

5) Repeat steps 3 and 4 until the analysis window reaches the end of historical data.

It is supposed that the fuzzy relationship of time t and time t + 1 is $A_i \rightarrow A_j$, and the analysis window size is n. A_i is the current state, and A_j is the next state. Some notations used are defined as follows:

$[A_j]$ corresponding interval u_j whose membership in A_j is maximum;
$L[A_j]$ lower bound of interval u_j ;
$U[A_j]$ upper bound of interval u_j ;
$M[A_j]$ middle value of interval u_j ;
E_t :actual value of time t;
F_{t+1} :forecasted value of time t + 1;

Algorithm 2:

1: R = 0, S = 0;
2: if n = 1
3: $F_{t+1} = M[A_j]$;
4: if n > 1
5: $D = \sum_{i=0}^{n-2} \left| E_{t-i} - E_{t-i-1} \right| /n-1$;
6: $X_{t+1} = E_t + D/2$, $XX_{t+1} = E_t - D/2$;
7: $Y_{t+1} = E_t + D$, $Y\,Y_{t+1} = E_t - D$;
8: $P_{t+1} = E_t + D/4$, $PP_{t+1} = E_t - D/4$;
9: $Q_{t+1} = E_t + 2 * D$, $QQ_{t+1} = E_t - 2 * D$;
10: $G_{t+1} = E_t + D/6$, $GG_{t+1} = E_t - D/6$;
11: $H_{t+1} = E_t + 3 * D$, $HH_{t+1} = E_t - 3 * D$;
12: if $X_{t+1} \geq L[A_j]$ and $X_{t+1} \leq U[A_j]$
13: R = R + X_{t+1}, S = S + 1;
14: if $XX_{t+1} \geq L[A_j]$ and $XX_{t+1} \leq U[A_j]$
15: R = R +XX_{t+1}, S = S + 1;
16: if $Y_{t+1} \geq L[A_j]$ and $Y_{t+1} \leq U[A_j]$
17: R = R + Y_{t+1}, S = S + 1;
18: if $Y\,Y_{t+1} \geq L[A_j]$ and $Y\,Y_{t+1} \leq U[A_j]$
19: R = R + $Y\,Y_{t+1}$, S = S + 1;
20: if $P_{t+1} \geq L[A_j]$ and $P_{t+1} \leq U[A_j]$
21: R = R + P_{t+1}, S = S + 1;
22: if $PP_{t+1} \geq L[A_j]$ and $PP_{t+1} \leq U[A_j]$
23: R = R + PP_{t+1}, S = S + 1;
24: if $Q_{t+1} \geq L[A_j]$ and $Q_{t+1} \leq U[A_j]$
25: R = R + Q_{t+1}, S = S + 1;
26: if $QQ_{t+1} \geq L[A_j]$ and $QQ_{t+1} \leq U[A_j]$
27: R = R + QQ_{t+1}, S = S + 1;
28: if $G_{t+1} \geq L[A_j]$ and $G_{t+1} \leq U[A_j]$
29: R = R + G_{t+1}, S = S + 1;
30: if $GG_{t+1} \geq L[A_j]$ and $GG_{t+1} \leq U[A_j]$
31: R = R + GG_{t+1}, S = S + 1;
32: if $H_{t+1} \geq L[A_j]$ and $H_{t+1} \leq U[A_j]$
33: R = R + H_{t+1}, S = S + 1;
34: if $HH_{t+1} \geq L[A_j]$ and $HH_{t+1} \leq U[A_j]$
35: R = R +HH_{t+1}, S = S + 1;
36: $F_{t+1} = R + M[A_j]/S + 1$;

It is supposed that the fuzzy relationship of time t and time t + 1 is $A_i \rightarrow \#$, and the analysis window size is n.

