Abstract— An iterative learning scheme consisting of feedforward learning controllers and hybrid feedback PD-PID controllers are developed for two-link flexible manipulator. The PD controllers ensure the hubs track desired trajectory through hub angle and joint velocity feedback while the PID controllers suppress link vibration through end-points acceleration feedback. The iterative learning control in the feedforward paths are used to improve overall performance of the robot (e.g., input tracking and vibration suppression) by predicting the desired controlled torque. The proposed learning scheme is simple, efficient and easy to implement even with existing controller. Numerical simulation was carried out in Matlab/Simulink environment to show the effectiveness of the proposed control schemes. The reduction in tracking error and system vibration shows that the predicted actuator torques converge to the desired torques as iteration number increases. The performance of the proposed controller in terms of input tracking and vibration suppression is presented and compared with hybrid PD-PID controller. To demonstrate the robustness of the proposed control schemes effect of payload variation is also studied.

Index Terms— Flexible Manipulator, Iterative Learning Control, PID Control, PID Tuning.

I. INTRODUCTION

Flexible manipulator systems (FMSs) offer several advantages compared to their rigid counterparts. The advantages include: faster manipulation due to reduced inertial, energy efficient, higher payload to weight ratio, and less overall cost [1–4]. Despite these advantages, control of FMSs is very difficult because of the distributed parameter nature of the systems. [5]. Proportional-integral-derivative (PID) controller constitutes over 90% of controllers used in industry [6]. PID controllers are cheap, robust over a wide range of operating condition, simple in structure and above all easy to implement [7]. Many PID based control schemes have been reported in literature [8–12]. Performance of fixed-gain PID controllers is limited in real time operations of robot motion control. With fixed-gain controller, steady-state error will continue to be present no matter the number of repetitive operations. High gain can reduce this error but cannot eliminate it due to the effect of unmodelled dynamics in high-order deformation modes [13]. Also there is limit to which gains can be increased because of actuator saturation. A number of adaptive approaches have been proposed in literature [5] to take care of shortcomings of the fixed-gain PID controllers. The major drawback of adaptive approaches is its high computational load requirement [14].

Iterative learning control (ILC) is a feedforward control technique to improve performance of a system doing repetitive tasks through reducing tracking error from trial to trial [15, 16]. The idea of ILC is that a skill can be improved and subsequently perfected through constant practices. Performance of systems that execute repetitive task can be improved by learning from previous execution [13, 14, 16]. The concept of ILC was credited to Arimoto et al. [17] to be the pioneer of this idea calling the learning control scheme improvement process. Advantages of ILC include: good transient performance, and robust to uncertain system dynamics [18]. ILC control strategy is favoured over adaptive control approaches because it alters the controlled input signal while the adaptive controller changes the controller itself [19]. ILC scheme estimate a compensation input for the next iteration such that the error in the next trial is reduced and converges to minimize tracking error. This requires a suitable learning strategy from the previous cycle to improve the performance in the next cycle. ILC strategy like other open-loop/feedback control technique requires accurate knowledge of the plant for its effective use [20]. The uses of ILC in combination with feedback control law have been reported in the literature [13, 21]. A properly designed feedforward controller has been proved to reduce complexity of required feedback controller [22]. The feedback controller ensures system stability [23] and the ILC will improve system performance.

Although ILC has received great attention in the last two decades, the literature is very scarce on it use in FMSs. Some of the works in this area include: [3, 13, 22-25]. Deluca and Panzieri [13] proposed a simple iterative learning scheme to compensate for gravity in two-link robot with flexible forearm. The proposed controller performs efficiently even with little knowledge of the value of gravitational force. This was proved through simulation and experimentation. Zain et al., [24], proposed a hybrid proportional (PD) with PID and ILC scheme for a single link flexible manipulator. A comparative assessment of the proposed scheme was made with PD-PID controller. The simulation result shows that the proposed controller performs better than PD-PID controller. An ILC scheme has been shown to effectively suppress vibration in single link flexible manipulator when used in combination with PD feedback controller [22]. The PD controller is use for the rigid body motion control and ILC for vibration suppression through acceleration feedback. The effectiveness of the
The proposed controller was demonstrated through simulation and experimentation. It was show that the controller performs better than PD-ILC without acceleration feedback. Zain et al. [4] also proposed PID controller for rigid body motion and PID which incorporate ILC for vibration control. The gains of ILC are optimized using genetic algorithm. Addition of ILC improves the performance of the robot.

