A Piecewise Constant Region Based Simultaneous Image Segmentation and Registration

Jung-Ha An and Yunmei Chen

***Abstract*—A new variational region based model for a simultaneous image segmentation and a rigid registration is proposed. The purpose of the model is to segment and register novel images simultaneously using a modified piecewise constant Mumford-Shah functional and region intensity values. The segmentation is obtained by minimizing a modified piecewise constant Mumford-Shah functional. A registration is assisted by the segmentation information and region intensity values. The numerical experiments of the proposed model are tested against synthetic data and simulated normal noisy human-brain magnetic resonance (MR) images. The preliminary experimental results show the effectiveness of the model in detecting the boundaries of the given objects and registering novel images simultaneously.**

***Index Terms*—Simultaneous segmentation and a rigid registration, a piecewise constant Mumford-Shah functional, region intensity, gamma-approximation, magnetic resonance images.**

I. Introduction

EXTRACTING the boundary and register the given images are the most important tasks in image processing, image analysis, and computer vision. There have been numerous techniques developed to solve image segmentation and registration problems. In the past, solutions of these two problems have been studied separately from each other.

Segmentation techniques have been developed to capture the object boundary by several different approaches; edge-based methods mainly using active contour models, region-based methods, or the combination of the two by using Geodesic Active Region models. Recently, the prior shape and intensity information has been also incorporated into deformable models [11], [10].

Image registration is the process of overlaying two or more images of the same scene taken at different times, and/or by different sensors. Area based methods or feature based methods have been developed to match given images. For the details of the image registration techniques, please refer to [27].

A joint segmentation and registration methods have been developed [24], [9], [6], [7], [8], [23], [2], [3], [22], [19], [14], [18], [21], [17]. An explicit combination of registration with segmentation has been developed in a variational framework through active contours [24], [25]. The algorithms of [24], [25] was extended in [12] to joint segmentation and of an object in two images. The morphing active contours algorithm is combined with the joint segmentation and registration with an application to CT scans for radiotherapy treatment planning [23].

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In this paper, a new variational region based model for a simultaneous image segmentation and a rigid registration is suggested. The purpose of the model is to segment and register given images simultaneously utilizing a modified piecewise constant Mumford-Shah functional and region intensity values. The model performs a simultaneous segmentation and registration in a similar way from [24], [25], [18]. But the model differs from [24], [25], [18] by using a region intensity and a modified Mumford-Shah functional [15]. The model in this paper is an extended work of [2] and a generalized to simultaneous segmentation and registration model from [4]. This paper is organized as follows: The region based model for a simultaneous segmentation and a rigid registration is proposed in section II. In section III, numerical results of the proposed model which were applied to synthetic and human brain MR images are shown. Finally, the conclusion follows and future work is stated in section IV.

II. Description of the Proposed Model

A. A Proposed Model

A new variational region based model for a simultaneous image segmentation and a rigid registration using a modified piecewise constant Mumford-Shah functional is suggested. The goal of the model is to segment and register given images simultaneously. The segmentation is obtained by minimizing a modified piecewise constant Mumford-Shah functional [15]. A global rigid registration is assisted by the segmentation information and region intensity value. A segmentation and registration process is obtained simultaneously in this model. Therefore, it is assumed that an image quality of I_1 is better than that of I_2 in the proposed model. The model is aimed to find θ, c_1, c_2, Υ, d_1, and d_2 by minimizing the energy functional:

$$\begin{aligned}
&E(\theta, c_1, c2, \Upsilon, d_1, d_2) = \\
&\lambda_1 \int_\Omega \{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2 (I_1(\bar{x})-c_1)^2 + \\
&\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2 (I_1(\bar{x})-c_2)^2 d\bar{x} + \\
&\lambda_2 \int_\Omega \{\frac{\epsilon_1|\nabla(\frac{\theta}{\epsilon})|^2}{\pi^2(1+(\frac{\theta}{\epsilon})^2)^2} + \frac{(\pi^2-4\arctan^2(\frac{\theta}{\epsilon}))^2}{\epsilon_1\pi^4}\} d\bar{x} + \\
&\lambda_3 \int_\Omega \{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2 (I_2(\Upsilon(\bar{x}))-d_1)^2 + \\
&\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2 (I_2(\Upsilon(\bar{x}))-d_2)^2) d\bar{x}, \qquad (1)
\end{aligned}$$

where I_1 and I_2 are novel images, Ω is domain, $\lambda_i > 0$ $(i = 1, 2, 3)$, ϵ, and ϵ_1 are parameters balancing the influences from the four terms in the model.