Algorithm 3

1: R = 0, S = 0;
2: if n = 1
3: Ft+1 = M[Ai];
4: if n > 1
5:$D = \sum_{i=0}^{n-2} \left| E_{t-i} - E_{t-i-1} \right| /n-1$;
6: Xt+1 = Et + D/2, XXt+1 = Et − D/2;
7: Yt+1 = Et + D, Y Yt+1 = Et − D;
8: Pt+1 = Et + D/4, PPt+1 = Et − D/4;
9: Qt+1 = Et + 2 * D, QQt+1 = Et − 2 * D;
10: Gt+1 = Et + D/6, GGt+1 = Et − D/6;
11: Ht+1 = Et + 3 *D, HHt+1 = Et − 3 *D;
12: if Xt+1 ≥ L[Ai] and Xt+1 ≤ U[Ai]
13: R = R + Xt+1, S = S + 1;
14: if XXt+1 ≥ L[Ai] and XXt+1 ≤ U[Ai]
15: R = R +XXt+1, S = S + 1;
16: if Yt+1 ≥ L[Ai] and Yt+1 ≤ U[Ai]
17: R = R + Yt+1, S = S + 1;
18: if Y Yt+1 ≥ L[Ai] and Y Yt+1 ≤ U[Ai]
19: R = R + Y Yt+1, S = S + 1;
20: if Pt+1 ≥ L[Ai] and Pt+1 ≤ U[Ai]
21: R = R + Pt+1, S = S + 1;
22: if PPt+1 ≥ L[Ai] and PPt+1 ≤ U[Ai]
23: R = R + PPt+1, S = S + 1;
24: if Qt+1 ≥ L[Ai] and Qt+1 ≤ U[Ai]
25: R = R + Qt+1, S = S + 1;
26: if QQt+1 ≥ L[Ai] and QQt+1 ≤ U[Ai]
27: R = R + QQt+1, S = S + 1;
28: if Gt+1 ≥ L[Ai] and Gt+1 ≤ U[Ai]
29: R = R + Gt+1, S = S + 1;
30: if GGt+1 ≥ L[Ai] and GGt+1 ≤ U[Ai]
31: R = R + GGt+1, S = S + 1;
32: if Ht+1 ≥ L[Ai] and Ht+1 ≤ U[Ai]
33: R = R + Ht+1, S = S + 1;
34: if HHt+1 ≥ L[Ai] and HHt+1 ≤ U[Ai]
35: R = R +HHt+1, S = S + 1;
36: Ft+1 = R/S;

Considering the sequence of flags n, we have the following rules.

Rule 1) If n_t = 1, 1 and 2 are selected as the analysis window sizes for forecasting at time t + 1.

Rule 2) If n_t = p and nt ≥ n_{t-1}, p and p + 1 are selected as the analysis window sizes for forecasting at time t + 1.

Rule 3) If n_t = p and $n_t < n_{t-1}$, p and p − 1 are selected as the analysis window sizes for forecasting at time t + 1.

Rule 4) If n_t >= n_{t-1} and Act_t >= Act_{t-1}, the forecasted value at time t + 1 is max(For3, For4).

Rule 5) If $n_t < n_{t-1}$ and $Act_t < Act_{t-1}$, the forecasted value at time t + 1 is min(For3, For4).

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Rule 6) If $n_t >= n_{t-1}$ and $Act_t <= Act_{t-1}$ or $n_t <= n_{t-1}$ and $Act_t >= Act_{t-1}$, the forecasted value at time t + 1 is (For3 + For4)/2.

Table 1
The results of fuzzification.