All the above works are done on single flexible link or two-link with flexible fore-link. In this study, a set-point regulation problem of two-link flexible manipulator performing a repetitive task is considered with payload variation. The dynamic model of the system is developed using Lagrange and Assume mode method. The model has been developed by [26]. A hybrid PD-PI controller for two-link flexible manipulator in [27], it will be extended to include iterative learning control scheme in order to improve its performance. The PD control law is applied at the joint for rigid body motion control using the hub angle and joint velocity feedback. The PID used endpoint acceleration feedback for vibration control. The feedforward ILC scheme acts on the previous joint error and controlled toque to learn and improve the tracking performance and vibration control. A fast learning iterative scheme is proposed that builds up the required command input torque for the disturbance rejection. Performance of the proposed controller is studied through simulation in Matlab/Simulink environment. The performance is compared with Hybrid PD-PI controller [27]; effect of payload variation on the proposed controller is also studied. The results are presented and discussed in detail.

The rest of the paper is organised as follows: the dynamics of the system is presented in section 2. Section 3 gives the controllers’ design schemes while, simulation results and discussion are presented in section 4. The concluding remark is presented in section 5.

II. MATHEMATICAL MODELLING

![FIG. 1. PLANAR TWO-LINK FLEXIBLE MANIPULATOR.](image)

A. Formulation of the recursive kinematics equations

The mathematical model of the planar two-link flexible manipulator shown in Fig. 1. has been developed by [26] using Lagrange and Assumed mode method. The links are modelled as Euler-Bernoulli beam with proper clamped-boundary conditions. Small elastic deflection is assumed and it is restricted to the plane of rigid motion. $(\hat{X}_0, \hat{Y}_0)$, $(X_i, Y_i)$, $(\hat{X}_i, \hat{Y}_i)$ are the inertial frame, the rigid body moving frame, and the flexible body moving frame associated to link $i$ respectively. $\theta_i$ is the rigid body motion (joint angle), and $y_i(x_i)$ is the transversal deflection of link $i$ ($0 \leq x_i \leq \ell_i$), where $\ell_i$ is the length of link $i$ , the rigid (joint) and the flexible rotation matrices $A_i$ and $E_i$ respectively are defined as:

$$A_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix}, \quad E_i = \begin{bmatrix} 1 & -y_i' \\ y_i' & 1 \end{bmatrix}$$

(1)

Where $y_i'$ is the transversal deflection of link $i$ (small deflections assumption). Global transformation matrix $W_i$, from the $(\hat{X}_0, \hat{Y}_0)$ to $(X_i, Y_i)$ be given by:

$$W_i = W_{i-1} E_{i-1} A_i = \hat{W}_{i-1} A_i , \quad \hat{W}_0 = I$$

The previous absolute position vectors $p_i$ along the deflected link $i$, is defined by recursive kinematics equations:

$$p_i = r_i + W_i p_i \quad r_{i+1} = r_i + W_i r_{i+1}$$

(3)

where $p_i = p_i(x_i) = (x_i, y_i(x_i))^T$ is the position of a point along the deflected link $i$, with respect to frame $(X_i, Y_i)$, and $r_{i+1} = r_{i+1}(\ell_i) = (\ell_i, y_i(\ell_i))^T$ being the position of the origin of frame $(X_{i+1}, Y_{i+1})$ with respect to frame $(X_i, Y_i)$. The absolute velocity of this point $\dot{p}_i$ on the links is

$$\dot{p}_i = \dot{r}_i + \hat{W}_i \dot{p}_i + W_i \dot{\hat{r}}_i; \quad \dot{r}_{i+1} = \dot{r}_i + \hat{W}_i \dot{r}_{i+1} + W_i \dot{r}_{i+1}$$

(4)

and $\dot{r}_{i+1} = \dot{p}_i(\ell_i)$, with $\dot{p}_i(x_i) = (0, \dot{y}_i(x_i))^T$. The links are assumed inextensible in the longitudinal direction. The rates of the recursions take the form of:

$$W_i = \hat{W}_{i-1} A_i + \hat{W}_{i-1} \dot{A}_i; \quad \dot{W}_i = \hat{W}_i E_i + W_i \dot{E}_i$$

(5)

