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EEG Signals Classification by S-shaped Radial Implicative Fuzzy Systems

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Abstract—**The paper introduces an EEG signals classifier for classification of vigilance level of a car driver. The classifier is based on the concept of radial implicative fuzzy system. The novel in the presented approach is accommodation of S-shaped fuzzy sets which can handle boundary regions of the relevant input space. Both structure and parameter learning of the system are referred to and corresponding classification abilities are presented.**

Index Terms—**EEG signals classification, microsleep detection, radial fuzzy system**

I. Introduction

The paper focuses on the use of radial implicative fuzzy systems for classification of electroencephalographic signals (EEG signals) [1]. Classification of this kind of signals requires application of special methodologies. Suitable classes of by nature inspired computational techniques are in an extensive use in recent decade. Namely, three methodologies are well known. These are artificial neural networks, fuzzy computing and evolutionary computing [2], [3], [4].

Here we demonstrate an application of a class of fuzzy systems which is closely linked to the concept of radial basis neural networks. The relation is due to the radial shape of employed fuzzy sets [5]. Moreover, so called S-shaped fuzzy sets are also employed in order to enhance description capabilities in boundary regions of input space.

A development of such a system consists of several phases. On main level these are phases of structure and parameter learning. In our approach the phase of structure learning consists of initial setting of a rule base of a fuzzy system. For this purpose GUHA method of exploratory data analysis is employed [6], [7]. The phase of parameter learning is performed by application of Levenberg-Marquard algorithm with the requirement on retention of coherence of the system. That is, with the requirement on permanent consistency of system's rule base [8].

Classification of a car driver vigilance level is the important issue in connection with the concept of smart cars [9], [10], [11]. In these cars there are installed different supporting devices which interoperate with a driver to help him to drive more safely or effectively.

One of the possible inputs for such a device is an on-line stream of EEG records taken directly from a driver head. The signals are analyzed and classified into several vigilance level classes. In our set up these are classes of *mentation, wakefulness* and *microsleep* corresponding gradually to the increasing level of drowsiness.

In next four sections we provide the description of our approach. Section II introduces the classifier based on the concept of radial implicative fuzzy system. Section III describes analyzed EEG data. Section IV deals with the methodology of learning of the classifier. The last Section V presents obtained results and concluding remarks.

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II. S-radial implicative fuzzy systems

In this section we gradually develop the concept of S-shaped radial (S-radial) fuzzy system. We start with the concept of implicative fuzzy system, which is further enhanced by employment of radial membership functions and incorporation of S-shaped radial fuzzy sets. The computational model is presented in the last subsection. The concept and relevant computational model were described in details in [5]. Here we give a brief description in order to reader obtain the basic impression about these systems.

A. Implicative fuzzy systems

Fuzzy systems are generally rule based computational models [3]. Each single rule represents an individual fuzzy relation in input-output space encoding a partial knowledge on a mapping of interest. These partial mappings are aggregated in the rule base of a system and accompanied by a kind of reasoning algorithm. The algorithm is usually called the inference engine. An engine produces for a given input an output which is typically a real number (MISO configuration), real vector (MIMO configuration), or output class or vector of classes. In what follows we will work with the system in MIMO configuration with tree output classes.

An individual rule is given by combination of antecedent and consequent part. That is why rules are commonly denoted as IF-THEN rules. Antecedents operates on multidimensional input space $\mathcal{R}^n, \boldsymbol{x} = (x_1, \ldots, x_n) \in \mathcal{R}^n$ and consequents (in our configuration) on three-dimensional unit cube $[0,1]^3$ representing membership degree into particular classes.

Mathematically, the j-th IF-THEN rule corresponds to fuzzy relation $R_j(\boldsymbol{x}, y)$ given as

$$R_j(\boldsymbol{x}, y) = A_{j1}(x_1) \star \cdots \star A_{jn}(x_n) \to B_j(y), \qquad (1)$$

where

$$A_j(\boldsymbol{x}) = A_{j1}(x_1) \star \cdots \star A_{jn}(x_n) \qquad (2)$$

is the antecedent composed by combination of n fuzzy sets with membership functions $A_{ji}(x_i)$. Each of these fuzzy sets operates on single dimension i. The combination is performed by a fuzzy conjunction which is represented by a t-norm $\star$ [3], [12]. Consequent fuzzy sets $B_j(y)$ are actually vectors from $[0,1]^3$.