Year	Actual enrollments	Fuzzy sets
1971	13055	A1
1972	13563	A1
1973	13867	A1
1974	14696	A2
1975	15460	A3
1976	15311	A3
1977	15603	A3
1978	15861	A3
1979	16807	A4
1980	16919	A4
1981	16388	A4
1982	15433	A3
1983	15497	A3
1984	15145	A3
1985	15163	A3
1986	15984	A3
1987	16859	A4
1988	18150	A6
1989	18970	A6
1990	19328	A7
1991	19337	A7
1992	18876	A6

Table 2
The detailed linguistic values corresponding to all intervals

Interval	Linguistic value	Fuzzy set
I1	Not many	A1
I2	Not too many	A2
I3	Many	A3
I4	Many many	A4
I5	Very many	A5
I6	Too many	A6
I7	Too many many	A7

IV. PARTICLE SWARM OPTIMIZATION

The particle swarm optimization (PSO) is a promising optimization approach developed by Kennedy and Eberhart [19], [20] .The PSO consists of a swarm of particles that search for the best position with respect to the corresponding best solution for an optimization problem in the virtual search space, just like the birds blocking or the fish grouping. Any particle remembers its personal best position it has been passed so far when it moves to another position. The moving method of a particle is described in the following equations:

$$v_i^{t+1}=w^t * v_i^t + c_1 * r_1 * (p_{best} - x_i^t) + c_2 * r_2 * (G_{best} - x_i^t) \quad (8)$$

$$x_i^{t+1}=x_i^t + v_i^t \quad (9)$$

In Eq. (8) and (9), the symbol V_{id} denotes the velocity of the particle id, and is limited to [$-V_{min}$, V_{max}] where V_{max} is a constant e-defined by user. The symbol x denotes the inertial weight coefficient. The symbols C1 and C2 denote the self confidence coefficient and the social confidence coefficient, respectively. In a standard PSO, the value of x decreases linearly during the whole running procedure, and C1 and C2 are constants [19]. The symbol Rand () denotes a function can generate a random real number between 0 and 1 under normal distribution. The symbols X_{id} and P_{id} denote the current position and the personal best position of the particle id, respectively. The symbol Pgbest denotes the best one of all personal best positions of all particles within the swarm. The whole running procedure of the standard PSO is described in Algorithm 4.

Algorithm 4. Standard PSO algorithm

1: initialize all particles' positions and velocity
2: while the stop condition (the optimal solution is found or the maximal moving steps are reached) is not satisfied do
3: for all particle id do
4: move it to another position according to Eqs. (8) and (9)
5: end for
6: end while

V. PROPOSED MODEL (ATVF-PSO)

A new forecast model, named ATVF-PSO, consisting of the adaptive fuzzy time series and the particle swarm optimization, is proposed in this paper. In the ATVF-PSO model, for the training phase, the use of the particle swarm optimization is to train all fuzzy forecast rules under all training data. Once all fuzzy forecast rules have been well trained, for the testing phase, we can use the ATVF-PSO model to forecast the new testing data. The detailed descriptions of the ATVF-PSO model are given in the following. Let the number of the intervals be n, the lower bound and the upper bound of the universe of discourse on historical data Y(t) be b_0 and b_n, respectively. A particle is a vector consisting of n-1 elements (i.e. $b_1, b_2, \ldots, b_i, \ldots, b_{n-2}$ and b_{n-1}, where $1 \leq i \leq n-1$ and $b_{i-1} < b_i$); based on these n-1 elements, define the n intervals as $I_1 = (b_0,b_1]$, $I_2 = (b_1,b_2]$, . . ., $I_i = (b_{i-1},b_i]$, . . ., $I_{n-1} = (b_{n-2},b_{n-1}]$ and In = $(b_{n-1},b_n]$, respectively. If a particle moves to another position, the elements of the corresponding new vector need to be sorted first to ensure that each element bi ($1 \leq i \leq n-1$) arranges in an ascending order. The graphical particle representation is given in Fig. 1. The ascending model exploits the intervals denoted by each particle to create an independent group of fuzzy forecast rules to forecast all historical training data and get the forecasted accuracy for each particle. The mean square error (MSE) value is used to represent the forecasted accuracy of a particle for the training phase. The lower the MSE value is, the better the forecasted accuracy is. The MSE function is defined in Eq (10), where the symbol Nforecasted denotes the number of the forecasted data, the symbol FD_i denotes the ith forecasted data and the symbol TD_i denotes the corresponding historical training data with respect to Fd_i.

$$MSE=\sum_{i=1}^{n}(FD_i-TD_i)^2/n \quad (10)$$

$$RMSE=(MSE)^{1/2} \quad (11)$$

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b1	b2	...	bi	...	bn-1

Fig. 1. The graphical particle representation.

