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2D Angle and Doppler Frequency Estimation in MIMO Radar

Wentao Shi, Jianguo Huang and Chengbing He

***Abstract*—A novel algorithm for the joint estimation of the Direction Of Departure (DOD)-Direction Of Arrival (DOA) and Doppler frequency for bistatic multiple-input multiple-output (MIMO) radar is proposed. The proposed algorithm uses the rotational factor produced by the time delay of sampling to construct an angle matrix by fully utilizing the properties of the auto-covariance and cross-covariance matrices. The eigenvalues of the angle matrix and the corresponding eigenvectors are related to the Doppler frequency and the DOD-DOA estimation separately. The ESPRIT without pairing method is used to obtain automatically paired DOD and DOA estimation. Simulation results are present to verify the effectiveness of the proposed algorithm.**

***Index Terms*—Direction Of Departure (DOD), Direction Of Arrival (DOA), multiple-input multiple-output (MIMO) radar, angle matrix, ESPRIT**

I. INTRODUCTION

THE idea of multiple-input multiple-output (MIMO) has been recently become a hot research for its potential advantages. MIMO radar has been proposed as a new radar system with various applications [1][2][3][4], which uses multiple antennas to transmit simultaneously several orthogonal waveforms and receives the reflected signals with multiple antennas.

According to the array configuration, MIMO radar can be subdivided into two categories, one of which has its transmit and receive antennas widely spaced and is called as statistical MIMO radar [1][4][5]. In this scenario, all the transmit array elements are widely separated and radiate independent signals to different look-directions of the target, which can overcome performance degradations caused by target scintillations. The other one is named as monostatic or bistatic MIMO radar [3][6][7], in which the transmit and receive antennas are closely spaced. The latter can obtain many advantages by exploiting waveform diversity. As a result of enhanced flexibility in the design of transmitting beampattern and waveform synthesis, the performance of multiple target detection and identification can be improved [8]. Furthermore, the maximum number of targets to be detected and located by the array is increased [6][9]. High-resolution spatial spectrum estimation is also obtained by extended array aperture with virtual sensors and narrower beams [2].

A bistatic MIMO radar scheme is proposed to identify and locate multiple targets, in which both the transmit array and receive array are uniform linear arrays. In [7], a two-dimensional (2-D) spatital spectrum estimation technique based on the Capon method is developed, in which the maximum number of localizable targets is the product of the number of receive and transmit elements minus one. However, the proposed method needs exhaustive search through all the 2-D space to find the DOA and DOD of the targets. In order to avoid the angle search, the ESPRIT algorithm [10]is applied to bistatic MIMO radar by exploiting the invariance property of the transmit and receive array, but an additional pair matching between DOAs and DODs of targets is required. Another similar algorithm is developed in [11] where the pairing is automatically obtained. In [12], the ESPRIT method is also applied to estimate target angles in bistatic MIMO radar. A closed formulation of the target DOA and DOD is obtained, which are automatically paired, and this method can cancel spatially colored noise. However, the number of targets which can be localized by this method is smaller than the number of receivers.

In this paper, we present a method for the joint estimation of Angle and Doppler frequency for bistatic MIMO radar. The proposed algorithm uses the rotational factor produced by the time delay of sampling to construct an angle matrix by fully utilizing the properties of the auto-covariance and cross-covariance matrices. The eigenvalues of the angle matrix and the corresponding eigenvectors are related to the Doppler frequency and the DOD-DOA estimation separately and respectively. The ESPRIT without pairing method in [11] is used to obtain angle estimation and the DOD, DOA and Doppler frequency can be obtained simultaneously and automatically paired.

This paper is organized as follows. The bistatic MIMO radar signal model is present in Section II. In Section III, the proposed algorithm is given in detail. The simulations and analysis of the results for the proposed method are given in Section IV. Finally, Section V concludes the paper.