The first term forces $\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2$, towards 0 if $I_1(\bar{x})$ is different from c_1 and towards 1 if $I_1(\bar{x})$ is close to c_1, for every $\bar{x}\in\Omega$. In a similar way, $\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2$, towards 0 if $I_1(\bar{x})$ is different from c_2 and towards 1 if $I_1(\bar{x})$ is close to c_2, for every $\bar{x}\in\Omega$.

The second term is an edge detection term. In the theory of Γ-convergence, the measuring an edge Γ length term in the Mumford-Shah model [13] can be approximated by a quadratic integral of an edge signature function $p(x)$ such that

$$\int_\Gamma dS=\int_\Omega(\epsilon_1|\nabla p|^2+\frac{(p-1)^2}{4\epsilon_1})d\bar{x}, \epsilon_1\ll 1$$

by Ambrosio and Tortorelli in 1990 [1]. This model is combined with double-well potential function $W(p)=p^2(1-p)^2$ which is quadratic around its minima and is growing faster than linearly at infinity, where $p\in H^1(\Omega)$. In [15], the following model is suggested:

$$\int_\Omega(9\epsilon_1|\nabla p_i|^2+\frac{(p_i(1-p_i))^2}{\epsilon_1})d\bar{x},$$

where for each $p_i\in H^1(\Omega), i=1,2$. Here $\epsilon_1\ll 1$ controls the transition bandwidth. As $\epsilon_1\to 0$, the first term is to penalize unnecessary interfaces and the second term forces the stable solution to take one of the two phase field values 1 or 0. For the details of phase field models and double-well potential functions, please refer [15] and [20]. In our model, the idea is followed from [15].

For each $\bar{x}\in\Omega$, the third term is forcing $\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2$, towards 0 if $I_2(\Upsilon(\bar{x}))$ is different from d_1 and towards 1 if $I_2(\Upsilon(\bar{x}))$ is close to d_1. In a similar way, $\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2$, towards 0 if $I_2(\Upsilon(\bar{x}))$ is different from d_2 and towards 1 if $I_2(\Upsilon(\bar{x}))$ is close to d_2. After minimizing these three terms, the best θ, Υ, c_1, c_2, d_1, and d_2 are obtained. In our proposed algorithm, only rigid transformation is considered. Therefore the rigid transformation vector $\Upsilon(\bar{x})$ is equal to $\mu R\bar{x}+T$, where μ is a scaling, R is a rotation matrix with respect to an angle Θ, and T is a translation.

B. Euler-Lagrange Equations of the Proposed Model

The evolution equations associated with the Euler-Lagrange equations in Equation (1) are

$$\begin{aligned}\frac{\partial\theta}{\partial t}&=-\lambda_1\{(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))(\frac{\epsilon^2}{\pi(\epsilon^2+\theta^2)})(I_1(\bar{x})-c_1)^2\\&-(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))(\frac{\epsilon^2}{\pi(\epsilon^2+\theta^2)})(I_1(\bar{x})-c_2)^2\}\\&+\lambda_2\{\frac{72\epsilon_1}{\pi^2}(div(\frac{\nabla\theta}{(1+\theta^2)^2})+\frac{2\theta|\nabla\theta|^2}{(1+\theta^2)^3})\}\\&+\frac{(\pi^2-4\arctan^2(\frac{\theta}{\epsilon}))\arctan(\frac{\theta}{\epsilon})}{4\epsilon_1\pi^4(1+(\frac{\theta}{\epsilon})^2)}\\&-\lambda_3\{(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))(\frac{\epsilon^2}{\pi(\epsilon^2+\theta^2)})(I_2(\Upsilon(\bar{x}))-d_1)^2\\&-(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))(\frac{\epsilon^2}{\pi(\epsilon^2+\theta^2)})(I_2(\Upsilon(\bar{x}))-d_2)^2\},\quad in\ \Omega\\\frac{\partial\theta}{\partial n}&=0,\quad on\ \partial\Omega,\end{aligned}$$