For the training phase, the ATVF-PSO model moves all particles to another position, respectively, according to Eqs. (8) and (9) and repeat the steps mentioned above to evaluate the forecasted accuracy of all particles until the pre-defined stop condition (the optimal solution is found or the maximal moving steps are reached) is satisfied. If the stop condition is satisfied, then all fuzzy forecast rules trained by the best one of all personal best positions of all particles are chosen to be the final result. For the testing phase, the ATVF-PSO model uses all well trained fuzzy forecast rules to forecast the new testing data. The detailed procedure of the ATVF-PSO model for the training phase and the testing phase is described in Algorithms 5 and 6, respectively.

Algorithm 5. The ATVF-PSO algorithm for the training phase

1: initialize all particles' positions and velocity
2: while the stop condition (the optimal solution is found or the maximal moving steps are reached) is not satisfied do
3: for all particle id do
4: fuzzify all historical training data according to all intervals defined by the current position of particle id(step 2).
5: find out all k-order (window size)fuzzy relationships according to all fuzzified historical training data with using algorithm 1.
7: forecast all historical training with using algorithm 2.
8: calculate the MSE value for particle id based on Eq. (10)
9: update the personal best position of particle id according to the MSE value mentioned above
10: end for
11: for all particle id do
12: move particle id to another position according to Eqs. (8) and (9)
13: end for
14: end while

Algorithm 6. The ATVF-PSO algorithm for the testing phase

the appropriate interval length and time order determine in training phase then with using rules 1 to 6 and algorithm 3 (deffuzifier) fuzzy time series is estimated.

VI. EXPRIMENTAL RESULTS

Method presented using MATLAB has been implemented and we use enrolments at the University of Alabama as a data set in this paper. Experimental results for ATVF-PSO model are compared with those of existing methods. Let the number of particles be 30, the maximal number of move for each particle be 100, the value of inertial weight (i.e. x) is linearly decreased from 1.4 to 0.4, the self confidence coefficient (C1) and the social confidence coefficient (C2) both be random, the velocity be limited to [-100,100] and the universe of discourse on the fuzzy time series be [13000, 20000], respectively. A comparison of the forecasted enrollments between ATVF-PSO and other models ATVF [12]. HPSO model [10]. The SC2 model [14], the HCL98 model [15], the C02 model [16]. the CC06H model [17] and the S07 model [18] is shown in Table 3. The experimental results mentioned above show that HPSO model is more precise than any existing methods for the training phase,

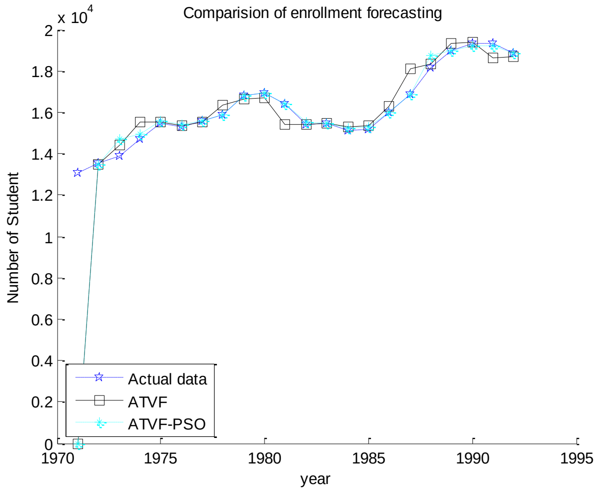

Fig. 2. Comparison of enrollment forecasting.

