Capacity and Optimal Power Allocation of Poisson Optical Communication Channels

Samah A. M. Ghanem, *Member, IAENG,* Munnujahan Ara, *Member, IAENG*

***Abstract*— in this paper, the channel capacity for different models of Poisson optical communication channels has been derived. The closed form expression for the single input single output (SISO) Poisson channel -derived by Kabanov in 1978, and Davis in 1980- will be investigated. In addition, we derive closed form expressions for the capacity of the parallel Poisson channel and the capacity of the multiple access Poisson Channel (MAC). The optimum power allocation is also derived for different models; results have been analyzed in the context of information theory and optical communications.**

***Index Terms*—MAC, Parallel Channels, Poisson Channels, Power Allocation, SISO.**

I. Introduction

THE optical communications applications at backbone as well as access networks dictates the need for finding closed form expressions of the information capacity. In particular, the capacity expressions of Poisson channels that model the application. Therefore, in this paper, we accomplish an information-theoretic approach to derive the closed form expressions for: the SISO Poisson channel already found by Kabanov [1] and Davis [2], the parallel multiple input multiple output (MIMO) Poisson channels, as well as for the MAC Poisson channel. Several contributions using information theoretic approaches to derive the capacity of Poisson channels under constant and time varying noise via martingale processes or via approximations using Bernoulli processes are in [1-5], to define upper and lower bounds for the capacity and the rate regions of different models in [6-7], to define relations between information measures and estimation measures [8], in addition to deriving optimum power allocation for such channels [4] [9]. However, this paper introduces a simple framework similar to [4] for deriving the capacity of Poisson with any model of consideration, in addition, it builds upon derivations for the optimal power allocation for SISO, Parallel, and MAC models, or any other Poisson channel model of consideration.

In Poisson channels, the shot noise is the dominant noise whenever the power received at the photodetector is high; such noise is modeled as a Poisson random process. In fact, such framework has been investigated in many researches, see [1-4], [6-10]. Capitalizing on the expressions derived on [1-2], [4] and on the results by [4], [8], we re-investigate the derivation process in a simple step by step way, we then obtain the optimal power allocation that maximizes the information rates.

The paper is organized as follows, Section I introduces the SISO Poisson channel, the notions used in the context of the paper, as well as the optimal power allocation that maximizes the capacity. Section II introduces the Parallel Poisson channel capacity expression as a normal generalization of the SISO setup, as well as the optimal power allocation. Section III introduces the MAC Poisson channel capacity as well as the optimal power allocation. Finally, we conclude the paper by some simulations and analytical results.

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II. The SISO Poisson Channel

Consider the SISO Poisson channel $\mathscr{P}$ shown in Fig.1. Let $N(t)$ represent the channel output, which is the number of photoelectrons counted by a direct detection device (photo-detector) in the time interval [0, T]. $N(t)$ has been shown to be a doubly stochastic Poisson process with instantaneous average rate $\lambda(t) + n$. The input $\lambda(t)$ is the rate at which photoelectrons are generated at time t in units of photons per second. And n is a constant representing the photo-detector dark current and background noise.

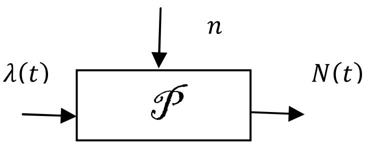

Fig. 1. The SISO Poisson channel model

A. Derivation of the Capacity of SISO Poisson Channels

Let $p(N_T)$ be the sample function density of the compound regular point process $N(t)$ and $p(N_T|S_T)$ be the conditional sample function of $N(t)$ given the message signal process $S(t)$ in the time interval [0,T]. Then we have,

$$p(N_T|S_T) = e^{-\int_0^T (\lambda(t)+n)\, dt + \int_0^T \log(\lambda(t)+n) dN(t)} \qquad (1)$$

$$p(N_T) = e^{-\int_0^T (\widehat{\lambda(t)}+n)\, dt + \int_0^T \log(\widehat{\lambda(t)}+n) dN(t)} \qquad (2)$$

We use the following consistent notation in the paper, $\widehat{\lambda(t)}$ is the estimate of the input $\lambda(t)$. $\mathbb{E}$ is the expectation operation over time. Therefore, the mutual information is defined as follows,

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$$I(S_T;N_T) = \mathbb{E}\left[log\left(\frac{p(N_T|S_T)}{p(N_T)}\right)\right] \quad (3)$$