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There are two main approaches to combination of antecedents with conseqents. In conjunctive systems combination is made by a t-norm, typically by the same which is emloyed for building the antecedent. In implicative systems the combination is made by genuine fuzzy implications $\to$ which are the generalizations of classical Boolen implication.

In our setting we work with residuated fuzzy implications which are derived from corresponding t-norms by process of residuation [3], [12]. Generally, for a t-norm $\star$ which is a mapping t from $[0,1]^2$ to $[0,1]$, i.e., $a, b \in [0,1]$, $t(a,b) \in [0,1]$ generalizing Boolean conjunction, we have the residuated implication $\to$ given as $i(a,b) = \sup_c \ t(a,c) \leq b$. The most common examples of t-norms and corresponding implications are product t-norm $t(a,b) = a \cdot b$ and Goguen implication $\to_P$ (derived from product): $a \to_P b = 1$, if $a \leq b$, otherwise $a \to_P b = b/a$; and minimum t-norm $t(a,b) = \min\{a,b\}$ and Gödel $\to_M$ implication (derived from minimum) $a \to_M b = 1$, if $a \leq b$, otherwise $a \to_M b = b$.

B. Radial fuzzy systems

Radial fuzzy systems use radial functions for representation of membership functions of employed fuzzy sets. Radial functions are well known from the area of radial basis neural networks [2]. A radial function $\Phi(\boldsymbol{x})$ is determined by its central point and one-dimensional (typically decreasing) function applied on a norm of a distance from this central point. Mathematically,

$$\Phi(\boldsymbol{x}) = act(\|\boldsymbol{x} - \boldsymbol{a}\|_{\boldsymbol{b}}), \tag{3}$$

where $\boldsymbol{x} \in \mathcal{R}^n$, $\boldsymbol{a}_j \in \mathcal{R}^n$ is the central point, $\|\cdot\|_{\boldsymbol{b}}$ is a scaled norm on $\mathcal{R}^n$ and act is a decreasing function $act(z) : [0,\infty) \to [0,1]$, such that $act(0) = 1$ and $\lim_{z\to\infty} act(z) = 0$.

In this paper the norm is considered to be scaled ℓ_p norm which is defined as ordinary ℓ_p norm for $p \in [1,\infty)$ and vector of scaling parameters $\boldsymbol{b} = (b_1, \ldots, b_n), b_i > 0$. Formally,

$$\|\boldsymbol{u}\|_{\boldsymbol{b}} = \left(\sum_i \frac{|u_i|^p}{b_i^p} \right)^{1/p}. \tag{4}$$

The choice $p = 2$ gives the scaled Euclidean norm.

The idea of radial fuzzy system is based on the notion of radial property. The radial property refers to the fact that the combination of individual fuzzy sets by a t-norm creates a multi-dimensional radial fuzzy set with the same shape. This property is not trivial in the sense that not all t-norms can be combined with arbitrary shapes of fuzzy sets. Formally, for a given act function and t norm $\star$ the following equation holds

$$act(|x_1 - a_1|_{b_1}) \star \cdots \star act(|x_n - a_n|_{b_n}) = act(\|\boldsymbol{x} - \boldsymbol{a}\|_{\boldsymbol{b}}). \tag{5}$$

Most prominent examples are combinations of product t-norm with Gaussian radial fuzzy sets and minimum t-norm with triangular shapes. The examples of these radial fuzzy sets are presented in Fig. 1. However, for example, the combination of triangular shape by product does not result into multi-dimensional triangular shape.

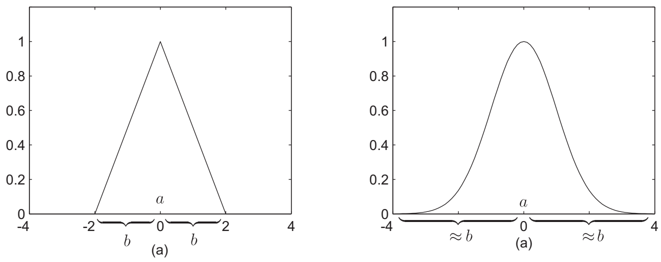

Fig. 1. One dimensional (a) triangular and (b) Gaussian radial fuzzy set.