II. SIGNAL MODEL

Consider a bistatic radar system consisting of an M-element transmit array and an N-element receive array, both of which are a uniform linear array (ULA) and all the elements are omnidirectional, shown in Figure 1. Δ_t and Δ_r are the inter-element spacing at the transmitter and receiver, respectively. M different orthogonal narrowband waveformsare emitted simultaneously at the transmit site.

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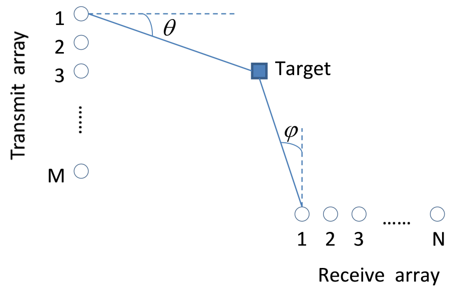

Fig. 1. Bistatic MIMO radar scenario.

The received signals can be matched by the transmitted waveforms. Assume that there are P uncorrelated targets at the same range, and the DOD and DOA of the pth target relative to the transmitter and the receiver are denoted by θ_p and φ_p ($p = 1, 2, \cdots, P$), respectively. The outputs of the matched filters at the receiver can be expressed as

$$x(t) = AS(t) + W(t) \tag{1}$$

where $A = [a_r(\varphi_1) \bigotimes a_t(\theta_1), \cdots, a_r(\varphi_P) \bigotimes a_t(\theta_P)]$ is an $NM \times P$ matrix composed of the P steering vectors. $a_r(\varphi_p) \bigotimes a_t(\theta_p)$ is the Kronecker product of the transmit and receive steering vectors for the pth target. $a_r(\varphi_p) = \left[1, e^{j2\pi\Delta_r \sin(\varphi_{\mathrm{p}})/\lambda}, \cdots, e^{j2\pi(N-1)\Delta_r \sin(\varphi_{\mathrm{p}})/\lambda}\right]^{\mathrm{T}}$ and $a_t(\theta_p) = \left[1, e^{j2\pi\Delta_t \sin(\theta_{\mathrm{p}})/\lambda}, \cdots, e^{j2\pi(M-1)\Delta_t \sin(\theta_{\mathrm{p}})/\lambda}\right]^{\mathrm{T}}$. λ denotes the wavelength. $S(t) = [s_1(t), s_2(t), \cdots, s_P(t)]^{\mathrm{T}}$ is a column vector consisting of the amplitudes and phases of the P sources at time t. $s_p(t) = \eta_p e^{j2\pi f_p t}$, where η_p is the reflection coefficient depending on the target RCS and f_p is the Doppler frequency. W(t) is an $NM \times 1$ complex Gaussian white noise vectors with zero mean and covariance matrix $\sigma^2 I$. $[\cdot]^{\mathrm{T}}$ denotes transpose operator.

III. The Proposed Method

Assume that the number of snapshots is L. Let

$$X_1 = [x(1), x(2), \cdots, x(L-1)] = AS_1 + W_1 \tag{2}$$

$$X_2 = [x(2), x(3), \cdots, x(L)] = AS_2 + W_2 \tag{3}$$

where $S_1 = [S(1), S(2), \cdots, S(L-1)]$, $S_2 = [S(2), S(3), \cdots, S(L)]$, $W_1 = [W(1), W(2), \cdots, W(L-1)]$, $W_2 = [W(2), W(3), \cdots, W(L)]$. Note that $S_2 = \Lambda S_1$, (3) can be expressed as

$$X_2 = A\Lambda S_1 + W_2 \tag{4}$$

where $\Lambda = \mathrm{diag}\left[e^{j2\pi f_1}, e^{j2\pi f_2}, \cdots, e^{j2\pi f_P}\right]$, diag denotes a diagonal matrix constructed by a vector.