Theorem1 (Kabanov'78[1]-Davis'80 [2]):

The capacity of the SISO Poisson channel is given by:

$$C = \frac{K}{P}(P+n)\log(P+n) + \left(1-\frac{K}{P}\right)nlog(n) - (K+n)\log(K+n) \quad (4)$$

Proof:

Substitute (1) and (2) in (3), we have,

$$I(S_T;N_T) = \mathbb{E}\left[-\int_0^T (\lambda(t) - \widehat{\lambda(t)})\,dt + \int_0^T \log\left(\frac{\lambda(t)+n}{\widehat{\lambda(t)}+n}\right)dN(t)\right]$$

Since $\mathbb{E}\left[\widehat{\lambda(t)}\right] = \mathbb{E}\left[\mathbb{E}[\lambda(t)|N_T]\right] = \mathbb{E}[\lambda(t)]$, it follows that,

$$I(S_T;N_T) = \mathbb{E}\left[\int_0^T \log\left(\frac{\lambda(t)+n}{\widehat{\lambda(t)}+n}\right)dN(t)\right]$$

And $N(t) - \int_0^T \log(\lambda(t)+n)$ is a martingale from theorems of stochastic integrals, see [10], [4] therefore,

$$I(S_T;N_T) = \mathbb{E}\left[\int_0^T (\lambda(t)+n)\log\left(\frac{\lambda(t)+n}{\widehat{\lambda(t)}+n}\right)dt\right]$$

$$= \int_0^T \mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)] - \mathbb{E}\left[(\lambda(t)+n)\log\left(\widehat{\lambda(t)}+n\right)dt\right]$$

$$= \int_0^T \mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)] - \mathbb{E}\left[\mathbb{E}[(\lambda(t)+n)]\log\left(\widehat{\lambda(t)}+n\right)|N_T\right]dt$$

$$= \int_0^T \mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)] - \mathbb{E}[\mathbb{E}[(\lambda(t)+n)|N_T]]\log\left(\widehat{\lambda(t)}+n\right)\,dt$$

$$= \int_0^T \mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)] - \mathbb{E}\left[\left(\widehat{\lambda(t)}+n\right)\log\left(\widehat{\lambda(t)}+n\right)\right] \quad (5)$$

See [4] for similar steps. In [8], it has been shown that the term $log\left(\frac{\lambda(t)+n}{\widehat{\lambda(t)}+n}\right)$ is the derivative of the mutual information corresponding to the integration of the estimation errors, it plays as a counter part to the well known relation between the mutual information and the minimum mean square error (MMSE) in Gaussian channels in [11].

The capacity of the SISO Poisson channel given in theorem (1) is defined as the maximum of (5) solving the following optimization problem subject to an average power constraint,

$$\max\ I(S_T;N_T) \quad (6)$$

Subject to, $\frac{1}{T}\mathbb{E}\left[\int_0^T \lambda(t)\,dt \le \sigma A\right]$ (7)

$$0 \le \lambda(t) \le \sigma A$$

With $\sigma A = P$, P is the maximum power. However, we can easily check that the mutual information is strictly convex via its second derivative with respect to $\lambda(t)$. Now solving $\max\left(\int_0^T \mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)] - \mathbb{E}\left[\left(\widehat{\lambda(t)}+n\right)\log\left(\widehat{\lambda(t)}+n\right)\right] - \frac{\xi}{T}\mathbb{E}[\lambda(t)]\right)$, with ξ as the lagrangian multiplier. The possible values of $\mathbb{E}[(\lambda(t)+n)\log(\lambda(t)+n)]$ must lie in the set of all y-coordinates of the closed convex hull of the graph y= $(x+n)\log(x+n)$. Hence, the maximum mutual information achieved using the distribution $p(\lambda = P) = 1 - p(\lambda = 0) = \alpha$. Where $0 \le \alpha \le 1$, so that $\mathbb{E}[\lambda(t)] = K$. So, we must have $\mathbb{E}[\lambda(t)] = \sum \lambda(t)p(\lambda)$. It follows that, $K = Pp(\lambda = P) = p\alpha$. Then, $\alpha = \frac{k}{P}$. And then the capacity in (4) is proved.