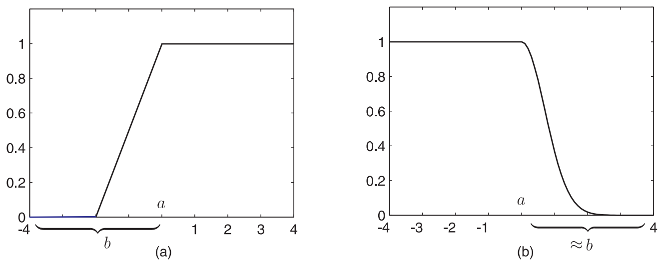

Fig. 2. One dimensional (a) S-shaped triangular and (b) Z-shaped Gaussian radial fuzzy set.

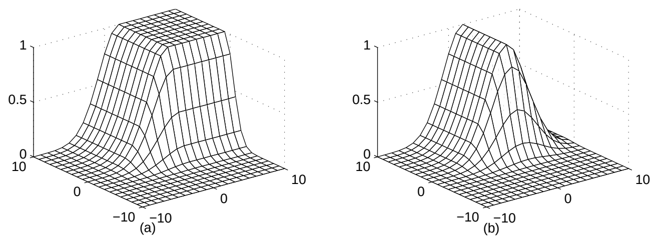

Fig. 3. Two dimensional S-shaped fuzzy sets.

C. S-shaped radial fuzzy systems

S-shaped fuzzy sets are the extension of the bell-shaped radial fuzzy sets to S-shaped or (Z-shaped) ones. The graphical examples are presented in Fig. 2 for S-shaped triangular and Z-shaped Gaussian fuzzy set.

Mathematically, these shapes are created by application of positive $u^{(+)}$ or negative part $u^{(-)}$ functions on u_i arguments. We have $u_i^{(+)} = u_i$ for $u_i \geq 0$ and $u_i^{(+)} = 0$ otherwise; $u_i^{(-)} = -u_i$ for $u_i \leq 0$ and $u_i^{(-)} = 0$ otherwise. Finally, the norm defined in terms of formula (4) i.e.,

$$\|\boldsymbol{u}\|_{s\boldsymbol{b}} = \left(\sum_i \frac{|u_i^{(+-)}|^p}{b_i^p} \right)^{1/p} \tag{6}$$

becomes to be scaled seminorm.

As it is shown in [14] the radial property is retained with scaled norm replaced by seminorm. Graphical examples in two dimensions are presented in Fig. 3.

D. Computational model

The computational model of radial and S-radial fuzzy implicative systems is presented here (the difference is in replacement of scaled norm by a scaled seminorm). First, the individual rules are combined into the form of single fuzzy relation which assembles into a fuzzy rule base. In the case of implicative systems individual rules are combined by fuzzy conjunction. The minimum is the most typical case.

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Formally, we have

$$RB(\boldsymbol{x}, y) = \min_j \{R_j(\boldsymbol{x}, y)\} = \min\{A_j(\boldsymbol{x}) \rightarrow B_j(y)\}. \quad (7)$$

The inference engine of implicative fuzzy systems actually seeks for outputs which are fully compatible with given input. For individual rules these are those classes y_k for which the following inequality holds: $A_j(\boldsymbol{x}) \leq B_j(y_k)$, i.e., these are those classes evaluating the fuzzy implication $A_j(\boldsymbol{x}) \rightarrow B_j(y_k)$ to one. Let us denote by $I_j(\boldsymbol{x})$ the set of such a classes. To obtain final output the intersection of these sets is taken in order to obtain the set of classes which is compatible with whole rule base. That is, the classes from this set are compatible with knowledge stored in the rule base of the fuzzy system.

If $I(\boldsymbol{x}) = \bigcap_j I_j(\boldsymbol{x})$ contains more than one class, the classification is taken as the class with the maximum sum of $B_j(y_k)$ values. Sum is taken over all js (rules).

The important question however is, how to assure that the intersection is non-empty for each possible input $\boldsymbol{x}$. This question of coherence is discussed in [5] and [14]. Here we only mention that there can be done study on this issue and it is possible to put conditions on parameters of fuzzy sets in order to the whole system be coherent.

III. EEG DATA

The electroencephalographic data (EEG data in short) which we analyze were obtained during experimental sessions held at the Joint Laboratory of System Reliability (JLSR) located at the Czech Technical University in Prague. The data were scanned via 19-th channel measuring cap from the driver's head as shown in Fig. 4.

Fig. 4. A driver with measuring cap with electrodes.