B. Lagrangian formulation

Computing the total kinetic energy $(T)$ and potential energy $(U)$, then Lagrangian $L$ is given by:

$$L = T - U$$

(6)

The total kinetic energy $T$ is given by:

$$T = \sum_{i=1}^n T_{hi} + \sum_{i=1}^n T_{li} + T_p$$

(7)

$T_{hi}, T_{li}, T_p$ are the kinetic energies of the hub $i$, link $i$, and the payload, respectively.

$$T_{hi} = \frac{1}{2} m_{hi} \dot{r}_i^T \dot{r}_i + \frac{1}{2} J_{hi} \dot{\theta}_i^2$$

(8)

$m_{hi}$ is mass of hub $i$, $J_{hi}$ is moment of inertia of the end-effector and hub $i$, respectively.

$$T_{li} = \frac{1}{2} \int_{x_i}^{x_{i+1}} \rho_i(x_i) \dot{p}_i(x_i) \dot{p}_i(x_i) dx_i$$

(9)

$\rho_i$ is the linear density of link $i$.

$$\dot{\theta}_i = \sum_{j=1}^n \dot{\theta}_j + \sum_{k=1}^n \dot{y}'_{ik}$$

(10)
\[ \dot{\alpha}_i \] is the absolute scalar angular velocity of frame \((X, Y)\).

Total potential energy \(U\) is:

\[
U = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{1}{2} \int_{0}^{t} (EI)_i (x_i) \left( \frac{d^2 y_i(x_i)}{dx_i^2} \right)^2 dx_i \tag{11}
\]

Where \(y_i\) is the deflection of link \(i\). Equation (13) is a partial differential equation satisfying the following boundary conditions:

\[ y_i(0, t) = 0, \quad y'_i(0, t) = 0, \quad i = 1, ..., n \]

Assuming a \(n_d\) number of modes, deflection of each link can be obtained by the method of separation of variables as:

\[ y_i(x_i, t) = \sum_{j=1}^{n_d} \phi_{ij}(x_i) \delta_j(t) \] \[ \tag{14} \]

Where \(\delta_j(t)\) are the time varying variable associated with the special mode shape function \(\phi_{ij}(x_i)\) of link \(i\). Solution of the two variables are as follows:

\[ \phi_{ij}(x_i) = C_{1ij} \sin(\beta_{ij} x_i) + C_{2ij} \cos(\beta_{ij} x_i) + C_{3ij} \sinh(\beta_{ij} x_i) + C_{4ij} \cosh(\beta_{ij} x_i) \] \[ \tag{15} \]

Where \(\delta_j(t) = \exp(j \omega_j t) = C_{5ij} \sin(\omega_j t) + C_{6ij} \cos(\omega_j t) \]

\[ \beta_{ij}^4 = \omega_j^2 \rho_i (EI)_i, \] \[ \tag{16} \]

\(\omega_j\) is the natural angular frequency of link \(i\) \(C_{1ij}\) to \(C_{6ij}\), are constants obtained from the following boundary conditions:

\[ y_i(0, t) = 0, \quad \dot{y}_i(0, t) = 0 \]

Which yields:

\[ C_{2ij} + C_{4ij} = 0, \quad C_{1ij} + C_{3ij} = 0 \] \[ \tag{18} \]

D. Dynamic equations of motion.

The dynamic model is formulated using Lagrange-Euler equation:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i \quad i = 1, ..., n
\] \[ \tag{19} \]

Solution of equation (19) yields the closed form equation:

\[ \mathbf{B}(q) \ddot{q} + \mathbf{h}(q, \dot{q}) + \mathbf{K} q = \tau \]

where \(q_i = (\theta_1, \theta_2, ..., \theta_{n_d})\), \(\tau\) is a vector of generalized forces applied at the joints of \(n\)-link. \(\mathbf{B}\) is a positive-definite symmetric inertia matrix, \(\mathbf{h}\) is a vector of Coriolis and centripetal forces, and \(\mathbf{K}\) is the diagonal stiffness matrix. Detailed derivation of the mathematical model can be found in [26]

III. CONTROLLER DESIGN

The control schemes involve two stages. The first stage is the hybrid PD-PID controller design for the two-link flexible manipulator. In the second stage, the hybrid PD-PID controller is extended to incorporate the ILC scheme. To be able to estimate and compare the overall performance of the controllers; performance index is calculated for the two control schemes