B. *Optimum Power Allocation for SISO Poisson channels*

To solve (6) subject to (7) in the following form,

$$\max\left(\frac{K}{P}(P+n)\log(P+n) + \left(1-\frac{K}{P}\right)nlog(n) - (K+n)\log(K+n) - \frac{\xi}{T}K\right) \quad (8)$$

Since (8) is concave with respect to K, using the lagrangian corresponding to the derivative of the objective with respect to K, and the Karush–Kuhn–Tucker (KKT) conditions, the optimal power allocation is the following,

$$K^* = (P+n)e^{-\left(1+\frac{\xi}{T}\right)} + \frac{n}{P}\log\left(1+\frac{P}{n}\right) - n \quad (9)$$

III. The MIMO Parallel Poisson Channel

Consider the MIMO parallel Poisson channel shown in figure (2).

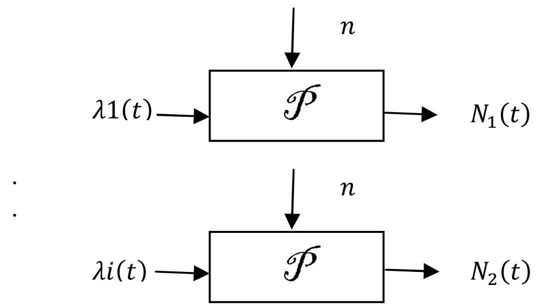

Fig. 2. The Parallel Poisson channel model

Consider a 2-fold parallel Poisson channel, then, $N_1(t)$ and $N_2(t)$ are doubly stochastic Poisson processes with instantaneous average rates $\lambda 1(t)+n$ and $\lambda 2(t)+n$ respectively.

A. *Derivation of the Capacity of Parallel Poisson Channels*

Let $p(N_1,N_2)$ and $p(N_1,N_2|S_1,S_2)$ be the joint density and conditional sample function of the compound regular point processes $N_1(t)$ and $N_2(t)$ respectively, given the message signal processes $S_1(t)$ and $S_2(t)$ in the time interval [0, T]. Then we have,

$$p(N_1,N_2|S_1,S_2) = p(N_1|S_1)p(N_2|2) \quad (10)$$

$$p(N_1,N_2) = p(N_1)p(N_2) \quad (11)$$

$p(N_1|S_1), p(N_2|2), p(N_1)$, and $p(N_2)$ are given by (1) and (2) respectively for each input $\lambda i(t)$. Therefore, the mutual information is defined as follows,

$$I(S_T;N_T) = \mathbb{E}\left[log\left(\frac{p(N_1|S_1)p(N_2|S_2)}{p(N_1)p(N_2)}\right)\right] \quad (12)$$

Theorem2:

The capacity of the 2-input parallel Poisson channel is given

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by the sum capacity of the independent SISO Poisson channels as follows:

$$C = \frac{K1}{P1}(P1+n)\log(P1+n) + \left(1-\frac{K1}{P1}\right) nlog(n) - (K1+n)\log(K1+n) + \frac{K2}{P2}(P2+n)\log(P2+n) + \left(1-\frac{K2}{P2}\right) nlog(n) - (K2+n)\log(K2+n) \qquad (13)$$

Proof:

Substitute (10) and (11) in (12), we have,

$$I(S_T;N_T) = \mathbb{E}\left[\int_0^T \log\left(\frac{\lambda 1(t)+n}{\widehat{\lambda 1(t)}+n}\right) dN1(t)\right] + \mathbb{E}\left[\int_0^T \log\left(\frac{\lambda 2(t)+n}{\widehat{\lambda 2(t)}+n}\right) dN2(t)\right]$$

Following the same steps of the proof of theorem1, we can easily find that,

$$I(S_T;N_T) = \int_0^T \mathbb{E}[(\lambda 1(t)+n)\log(\lambda 1(t)+n)] - \mathbb{E}\left[\left(\widehat{\lambda 1(t)}+n\right)\log\left(\widehat{\lambda 1(t)}+n\right)\right] + \mathbb{E}[(\lambda 2(t)+n)\log(\lambda 2(t)+n)] - \mathbb{E}\left[\left(\widehat{\lambda 2(t)}+n\right)\log\left(\widehat{\lambda 2(t)}+n\right)\right] \quad (14)$$

Hence, the maximum mutual information achieved using the distribution of any input i such that $p(\lambda i = P) = 1 - p(\lambda i = 0) = \alpha i$. Where $0 \le \alpha i \le 1$ so that $\mathbb{E}[\lambda i(t)] = Ki$. So, we must have $\mathbb{E}[\lambda i(t)] = \sum \lambda i(t) p(\lambda i)$. It follows that, $Ki = Pip(\lambda i = Pi) = p\alpha i$. Then, $\alpha i = \frac{ki}{Pi}$. Therefore, the capacity in (13) is proved.