Volunteer drivers, mainly students or professional drivers, underwent different driving scenarios in test car provided by AUTO ŠKODA automaker. The car is surrounded by projection screen and its actuators are interconnected with the computer controlling the projection. During driving scenarios EEG signals were recorded and classified into appropriate classes by experts on the basis of objective (reaction time on a random acoustic signal) and subjective marks (face grimace, closing eyes).

The resulting database consists of 766 records classified into the following three classes:

- *mentation* (218 cases) - this class corresponds to a higher mental activity and higher vigilance level (the driver is performing simple counting)
- *wakefulness* (210 cases) - the driver is asked not to do any mental activity and to try to be relaxed while still driving
- *microsleep* (338 cases) - records classified into this class correspond to situation when the driver has long reaction time on an acoustic signal, get off the road or even got asleep

The data are preprocessed by transformation from time to frequency domain by means of Welch's Fourier transform. Each time record is transformed into 30-dimensional vector. In each dimension intensity of signal at each of frequencies from 1 to 30 Hz is stored. Graphically this process is demonstrated in Fig. 5

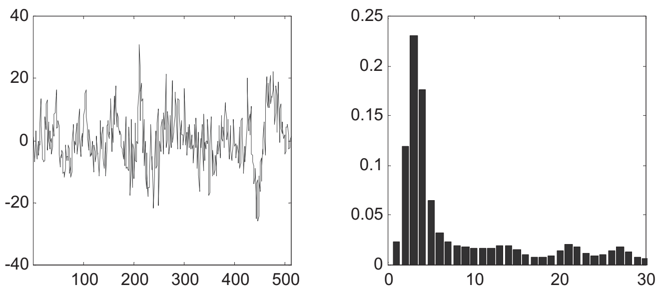

Fig. 5. EEG spectrum in time (left) and frequency (right) domain.

IV. CONSTRUCTION OF CLASSIFIER

In this section we introduce both phases of structure and parameter learning of S-radial implicative fuzzy system for classification of EEG spectrograms.

A. Structure learning

Structure learning is the process of selection of number of IF-THEN rules and initial setting of their parameters. For this purpose we use GUHA method of exploratory data analysis [6], [7].

The method as other similar ones works with an initial data matrix. In our setting, in the matrix each row corresponds to one record and columns to 30 frequencies + 1 column indicates the classification. Each column is endowed by set of its categories which are in fact intervals establishing the covering of range of signal activity in each of frequencies. Categories in the classification column correspond to individual classes of driver's vigilance.

The process of categorization transforms a data matrix into the dichotomized matrix of zero and ones with the same number of rows. Columns then correspond to individual categories. In each cell, there is value one if the intensity at corresponding frequency falls into the corresponding category, otherwise there is zero.

The exploratory engine of the method works with hypotheses on data and checks by statistical test their validities. The tests are in fact tests on contingency tables corresponding to individual hypotheses. A GUHA hypothesis consists of antecedent and consequent which are bounded by a generalized quantifier.

Ancedents and consequents (together called cedents) are in fact Boolean conjunctions of individual categories of given length. Each cedent then can be evaluated by one or zero with respect to the values stored in corresponding

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TABLE I
FOUR-FOLD TABLE

	consequent (C)	non(C)
antecedent (A)	a	b
non(A)	c	d

cells of dichotomized initial data matrix and according to rules of classical Boolean logic. After the evaluation over whole dichotomized data matrix we obtain the following contingency table (four-fold table) I.

In this table a is the number of records satisfying simultaneously both antecedent and consequent, b is the number of records satisfying A but not satisfying C, c the number of records not satisfying A but satisfying C, and finally d is the number of records simultaneously not satisfying antecedent and consequent. Clearly, $a + b + c + d$ gives the number of records (rows) in the database, hence in our case 766.

A generalized quantifier is given by its associated function which is a Boolean function operating on four-fold tables mapping them into one or zero. If the evaluation is one then the hypothesis is taken as valid in data (supported by statistical test) and unvalid if the evaluation is zero.

As the example we present FEQ (founded equivalence) quantifier, which has two parameters $base \in \mathcal{N}$ and $cp \in (0, 1]$.

$$\mathrm{FEQ}_{base,\,cp}(a, b, c, d) =$$

$$= \begin{cases} 1 & \text{if } a \geq base \text{ and } \frac{a+c}{a+b+c+d} \geq cp, \\ 0 & \text{otherwise.} \end{cases}$$

Parameter $base$ specifies minimal number of records satisfying A&C, usually $base = 10\%$ of the total number of records in data matrix and $cp \in [0, 1]$ specifies the strength of equivalence relation between Boolean properties coded by antecedent and consequent.