some experimental results of the forecasting models for the testing phase are listed in Table 4. We also use the RMSE values to evaluate the forecasted accuracy. Based on the historical data for the past years, we can forecast the new enrollment for the next year only. For example, the historical data of enrollments under years 1971–1989, is used to forecast the new enrollment of year 1990. To forecast the new enrollment of year 1991, it is then based on the enrollments under years 1971– 1990 Among the existing methods. We then only show the comparison of the forecasted accuracy with CC06F model, HPSO model ATVF and ATVF-PSO in Table 4. ATVF-PSO model is more precise than CC06F model, ATVF model and HPSO model at all. In contrast to the discussions above, Fig. 2 compare the different enrollment forecasting methods ATVF and ATVF-PSO under the same intervals.

Table 3
A comparison of the forecasted enrollments with different number of intervals and different order of fuzzy time series.

Year	Actual data	SC2	HCL98	S07	C02	ATVF	ATVF-PSO
1971	13055						
1972	13563					13506.1	13494
1973	13867					14436.5	14681.5
1974	14696	14286	14500			15500	14934.5
1975	15460	14700	15361	15500		15519.7	15590
1976	15311	14800	16260	15468	15500	15374.1	15422.9
1977	15603	15400	15511	15512	15500	15542.9	15603
1978	15861	15500	16003	15582	15500	16351.7	15861
1979	16807	15500	16261	16500	16500	16664.1	16807
1980	16919	16800	17407	16361	16500	16680.8	16919
1981	16388	16200	17119	16362	16500	15435.5	16388
1982	15433	16400	16188	15744	15500	15433	15553.9
1983	15497	16800	14833	15560	15500	15497	15497
1984	15145	16400	15497	15498	15500	15295.5	15255.6
1985	15163	15500	14745	15306	15500	15366.6	15276.5
1986	15984	15500	15163	15442	15500	16306.3	15984
1987	16859	15500	16384	16558	16500	18090.8	16859
1988	18150	16800	17659	17187	18500	18350.2	18736.2
1989	18970	19300	19150	18475	18500	19316.2	18970
1990	19328	17800	19770	19382	19500	19387.5	19204.6
1991	19337	19300	19928	19487	19500	18647.5	19212.5
1992	18876	19600	19537	18744	18500	18711.6	18876
RMSE		880.7	566.9	365.6	294.4	455.67	230.4

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Table 4
A comparison of the forecasted accuracy in the HPSO model and the CC06F model, ATVF and ATVF-PSO models under different number of intervals Methods Number of intervals.

Number of intervals Model	8	9	10
CC06F	364.64	310.23	292.37
HPSO	346.35	300.87	246.41
ATVF	400.92	450.3	330.4
ATVF-PSO	317.82	290.70	141.90

A comparison of the forecasted accuracy (i.e. the RMSE value eq(11)) with ATVF-PSO model, ATVF model and HPSO under different number of intervals are listed in Table 5, All forecasting models are well trained by historical training data to forecast the new testing data (i.e. the enrollments of years 1990, 1991, and 1992), and to use the RMSE values to evaluate the forecasted accuracy.

Table 5
A comparison of the forecasted results produced by the ATVF model , ATVF-PSO model and the HPSO model.

number of intervals = 7

Year	Actual data	HPSO	ATVF	ATVF-PSO
1990	19328	18988	18970	19657
1991	19337	19167	19306	18638
1992	18876	19265	19315	18703
RMSE		314	327.5	168.3

VII. CONCLUSION

In this paper, we have proposed a new hybrid forecast model (named ATVF-PSO) based on the particle swarm optimization and the Adaptive time variant model for fuzzy time series to forecast enrolments of the University of Alabama. This technique adjusts the length of each interval in the universe of discourse for forecasting. The experimental results show this model have better forecasting accuracy than previous ones. We will decide to use multi factor forecasting based on the described scheme in the further research.

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