A. Hybrid PD-PID Controller Design.

The control objective is to design PD collocated controllers for each of the links in Fig 1. so that the hub angles follow the reference trajectories. Also non-collocated PID controllers need to be designed so that the vibrations of the system are eliminated simultaneously through end-point acceleration feedback. The hybrid PD-PID control structure is shown in Fig. 2. For the collocated PD controller, the control input is given by:

\[ u_{PID}(t) = A_{i1}(K_{Pj}(\dot{\theta}_d(t) - \dot{\theta}_i(t)) - K_{Ij}(\theta_i(t) - \theta_d(t))) \] \[ \tag{21} \]

Where \(u_{PID}\) is PD control input, \(\theta_d\) and \(\theta_i\), \(A_{i1}\), \(K_{Pj}\) and \(K_{Ij}\) are desired hub angle, actual hub angle, amplifier, proportional and derivative gains respectively.

For the non-collocated PID controller design, a PID controller uses end-point elastic acceleration for vibration suppression of each of the links because of the coupling effects. The control input for the PID is as follows:

\[ u_{PID}(t) = \left( k_{py} e(t) + k_y \int e(t) dt + k_{dj} \frac{de(t)}{dt} \right) \] \[ \tag{22} \]

\(e(t) = \alpha_{id}(t) - \alpha_i(t)\) \[ \tag{23} \]

where \(u_{PID}\) is the PID controller input. \(K_{py}\), \(K_y\) and \(k_{dy}\) are the proportional, integral and derivative gains. \(\alpha_{id}(t)\) and \(\alpha_i(t)\) are desired and actual end-point acceleration. \(\alpha_{id}(t)\) is set to zero since the objective is to have zero acceleration in the system. Total control input \(\tau_i(t)\) is given by:

\[ \tau_i(t) = u_{PID}(t) + u_{PID}(t) \] \[ \tag{24} \]

Detailed information on the design of hybrid PD-PID controller for the two-link flexible manipulator can be found in [27].

B. Iterative Learning Control Scheme

To improve the performance of the hybrid PD-PID controller described in section 3.1 above according to [27], ILC scheme is proposed to be incorporated with the hybrid PD-PID controller. The structure of the proposed control law is shown in Fig. 3. a serial architecture is employed in
this study. Advantage of this type of architecture is its ease of implementation even with an existing controller [19] and does not affect the stability of an existing system. There are different types of learning algorithms in literature they include: P-type, D-type, PI-type, PD-type and PID type [15,16,28]. P-type learning algorithm is applied to linear systems, the PD-type is applied to non linear systems and the PID-type is applied to low-order systems The P-type learning algorithm is the most widely used method in industry because of its robustness to noise [29]. In this study a P-type learning algorithm is employed with modification of including a learning filter to iterative control input and using squared previous error to give better tracking error convergence. This simple algorithm gives a very good performance.

The performance index of the controllers is estimated in terms of input tracking, vibration suppression, torque required and end-point residual. Performance index J is given by:

\[
J = \frac{1}{t_{f}} \int_{0}^{t_{f}} \left[ (J_1 + J_2) \right] dt
\]

(26)

where: \( J \) is the performance index, \( t_{f} \), \( \theta_{i_{\text{max}}} \), \( \sigma_{i_{\text{max}}} \), \( \alpha_{i_{\text{max}}} \), and \( \tau_{i_{\text{max}}} \) are the final simulation time, maximum hub angle, hub velocity, link deflection, tip acceleration and torque of link \( i \) respectively. \( \theta, \sigma, \alpha, \tau \) are the hub angle, hub velocity, link deflection, tip acceleration and torque of link \( i \) respectively.

\[
J_1 = \left( \frac{\theta_{i_{\text{f}}} - \theta_{i_{\text{d}}}}{\theta_{i_{\text{max}}}} \right)^2 + \left( \frac{\sigma_{i_{\text{f}}} - \sigma_{i_{\text{d}}}}{\sigma_{i_{\text{max}}}} \right)^2 + \left( \frac{\alpha_{i_{\text{f}}} - \alpha_{i_{\text{d}}}}{\alpha_{i_{\text{max}}}} \right)^2
\]

(27)

\[
J_2 = \left( \frac{\theta_{i_{\text{f}}} - \theta_{i_{\text{d}}}}{\theta_{i_{\text{max}}}} \right)^2 + \left( \frac{\alpha_{i_{\text{f}}} - \alpha_{i_{\text{d}}}}{\alpha_{i_{\text{max}}}} \right)^2
\]

