Modeling and Analysis of the Dynamics of Load Carrying System

Bogdan Posiadala, Mateusz Tomala

Abstract—**In this work, the problem of modeling and analyzing of dynamic effects of crane's load carrying system has been considered. The initial problem of the motion of load carried by the forest crane during the working cycle has been formulated and solved. Friction effects are taken into consideration. The workout calculation model enables one to analyze the motion of the load under influence of kinematical forcing and the free vibration of the load as the result of the previous forced motion. The exemplary results of the numerical simulation are presented.**

Index Terms— **analysis, crane, dynamics, friction, modeling, multibody system.**

I. Introduction

The problems of modeling and analysis of dynamical phenomenon of heavy construction machines and their components have been a subject of many works. In paper [1], B. Posiadala et al. have presented the formulation and solution of initial value problem representing the motion of load carried by a truck crane. The load is considered as a particle. The motion of the load under influence of kinematic forcings and its further free motion have been investigated. In works [2], [3], B. Posiadala has developed the model of the system by including the influence of flexibility of the rope and crane supporting system. The model of dynamics of the system, representing the main machine elements, including the support system, coupled with the load has been obtained. In paper [4], A. Maczynski has presented the model and the investigation results concerning the minimizing of load oscillations in rotary cranes.

In works [5], [6], B. Posiadala at al. have presented the formulation and solution of initial value problem representing the motion of the load carrying system considered as the systems of rigid bodies. In this work, this model has been developed by including friction effects in revolute joints of the system. The formulation and solution of the initial problem, based on the developed model, as well as the exemplary results of numerical simulations are presented.

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II. Computational Model

In this work, the load carrying system of a forest crane has been investigated and its kinematical configuration has been taken into account. The computational model is shown on Fig. 1. The system consists of three rigid bodies connected with each other by revolute joints *P* and *Q*. The body I is connected with the end of the boom by the revolute joint Ω.

The Cartesian coordinate system *OXYZ* and four generalized coordinates: φ, θ_1, θ_2 and Φ have been introduced to develop the mathematical model. The motion of each body of the system can be described using these coordinates and their time derivatives. The φ coordinate is one of control parameters of the crane. This coordinate describes a position of lifting plane and is closely related to $\mathbf{r}_\Omega$ vector components. The problem of formulating components of this vector in global frame as functions of time has been presented in works [1]-[2]. In these works, typical groups of cranes and typical working cycles have been taken into consideration. The Φ coordinate describes the rotation angle of gripper and load and also is a control parameter. A correct formulation of φ, Φ and $\mathbf{r}_\Omega$ functions opens the way to model any working cycle of the crane. Angular coordinates θ_1 and θ_2 describe a dynamic response of the system and they are obtained as the solution of initial value problem.

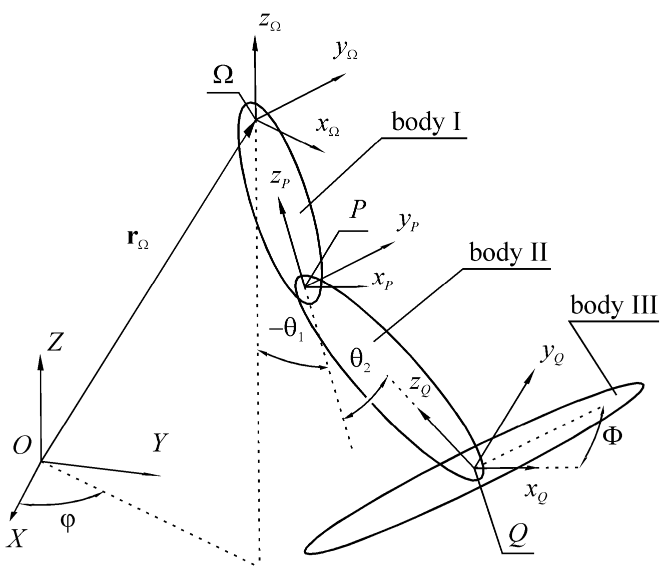

Fig. 1. Computational model of load carrying three-element system.

Additionally, three local frames have been introduced into the model and their origins are placed in the revolute joints of the system. Every frame is oriented in such way that one of its axes is a pivot. For the Ω joint the axis of rotation is

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y_Ω, for P joint – x_P axis. This way of defining of the local frames simplifies the consideration of the friction effects in each revolute joint.

III. Formulation and Solution of Initial Value Problem

The equations of the system motion have been obtained using Lagrange equations of the second kind. The system has two angular degrees of freedom: θ_1 and θ_2. Lagrange equations of the second kind for the considered system can be written as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_k}\right) - \frac{\partial L}{\partial \theta_k} = M_k, \quad k = 1,2\,, \tag{1}$$

where L is the Lagrangian and M_k (k = 1, 2) are computational moments which act about Ω and P points, respectively. The Lagrangian is defined as a total kinetic energy of the system minus its total potential energy:

$$L = E_K - E_P = \sum_{i=1}^{3}\left(E_K^{(i)} - E_P^{(i)}\right). \tag{2}$$

The equations of motion of the system consist the system of ordinary differential equations (ODE). Computational moments of forces M_k depend only on the instantaneous configuration of the system and on friction law which is taken into account. In this work, simple Coulomb friction law is considered and according to this the force of friction is a linear function of the radial force in the joint:

$$\left|\mathbf{T}^{(k)}\right| = \mu\left|\mathbf{N}^{(k)}\right|, \tag{3}$$

where μ is a generalized friction coefficient. Additionally, different coefficients of static and kinetic friction are taken into consideration. According to the Karnopp assumption [7], the interval of small velocities <-ε,ε> is considered. In this interval it is assumed that the velocity is equal to zero and static friction coefficient is taken into account. The simplified formula describing the friction coefficient versus generalized velocity is shown in the Fig. 2.