$$\frac{\partial c_1}{\partial t}=\lambda_1\int_\Omega\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2(I_1(\bar{x})-c_1)d\bar{x},$$

$$\frac{\partial c_2}{\partial t}=\lambda_1\int_\Omega\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2(I_1(\bar{x})-c_2)d\bar{x},$$

$$\begin{aligned}\frac{\partial\mu}{\partial t}&=-\lambda_3\int_\Omega\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon}))\}^2\\&(I_2(\Upsilon(\bar{x}))-d_1)(\nabla I_2(\Upsilon(\bar{x})))R\bar{x}d\bar{x}-\\&\lambda_3\int_\Omega\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2\\&(I_2(\Upsilon(\bar{x}))-d_2)(\nabla I_2(\Upsilon(\bar{x})))R\bar{x}d\bar{x},\end{aligned}$$

$$\begin{aligned}\frac{\partial\Theta}{\partial t}&=-\lambda_3\int_\Omega\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2\\&(I_2(\Upsilon(\bar{x}))-d_1)(\nabla I_2(\Upsilon(\bar{x})))\mu\frac{dR}{d\Theta}\bar{x}d\bar{x}-\\&\lambda_3\int_\Omega\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2\\&(I_2(\Upsilon(\bar{x}))-d_2)(\nabla I_2(\Upsilon(\bar{x})))\mu\frac{dR}{d\Theta}\bar{x}d\bar{x},\end{aligned}$$

$$\begin{aligned}\frac{\partial T}{\partial t}&=-\lambda_3\int_\Omega\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2\\&(I_2(\Upsilon(\bar{x}))-d_1)(\nabla I_2(\Upsilon(\bar{x})))d\bar{x}-\\&\lambda_3\int_\Omega\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2\\&(I_2(\Upsilon(\bar{x}))-d_2)(\nabla I_2(\Upsilon(\bar{x})))d\bar{x},\end{aligned}$$

$$\frac{\partial d_1}{\partial t}=\lambda_3\int_\Omega\{\frac{1}{2}(1+\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2(I_2(\Upsilon(\bar{x}))-d_1)d\bar{x},$$

$$\frac{\partial d_2}{\partial t}=\lambda_3\int_\Omega\{\frac{1}{2}(1-\frac{2}{\pi}\arctan(\frac{\theta}{\epsilon})\}^2(I_2(\Upsilon(\bar{x}))-d_2)d\bar{x},$$

where R is the rotation matrix in terms of the angle Θ.

III. Numerical Results

The numerical results with applications to the synthetic image and simulated normal human brain MR images are shown in this section. The simulated normal human brain MR images are obtained from *http://www.bic.mni.mcgill.ca/brainweb*.

The Equation (1) was solved by finding a steady state solution of the evolution equations. The evolution equations are associated with the Euler-Lagrange equations of (1). A finite difference scheme and the gradient descent method is applied to discretize the evolving equations.

The experimental results of the proposed model with applications to the synthetic data and simulated brain MR images are shown in Figures 1, 2, 3, 4, 5, 6 and 7.

The numerical result with an application to the synthetic data is shown in Figure 1. The first one in Figure 1 is the given image I_1. An image I_2 was created by adding noise, loss of information, and rotation from an image I_1.

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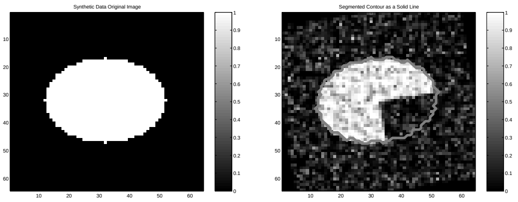

Fig. 1. First: A Synthetic Image I_1 and Second: The Segmented Synthetic Image I_2 with Noise, Loss of Information, and Rotation with a Solid Line