(28)

TABLE 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{\text{h1}} ) = ( J_{\text{h2}} )</td>
<td>Mass moment of inertia of the hub 0.1 kgm²</td>
</tr>
<tr>
<td>G</td>
<td>Gear ratio 1</td>
</tr>
<tr>
<td>( M_1 )  = ( m_1 )</td>
<td>Mass of the link 0.1 kg</td>
</tr>
<tr>
<td>( M_p )</td>
<td>Mass of pay load 0.1 kg</td>
</tr>
<tr>
<td>( J_{\text{ot1}} ) = ( J_{\text{ot2}} )</td>
<td>Mass moment of inertia of the link about its hub 0.0083 kgm²</td>
</tr>
<tr>
<td>( J_p )</td>
<td>Mass moment of inertia of the end effector 0.0005 kgm²</td>
</tr>
</tbody>
</table>

IV. SIMULATION RESULTS

To demonstrate the efficiency of the proposed control scheme, test was carried out through simulation within Matlab/Simulink environment and the results are presented. The parameter of the two-link flexible manipulator is given in Table 1 [26]. The manipulator is expected to track a unit step input. The hub angle, end-point acceleration and the controlled toques are presented in Fig. 5. After careful tuning, the PD and the PID controller gains are given in Table 2 [27]. The ILC gains are obtained using try and error [28] the gains are tuned systematically because there is no available method of tuning the ILC gains [19] and the gains are presented in table 3.

The response obtained using PD-PID-ILC controller is compared with PD-PID controller (see Fig. 5.) It is observed that there is an improvement in performance of PD-PID with ILC. The hub angle tracking is faster with the proposed controller (Fig. 5a and 5b) showing that the proposed controller has truly learnt from past errors and past inputs as there is significant reduction in tracking error. There is no overshoot, and the system settles more quickly with the proposed controller than PD-PID control.
The steady state error of -0.0035rad and 0.0088rad for link 1 and 2 respectively was obtained using PD-PID while -0.0028rad and 0.0083rad with the proposed controller as shown in Fig. 5c and 5d. The tracking error (Fig. 5c and 5d) show that the learning algorithm is a very powerful one. Figure 5e and 5f shows the end-point acceleration, though the amplitude of the proposed controller is higher than PD-PID control but the system settles faster in about 3s compared to about 5s in PD-PID control. Also the applied torques in Fig.5g and 5h, the actuators settle in less than 4s compared to more than 5s in PD-PID control. This shows the effectiveness of the proposed controller.

To further demonstrate the robustness of the proposed PD-PID-ILC controller over the PD-PID controller, effect of changing the payload from the nominal value of 100g to 80g is studied and the results are presented in Fig. 6. It is shown that the two control schemes are robust to payload variation but better performance is observed with the proposed control scheme which is similar to the results shown in Fig. 5, this is an indication that the control scheme is cable of reducing system errors even in the face of payload variation. The overall performance index was calculated using equation (28) to compare the performance of the two control algorithms. Performance index of 0.416 is obtained for PD-PID-ILC control scheme which is better than 0.497 obtained for PD-PID control scheme. The performance index reduces as the learning step increases (see Fig. 7.)

V. CONCLUSION

Hybrid PD-PID-ILC control algorithm has been developed for two-link flexible manipulator. The hybrid PD-PID-ILC controller has been compared with hybrid PD-PID controller. The PD controller is used for rigid body motion control using hub angle and hub velocity feedback. The PID controller uses end-point acceleration for vibration suppression. The feedforward ILC learning algorithm incorporated in the PD-PID controller uses the square of the previous hub angle tracking error and previous control input to improve the overall performance of the system. A simple P-type learning algorithm usually employ for linear system was modified and used in this study. The ILC scheme is used to estimate required inputs compensation for each iteration such that the overall error is reduced and converges to minimize tracking error. The proposed control scheme has been tested through simulation in Matlab/Simulink environment. The results have demonstrated that a better performance is achieved with the proposed controller.

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REFERENCES


Fig. 5. Time history of hub angle, tracking error, end-point acceleration and applied torque with 100g payload

Fig. 6. Time history of hub angle, tracking error, end-point acceleration and applied torque with 80g payload.

Fig. 7. Time history of performance index