The auto-covariance matrix of X_1 is given by

$$R_{X_{11}} = E[X_1 X_1^{\mathrm{H}}] = AR_S A^{\mathrm{H}} + \sigma^2 I \tag{5}$$

where $R_S = E[S_1 S_1^{\mathrm{H}}]$ is the covariance matrix of signals, $[\cdot]^{\mathrm{H}}$ denotes complex conjugate transpose and E represents the expectation operator. The cross-covariance matrix of X_1 and X_2 is given by

$$R_{X_{21}} = E[X_2 X_1^{\mathrm{H}}] = A\Lambda R_S A^{\mathrm{H}} \tag{6}$$

As discussed in [13], (5) can be rewritten as

$$R_{X_{11s}} = R_{X_{11}} - \sigma^2 I = AR_S A^{\mathrm{H}} \tag{7}$$

Obviously, the rank of $R_{X_{11s}}$ is equal to P, the number of signals. Let $\{\mu_1 \geq \mu_2 \geq \cdots \geq \mu_K\}$ and $\{\nu_1, \nu_2, \cdots, \nu_K\}$ be the eigenvalues and corresponding eigenvectors of the matrix $R_{X_{11s}}$, respectively.

$$R_{X_{11s}} = \sum_{k=1}^{K} \mu_k \nu_k \nu_k^{\mathrm{H}} \tag{8}$$

with the assumptions that $K > P$.

From (7) we obtain

$$R_S A^{\mathrm{H}} = (A^{\mathrm{H}} A)^{-1} A^{\mathrm{H}} R_{X_{11s}} \tag{9}$$

Substitute (9) into (6)

$$R_{X_{21}} = A\Lambda (A^{\mathrm{H}} A)^{-1} A^{\mathrm{H}} R_{X_{11s}} \tag{10}$$

Define a matrix R referred to as the angle matrix

$$R = R_{X_{21}} R_{X_{11s}}^{\#} \tag{11}$$

where $R_{X_{11s}}^{\#}$ denotes the pseudoinverse of $R_{X_{11s}}$, and $R_{X_{11s}}^{\#}$ is constructed by the non-zero eigenvalues μ_p, and corresponding eigenvectors ν_p of $R_{X_{11s}}$.

$$R_{X_{11s}}^{\#} = \sum_{p=1}^{P} \mu_p^{-1} \nu_p \nu_p^{\mathrm{H}} \tag{12}$$

From (10) we obtain

$$\begin{aligned} R_{X_{21}} R_{X_{11s}}^{\#} A &= A\Lambda (A^{\mathrm{H}} A)^{-1} A^{\mathrm{H}} R_{X_{11s}} R_{X_{11s}}^{\#} A \\ &= A\Lambda (A^{\mathrm{H}} A)^{-1} A^{\mathrm{H}} A \\ &= A\Lambda \end{aligned} \tag{13}$$

Substitute (11) into (13), we have

$$RA = A\Lambda \tag{14}$$

Thus, we can estimate the Doppler frequencies and DOD-DOA from the P nonzero eigenvalues $[\mu_1, \mu_2, \cdots, \mu_P]$ and the corresponding eigenvectors $[\nu_1, \nu_2, \cdots, \nu_P]$ of the angle matrix R, respectively. The Doppler frequency of the pth signal is

$$f_p = \arg\left(\frac{\mu_p}{2\pi}\right) \tag{15}$$

The signal subspace matrix U_S with size $NM \times P$ is composed of eigenvectors $[\nu_1, \nu_2, \cdots, \nu_P]$ corresponding to the largest P eigenvalues of angle matrix R. Therefore, there exists a unique non-singular T such that $U_S = AT$. Define a new $NM \times P$ matrix $A' = [a_t(\theta_1) \bigotimes a_r(\varphi_1), \cdots, a_t(\theta_P) \bigotimes a_r(\varphi_P)]$. Then the matrix A' is row equivalent to A, and there exists an $NM \times NM$ transformation matrix B corresponding to the finite number of row interchange operations such that $A' = BA$. Assume that $U_S' = BU_S = BAT = A'T$, where U_S' also is the $NM \times P$ signal subspace matrix formed from U_S by the same row interchanged operations as A' is formed from A.