The effectiveness of GUHA method lies in its exploratory nature when during one GUHA run huge number of hypotheses is constructed and validated. An user of the method fixes lengths of antecedents and consequents together with the number of categories. This makes the exponential rise of the number of hypotheses tested. Typically, hundreds of thousands of hypotheses are tested during one run.

B. Application on EEG data

The application on EEG data is almost straightforward. In the categorization phase we select intervals corresponding to three categories of low, medium and high intensity of signal at given frequency. The selection of intervals is based on equifrequent criterion. That is, in each category there is (almost) equal numbers of records. The length of antecedents is set to 4 which is inspired by common aggregation of frequencies into 4 bands (delta, theta, alpha, beta). The length of consequent is set naturally to one which corresponds to individual categories.

The quantifier used is the above introduced founded equivalence quantifier which corresponds to the Fisher independence test. The interpretation of such a valid hypothesis is that under the validity of antecedent (i.e., the evaluation is one) the conditional probability of consequent is greater than unconditional one, i.e., validity of antecedent supports the validity of consequent and vice versa.

The output of GUHA run is a set of valid hypotheses sorted according to their strength (here it is the value of $(a + c)/766$ statistics). The hypotheses indentify the categories connected with corresponding classes in consequents.

The transformation of found valid hypotheses on parameters of the classifier follows. Central points of categories in antecedents are mapped to central points of fuzzy sets. For low and high categories S-radial fuzzy sets are employed. For medium category a proper radial fuzzy set is employed. Width parameters are set as halves of corresponding intervals. In consequent fuzzy set, the membership degrees are set to one for the class in the consequent of hypothesis, for other to zero.

C. Parameter learning

The process of parameter learning is the process of adjusting of initially set parameters. In our application we have used algorithm described in [13]. The algorithm is based on the Levenberg-Marquardt method. In each step parameters are adjusted in such way that the global error is decreased but the update of parameters is performed only if the coherence of the system is retained.

The coherence of the system is important issue of soundness of rules in the rule base. The case which applies in our example follows. Let the output of one rule is wakefulness class and the output of other rule is microsleep. Then the intersection of outputs is empty set and the classifier cannot take a decission. Hence coherence of the system is very important issue which in fact turns into the inner consitency of input-output relation represented by a rule base of the system.

The question of coherence is handled in several papers [8], [14]. The important result is that checking sufficient conditions for coherence is quadratic in number of rules hence it does not cause a serious decrease in the computational capability of the learning algorithm.

V. RESULTS AND CONCLUSIONS

The process of GUHA structure learning finished with the rule base consisting of 11 rules. Three related to mentation class, three to wakefulness class and five to microsleep class. Antecedents of rules operates on 4 dimensional space, outputs on 3 dimensional (actually on membership degrees of individual classes). Initially set parameters were adjusted by the referred parameter learning algorithm.

The above process of development and learning of classification system was set up in 10-fold cross validation design. That is, the whole database was split on 10-th parts each with proportion of individual classes retained; then each nine tenth of records were selected to form training data and one tenth to form testing data.

In table II we have the averaged rounded result concerning the accuracy of the classifier.

The global accuracy of our classifier is 79.2% (607/766). If only pure radial fuzzy sets are used the accuracy is 69.5%. So employment of S-shaped radial fuzzy sets brings an advantage without need for growing number of parameters. The main reason for this observation is that boundary regions

TABLE II
ACCURACY OF S-RADIAL FUZZY CLASSIFIER

real class ↓	mentation	wakefulness	microsleep
mentation	170	13	35
wakefulness	62	144	4
microsleep	38	6	293

of input space can be covered by only one S-shaped fuzzy set in contrary to a chain of pure radial fuzzy sets.

The other specific property of our approach is that the developed classifier has the logical structure of rules. When parameter learning is performed the coherence is retained. That is, the rules are created in such a way they do not constitute a contradictory rule base. Such a rule base can be further analyzed by means of fuzzy logic in narrow sense [12].

One possible way of application of coherent rule based systems is to incorporate them as knowledge bases in artificial agents which operate on different parts of domain of interest (for example exploration of a maze). Due to the construction of the rule base and implicational structure of individual rules this can be easily done.

The established methodology also allows incorporate a logical structure to radial neural networks, which is an interesting topic because neural networks generally represent rule-free computational models. This is the direction of our future research.

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