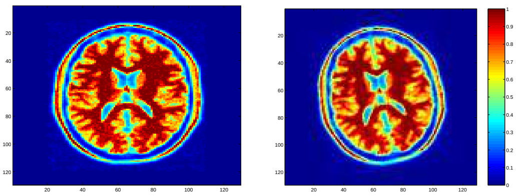

Fig. 2. First: A Novel Noisy Image I_1 and Second: A Novel Rotated Noisy Image I_2

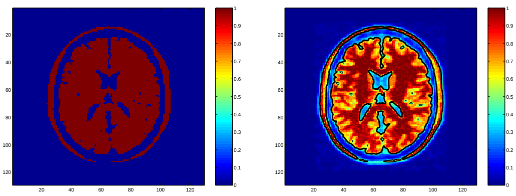

Fig. 3. First: The Segmented Simulated Brain Image of I_1 and Second: The Segmented Simulated Brain Image as a Solid Line of I_1

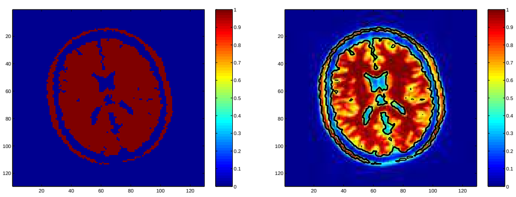

Fig. 4. First: The Segmented Simulated Brain Image of I_2 and Second: The Segmented Simulated Brain Image as a Solid Line of I_2

The second one in Figure 1 is the experimental result of the proposed model to an image I_2 as a solid line.

Simulated MRI volumes for normal brain with T1 modality are obtained for Figures 2, 3, 4, 5, 6, and 7. Slice thickness is $1mm$ with 9% noise and 20% non-uniformity intensity level. A given image I_2 is created by rotating an noisy image I_1.

In Figure 2 and Figure 5, the first one is the given noisy image I_1 and the second one is the given rotated image I_2 from I_1.

In Figure 3 and Figure 6, the first image is acquired by $H(\theta)$, where $H(\theta) = 1, \ if \ \theta \geq 0$ and $H(\theta) = 0 \ if \ \theta < 0$, and the second one is the segmented image I_1 with a solid line.

In a similar way, in Figure 4 and Figure 7, the first image is acquired by $H(\theta(\Upsilon))$ and the second one is the segmented image I_2 with a solid line.

IV. Conclusion

A new variational region based model for a simultaneous image segmentation and a rigid registration using a modified

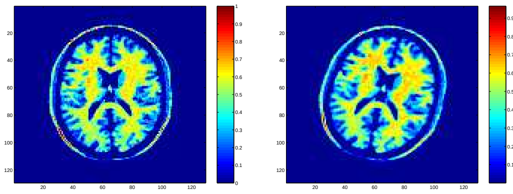

Fig. 5. First: A Novel Noisy Image I_1 and Second: A Novel Rotated Noisy Image I_2

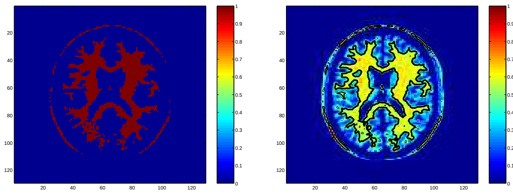

Fig. 6. First: The Segmented Simulated Brain Image of I_1 and Second: The Segmented Simulated Brain Image as a Solid Line of I_1

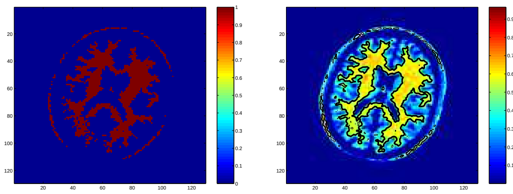

Fig. 7. First: The Segmented Simulated Brain Image of I_2 and Second: The Segmented Simulated Brain Image as a Solid Line of I_2

piecewise constant Mumford-Shah functional is presented. The goal of the model is to segment and register given images simultaneously utilizing a modified piecewise constant Mumford-Shah functional and region intensity values. The numerical results from Figure 1 to Figure 7 show the effectiveness of the model in detecting the boundaries of the given objects and registering images simultaneously. In addition, the numerical results show the robustness of the proposed model to noise, artifact, and loss of information. This model can be extended to 3D case and any other types of images. Generalization of the model to non-rigid registration is needed for the future work.

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