B. Optimum Power Allocation of Parallel Poisson Channels

We need to solve the following optimization problem,

$$\max\left(\frac{K1}{P1}(P1+n)\log(P1+n) + \left(1-\frac{K1}{P1}\right) nlog(n) - (K1+n)\log(K1+n) + \frac{K2}{P2}(P2+n)\log(P2+n) + \left(1-\frac{K2}{P2}\right) nlog(n) - (K2+n)\log(K2+n) - \frac{\xi}{T}(K1+K2)\right) \qquad (15)$$

Subject to,

$$\frac{1}{T}\mathbb{E}\left[\int_0^T \lambda 1(t) \le \sigma A\right] \qquad (16)$$

$$\frac{1}{T}\mathbb{E}\left[\int_0^T \lambda 2(t) \le \sigma A\right]$$

$$0 \le \lambda 1(t) \le \sigma A\,,\ 0 \le \lambda 2(t) \le \sigma A.$$

Using the lagrangian corresponding to the derivative of the objective with respect to K, and the Karush–Kuhn–Tucker (KKT) conditions, the optimal power allocation follows the optimal power allocation for the SISO setup in (9). See [9] for similar results related to optimum power allocation for a 2-fold Parallel Poisson channel where the power constraint was the sum of both average input powers.

IV. THE MAC POISSON CHANNEL

Consider the MAC Poisson channel shown in figure (2).

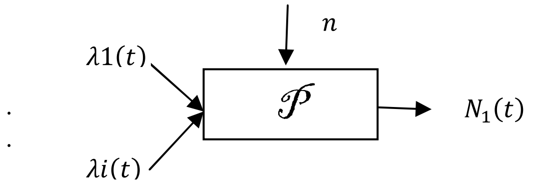

Fig. 3. The MAC Poisson channel model

Consider a 2-input MAC Poisson channel, then, $N_1(t)$ is a doubly stochastic Poisson processes with instantaneous average rates $\lambda 1(t) + \lambda 2(t) + n$.

A. Derivation of the Capacity of MAC Poisson Channels

Let $p(N_1)$ and $p(N_1|S_1,S_2)$ be the joint density and conditional sample function of the compound regular point processe $N_1(t)$ given the message signal processes $S_1(t)$ in the time interval [0, T]. Then we have,

$$p(N_1|S_1,S_2) = e^{-\int_0^T(\lambda 1(t)+\lambda 2(t)+n)\,dt + \int_0^T \log(\lambda 1(t)+\lambda 2(t)+n)dN(t)} \qquad (17)$$

$$p(N_1,N_2) = e^{-\int_0^T(\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n)\,dt + \int_0^T \log(\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n)dN(t)} \qquad (18)$$

Therefore, the mutual information is defined as follows,

$$I(S_T;N_T) = \mathbb{E}\left[log\left(\frac{p(N_1|S_1,S_2)}{p(N_1)}\right)\right] \qquad (19)$$

Theorem3:

The capacity of the 2-input MAC Poisson channel is given by:

$$C = \left(\frac{K1}{P}+\frac{K2}{P}\right)(P+n)\log(P+n) + \left(1-\frac{K1}{P}\right) nlog(n) + \left(1-\frac{K2}{P}\right) nlog(n) - (K1+K2+n)\log(K1+K2+n) \qquad (20)$$

Proof:

Substitute (17) and (18) in (19), we have,

$$I(S_T;N_T) = \mathbb{E}\left[-\int_0^T (\lambda 1(t) - \widehat{\lambda 1(t)})\,dt - \int_0^T (\lambda 2(t) - \widehat{\lambda 2(t)})\,dt + \int_0^T \log\left(\frac{\lambda 1(t)+\lambda 2(t)+n}{\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n}\right) dN(t)\right]$$