Now, we use the method in [11] to estimate the DOD and DOA. Define A_{r1} and A_{r2} be the $M(N-1) \times P$ submatrices of A consisting of the first and the last $M(N-1)$ rows of A, respectively. Similarly, let A_{t1} and A_{t2} be the $N(M-1) \times P$ submatrices of A' consisting of the first and the last $N(M-1)$ rows of A', respectively. Then

$$A_{r2} = A_{r1} \Phi_r \tag{16}$$

$$A_{t2} = A_{t1} \Phi_t \tag{17}$$

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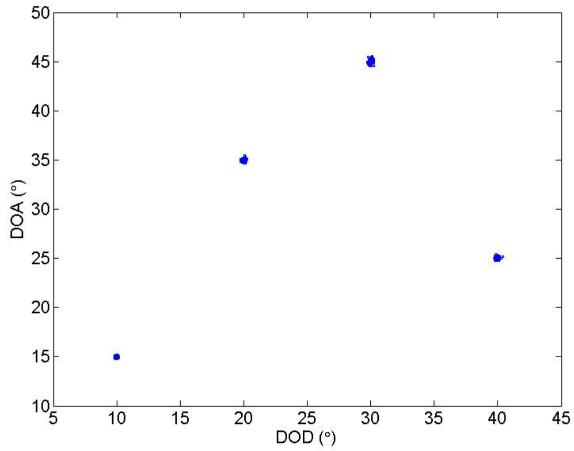

Fig. 2. Angle estimation of proposed method.

where
$\Phi_r = \mathrm{diag}[e^{j2\pi\Delta_r \sin\varphi_1/\lambda}, e^{j2\pi\Delta_r \sin\varphi_2/\lambda}, \cdots, e^{j2\pi\Delta_r \sin\varphi_P/\lambda}]$ and $\Phi_t = \mathrm{diag}[e^{j2\pi\Delta_t \sin\theta_1/\lambda}, e^{j2\pi\Delta_t \sin\theta_2/\lambda}, \cdots, e^{j2\pi\Delta_t \sin\theta_P/\lambda}]$. Let U_{r1}, U_{r2} be the $M(N-1) \times P$ submatrices formed from U_S in the same way as the A_{r1} and A_{r2} are formed from A. Then the diagonal elements of Φ_r are the eigenvalues of $\Psi_r = T^{-1}\Phi_r T$, which satisfy

$$U_{r2} = U_{r1}\Psi_r \tag{18}$$

Then we have

$$\Psi_r = U_{r1}^{\#} U_{r2} \tag{19}$$

Suppose that $Q = T^{-1}$, where the pth column of Q is an eigenvector corresponding to the pth eigenvalue of Ψ_r. Then we obtain the relationship

$$\hat{\Phi}_r = Q^{-1}\Psi_r Q \tag{20}$$

The eigenvalue decomposition (EVD) of Ψ_r yields $\hat{\Phi}_r = \hat{Q}^{-1}\Psi_r\hat{Q}$, where $\hat{\Phi}_r$ is a diagonal matrix, and the columns of $\hat{Q}$ are the eigenvectors of Ψ_r. Define

$$\hat{A}' = U_S \hat{T}^{-1} = U_S \hat{Q} \tag{21}$$

Then we can form $\hat{A}'_{t1}$ and $\hat{A}'_{t1}$ from $\hat{A}'$ in the same way as the A_{t1} and A_{t2} are formed from A'. From (18), we have

$$\hat{\Phi}_t = \hat{A}'^{\#}_{t1}\hat{A}'_{t2} \tag{22}$$

Therefore, the estimates of the transmit angle and the receive angle of the same signal can be acquired from the diagonal elements of $\hat{\Phi}_t$ and $\hat{\Phi}_r$ at the same position.

$$\theta_p = \arcsin\left(\frac{\lambda}{2\pi\Delta_t}\arg\left(\hat{\Phi}_t(p,p)\right)\right) \tag{23}$$

$$\varphi_p = \arcsin\left(\frac{\lambda}{2\pi\Delta_r}\arg\left(\hat{\Phi}_r(p,p)\right)\right) \tag{24}$$