Since $\mathbb{E}\left[\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}\right] = \mathbb{E}\left[\mathbb{E}[\lambda 1(t)+\lambda 2(t)|N_T]\right] = \mathbb{E}[\lambda 1(t)+\lambda 2(t)]$, it follows that,

$$I(S_T;N_T) = \mathbb{E}\left[\int_0^T \log\left(\frac{\lambda 1(t)+\lambda 2(t)+n}{\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n}\right) dN(t)\right]$$

And $N(t) - \int_0^T \log(\lambda 1(t)+\lambda 2(t)+n)$ is a martingale from theorems of stochastic integrals, see [10], [4] therefore,

$$I(S_T;N_T) = \mathbb{E}\left[\int_0^T (\lambda 1(t)+\lambda 2(t)+n)\log\left(\frac{\lambda 1(t)+\lambda 2(t)+n}{\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n}\right) dt\right]$$

$$= \int_0^T \mathbb{E}[(\lambda 1(t)+\lambda 2(t)+n)\log(\lambda 1(t)+\lambda 2(t)+n)] - \mathbb{E}\left[(\lambda 1(t)+\lambda 2(t)+n)\log\left(\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n\right)dt\right]$$

$$= \int_0^T \mathbb{E}[(\lambda 1(t)+\lambda 2(t)+n)\log(\lambda 1(t)+\lambda 2(t)+n)] - \mathbb{E}\left[\mathbb{E}[(\lambda 1(t)+\lambda 2(t)+n)]\log\left(\widehat{\lambda 1(t)}+\widehat{\lambda 2(t)}+n\right)|N_T\right]dt$$

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$$= \int_0^T \mathbb{E}[(\lambda 1(t) + \lambda 2(t) + n)\log(\lambda 1(t) + \lambda 2(t) + n)] - \mathbb{E}[\mathbb{E}[(\lambda 1(t) + \lambda 2(t) + n)|N_T]]\log\big(\widehat{\lambda 1(t)} + \widehat{\lambda 2(t)} + n\big)\, dt$$

$$= \int_0^T \mathbb{E}[(\lambda 1(t) + \lambda 2(t) + n)\log(\lambda 1(t) + \lambda 2(t) + n)]$$

$$-\mathbb{E}\left[\big(\widehat{\lambda 1(t)} + \widehat{\lambda 2(t)} + n\big)\log\big(\widehat{\lambda 1(t)} + \widehat{\lambda 2(t)} + n\big)\right] \quad (21)$$

The capacity of the MAC Poisson channel given in theorem (3) is defined as the maximum of (21) solving the following optimization problem subject to an average power constraint,

$$\max\ I(S_T; N_T) \quad (22)$$

Subject to, $\frac{1}{T}\mathbb{E}\left[\int_0^T \lambda 1(t) + \lambda 2(t)\, dt \ \le\ \sigma P\right]$ (23)

$$0 \le \lambda 1(t) \le \sigma P$$
$$0 \le \lambda 2(t) \le \sigma P$$

Now, solving $\max\Big(\int_0^T \mathbb{E}[(\lambda 1(t) + \lambda 2(t) + n)\log(\lambda 1(t) + \lambda 2(t) + n)] - \mathbb{E}\left[\big(\widehat{\lambda 1(t) + \lambda 2(t)} + n\big)\log\big(\widehat{\lambda 1(t)} + \widehat{\lambda 2(t)} + n\big)\right] - \frac{\xi}{T}\mathbb{E}[\lambda 1(t) + \lambda 2(t)]\Big)$, with ξ as the lagrangian multiplier. The possible values of $\mathbb{E}[(\lambda 1(t) + \lambda 2(t) + n)\log(\lambda 1(t) + \lambda 2(t) + n)]$ must lie in the set of all y-coordinates of the closed convex hull of the graph y=$(x1 + x2 + n)\log(x1 + x2 + n)$. Suppose that the maximum power for both inputs is $\sigma A1 + \sigma A2 = P$. Hence, the maximum mutual information achieved using the distribution $p(\lambda = P) = 1 - p(\lambda = 0) = \alpha$. Where $0 \le \alpha \le 1$ so that $\mathbb{E}[\lambda 1(t)] = K1$, $\mathbb{E}[\lambda 2(t)] = K2$. So, we must have $\mathbb{E}[\lambda 1(t) + \lambda 2(t)] = \sum(\lambda 1(t)p(\lambda 1) + \lambda 2(t))p(\lambda 2)$. It follows that, $K1 = Pp(\lambda 1 = P) = p\alpha$. $K2 = Pp(\lambda 2 = P) = p(1-\alpha)$.Then, $\alpha = \frac{k1}{P}$ and $1 - \alpha = \frac{k2}{P}$ And then the capacity in (20) is proved.

Note that we also have $K3 = \sigma A1 p(0 \le \lambda 1(t) \le \sigma P) + \sigma A2 p(0 \le \lambda 2(t) \le \sigma P) = \sigma A1\alpha + \sigma A2(1-\alpha)$, however, $K3$ is not considered in the capacity equations since we only need the maximum and the minimum powers for both $\lambda 1(t)$ And $\lambda 2(t)$ to get the maximum expected value.

B. *Optimum Power Allocation of MAC Poisson Channels*

To solve (22) subject to (23) in the following form,

$$\max\Big(\Big(\frac{K1}{P} + \frac{K2}{P}\Big)\ (P+n)\log(P+n) + \Big(1 - \frac{K1}{P}\Big)nlog(n) + \Big(1 - \frac{K2}{P}\Big)nlog(n) - (K1 + K2 + n)\log(K1 + K2 + n) - \frac{\xi}{T}(K1 + K2)\Big) \quad (24)$$

Using the lagrangian corresponding to the derivative of the objective with respect to K, and the Karush–Kuhn–Tucker (KKT) conditions, the optimal power allocation is the solution of the following equation,

$$K1^* + K2^* = (P+n)e^{-\left(1 + \frac{\xi}{T}\right)} + \frac{n}{P}\log\Big(1 + \frac{P}{n}\Big) - n \quad (25)$$

The optimum power allocation solution introduce the fact that orthogonalizing the inputs via time or frequency sharing will achieve the capacity, therefore it comes the importance for interface solutions to aggregate different inputs to the Poisson channel.

We can also differentiate (24) with respect to the maximum power P at which the capacity of the 2-input MAC Poisson channel is achieved with the optimal P is the solution of,

$$(K1 + K2)P^2 + (K1 + K2)nP + (K1 + K2)n(P + n)log\left(\frac{n}{P+n}\right) = 0 \quad (26)$$

V. DISCUSSION

A. *Mathematical Analysis*

The solutions provided in the paper show that the capacity of Poisson channels is a function of the average and peak power of the input. It can be easily seen that similar to the Gaussian Parallel channels; Poisson parallel channels have the characteristic that their throughput is the sum of their independent SISO channels. For the MAC Poisson channel, in [7] the authors tackle the capacity of MAC Poisson channels. However, they pointed out an interesting observation that we can also see here via theorem3; that is; in contrary to the Gaussian MAC, in the Poisson MAC the maximum throughput is bounded in the number of inputs, and similar to the Gaussian MAC in terms of achieving the capacity via orthogonalizing the inputs. We can also see that the maximum power is a function of the average power that both can be optimized to maximize the capacity.

B. *Simulation Analysis*

Fig.4. shows the capacity of the SISO, parallel, and MAC Poisson channels with respect to the average power and under a maximum power P=10, and dark current n=0.1, it can be easily noticed through the mathematical results as well as the simulations that the capacity of parallel Poisson channels is exactly double the capacity of the SISO Poisson channels if we consider the average power K1=K2=K and the maximum power constraint is met and equal for both channels, i.e. P1=P2=P. However, on the one hand, it is clear that at the low average power regime, the MAC Poisson channel capacity under same conditions lie between both channels. While it decays as the average power increases if inputs are not orthogonal.

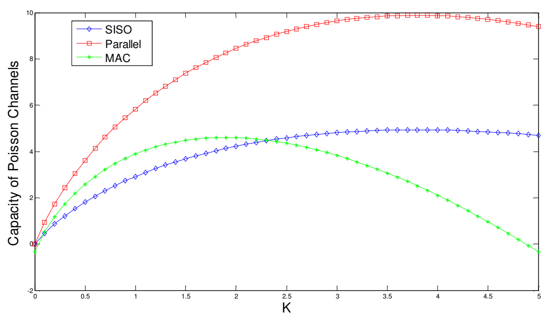

Fig.4. Capacity of Poisson Channels (photons/sec) versus the average power K.