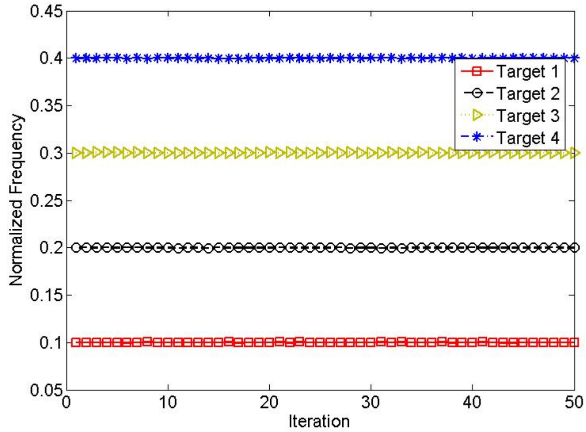

Fig. 3. Estimation of Doppler Frequency.

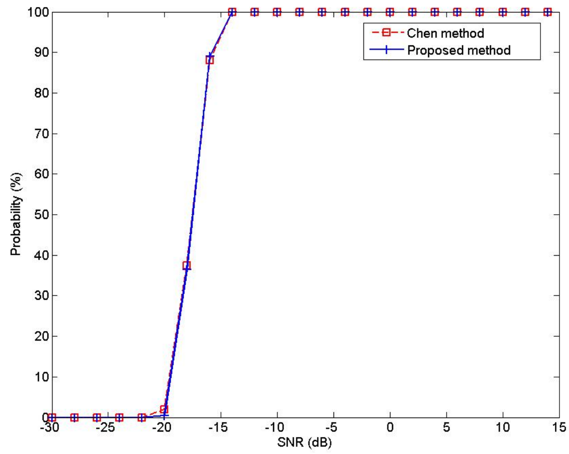

Fig. 4. Resolution Probability.

IV. Simulation Results

In this section, we present the simulation results in order to illustrate the performance of the proposed algorithm.

Consider a bistatic MIMO radar consisting of $M = 8$ transmit elements and $N = 6$ elements in the receiver array, both of which are spaced by a half wavelength. The number of snapshot is $L = 100$ and $SNR = 10dB$. Assume that there exit $P = 4$ uncorrelated targets located at the angles $(\varphi_r, \theta_t) = (15°, 10°), (25°, 40°), (35°, 20°), (45°, 30°)$ and their RCSs are given by $\eta_1 = \eta_2 = \eta_3 = \eta_4 = 1$. The normalized Doppler frequencies of the four targets are 0.1, 0.2, 0.3 and 0.4. The number of Monte-Carlo iterations is fixed at 100. The root mean squared error (RMSE) of DOD and DOA estimation are defined as

$$RMSE_{DOD} = \frac{1}{P}\sum_{p=1}^{P}\sqrt{E(\hat{\theta}_p - \theta_p)^2} \tag{25}$$

and

$$RMSE_{DOA} = \frac{1}{P}\sum_{p=1}^{P}\sqrt{E(\hat{\varphi}_p - \varphi_p)^2} \tag{26}$$

respectively, where $\hat{\theta}_p$ and $\hat{\varphi}_p$ are the DOD and DOA estimates for the same target, and their actual values are θ_p and φ_p, respectively.