On the other hand, for a different setup where one input average power is lowest and the other input power is maximum, i.e. a time or frequency shared inputs, it turns out that the MAC capacity is higher than that of Parallel channel, this is due to the fact that the dark noise is much more influencing the Parallel setup than that in the MAC setup. On the other hand, compared to the Gaussian MAC,

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similar situation exist; when equal powers are used, the capacity faces a decay to zero in the total achievable rate of the MAC, while when they differ i.e. orthogonal, the capacity moves into maximum.

Fig. 5 shows the capacity of SISO, Parallel, and MAC Poisson channels with respect to the detector dark current, it shows that the capacity is a decreasing function with respect to n; however, for the MAC the capacity increases after a certain point with respect to n. We can also see via Fig. 4 and Fig.5 that the main two factors in the MAC capacity is the orthognalization and the maximum power, while increasing the average power for one or the two inputs will not add positively to the capacity.

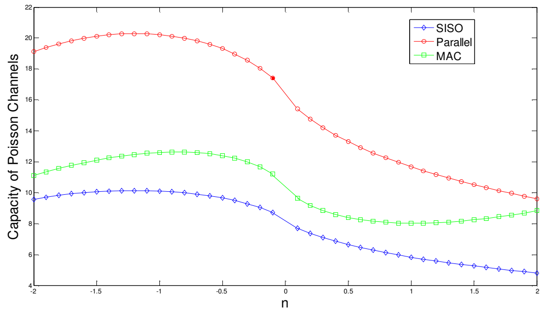

Fig.5. Capacity of Poisson Channels versus the dark current n, (photons/sec).

Fig. 6. Shows the optimum power allocation results, it can be deduced via the mathematical formulas as well as the simulations that the power allocation is a decreasing value with respect to the dark current for all Poisson channels. It means that the power allocation for the Poisson channels in some way or another follows a waterfilling alike interpretation in the Gaussian setup where less power is allotted to the more noisy channels [12]. However, it's well known that the optimum power allocation is an increasing function in terms of the maximum power.

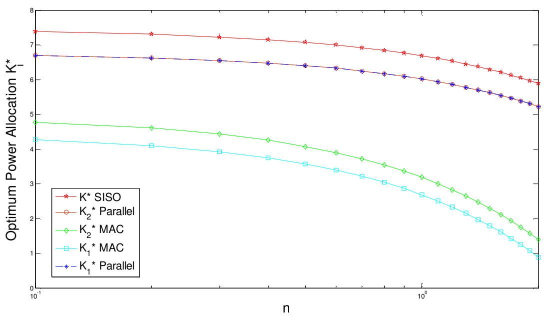

Fig.6. Optimum Power Allocation versus the dark current n, (photons/sec).

A. *General Analysis*

Here, we will introduce some important points about the capacity of Poisson channels in comparison to Gaussian channels within the context of this paper. Firstly, in comparison to the Gaussian capacity, the channel capacity of the Poisson channel is maximized with binary inputs, i.e. [0, 1], while the distribution that achieves the Gaussian capacity is a Gaussian input distribution. Secondly, the maximum achievable rates for the Poisson channel is a function of its maximum and average powers due to the nature of the Poisson processes that follows a stochastic random process with martingale characteristics, while in Gaussian channels, the processes are random and modeled by the normal distribution. Thirdly, the optimum power allocation for the Poisson channels is very similar for different models depending on the defined power constraints, and in comparison to the Gaussian optimum power allocation; it follows a similar interpretation to the waterfilling, at which more power is allocated to stronger channels, i.e. power allocation is inversely proportional to the more noisy channel. However, although the optimal inputs distribution for the Poisson channel is a binary input distribution, the optimal power allocation is a waterfilling alike, i.e. unlike the Gaussian channels with arbitrary inputs where it follows a mercury-waterfilling interpretation to compensate for the non-Gaussianess in the binary input [13].

VI. CONCLUSION

In this paper, we show via information theoretic approach that the capacity of optical Poisson channels is a function of the average and maximum power of the inputs, the capacity expressions have been derived as well as the optimal power allocation for different channel models. It is shown -through the limitation on users within the capacity of the Poisson MAC- that the interface solutions for the aggregation of multiple users/channels over a single Poisson channel is of great importance. However, a technology like orthogonal frequency division multiplexing (OFDM) for optical communications stands as one interface solution. While it introduces attenuation via narrow filtering, etc. it therefore follows the importance of optimum power allocation which can mitigate such effects, hence, we build up optimum power allocation derivations.

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