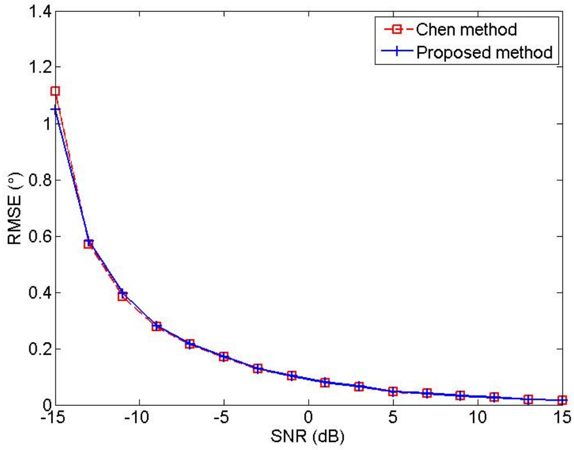

Fig. 5. RMSE in DOD estimation versus SNR

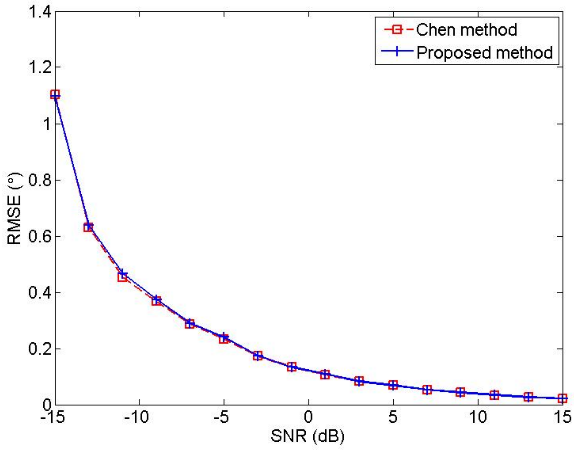

Fig. 6. RMSE in DOA estimation versus SNR

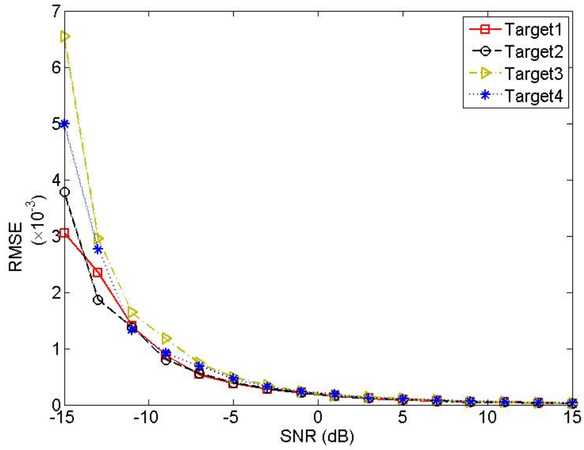

Fig. 7. RMSE in Doppler Frequency estimation from versus SNR

Figure 2 presents the obtained result by using the proposed method from 100 Monte-Carlo simulations for all four target with $SNR = 10dB$. We observe that the target directions are well localized and these directions are automatically paired.

Figure 3 shows the Doppler frequencies estimation results of 50 Monte-Carlo simulation with $SNR = 10dB$ for all four target, where the Doppler frequencies can be estimated very well.

Figures 4, 5 and 6 present the average of RMSE of the estimated target DOD and DOA by the proposed algorithm and the Chen's ESPRIT algorithm proposed in [11] from 100 Monte-Carlo simulations. We observe that the robustness to noise of the two algorithms are almost the same. The proposed algorithm can achieve the same performance of direction estimation which achieved by the Chen's ESPRIT algorithm.

Figures 7 shows the RMSE of the Doppler frequency of the four targets using the proposed algorithm, in which the simulation parameters are the same as in Figure 5.

V. Conclusion

In this paper, a novel algorithm for the joint estimation of the Direction Of Departure (DOD)-Direction Of Arrival (DOA) and Doppler frequency for bistatic multiple-input multiple-output (MIMO) radar is proposed by exploiting the rotational factor produced by time delay sampling. The properties of the auto-covariance and cross-covariance matrices are used to construct an angle matrix. Then, the eigenvalues of the angle matrix and the corresponding eigenvectors are related to the Doppler frequency and the DOD-DOA estimates separately. Finally, the ESPRIT without pairing method is used to obtain automatically paired DOD and DOA estimation. The simulation results are presented to verify the effectiveness of the proposed algorithm.

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