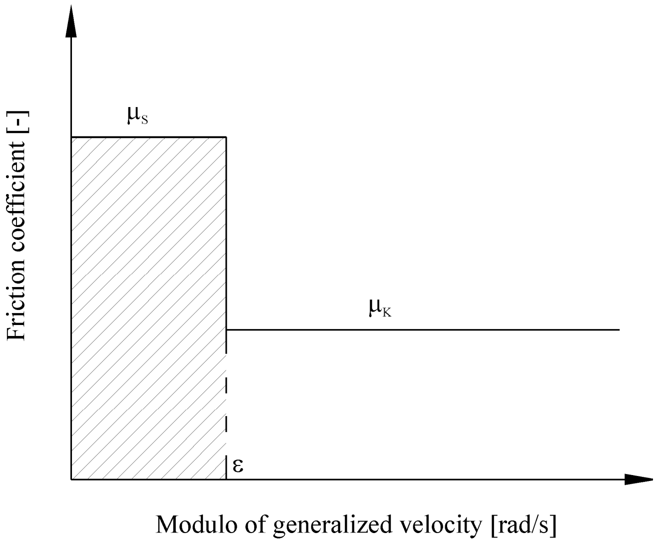

Fig. 2. The friction coefficient versus generalized velocity.

In the considered model the clearance or deformation effects have been neglected, thus the motion parameters depend only on generalized coordinates (φ, Φ, θ_1 and θ_2) and their time derivatives. Components of reaction forces (Fig. 3) could be obtained using Newton equations:

$$m_i\mathbf{a}^{(i)} = \mathbf{S}^{(i)},\ i = 1,2,3\,, \tag{4}$$

where m_i is i-th body's mass, $\mathbf{a}^{(i)}$ is the absolute acceleration of center of mass of the body and $\mathbf{S}^{(i)}$ is the total force acting on it.

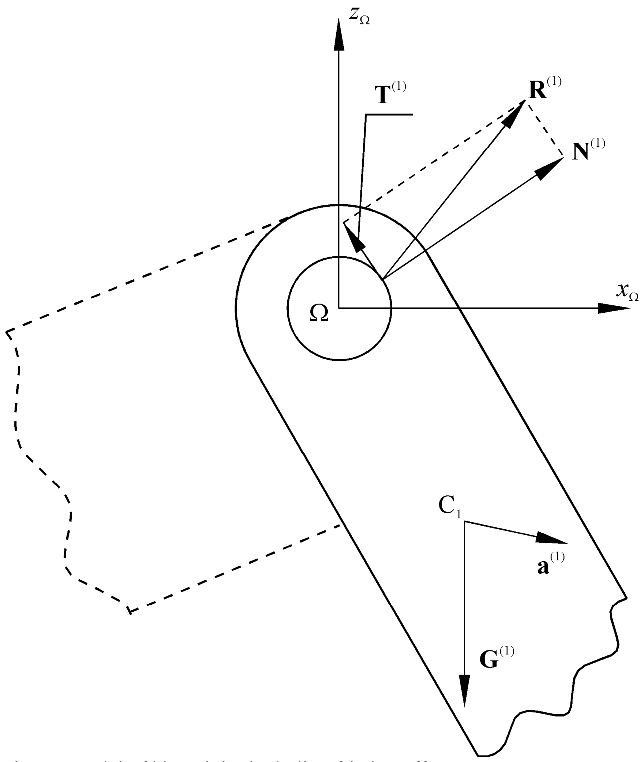

Fig. 3. Model of hinge joint including friction effects.

Total forces can be written as following sums:

$$\begin{aligned}\mathbf{S}^{(1)} &= \mathbf{R}^{(1)} + \mathbf{R}^{(2)} + \mathbf{G}^{(1)},\\ \mathbf{S}^{(2)} &= \mathbf{R}^{(3)} - \mathbf{R}^{(2)} + \mathbf{G}^{(2)},\\ \mathbf{S}^{(3)} &= -\mathbf{R}^{(3)} + \mathbf{G}^{(3)},\end{aligned} \tag{5}$$

where $\mathbf{G}^{(i)}$ is i-th body's gravity force. If reaction forces $\mathbf{R}^{(i)}$ and their components in a global frame are known, each reaction force component (radial or axial) can be easily defined. For i-th joint, a resultant reaction in rotation plane is a sum of tangent and radial reaction forces:

$$\mathbf{R}^{(k)} = \mathbf{T}^{(k)} + \mathbf{N}^{(k)}. \tag{6}$$

Equation (6) is a vector equation and can be rewritten into the scalar form using proper components. If one projects this equation onto the proper rotation plane ($\Omega x_\Omega z_\Omega$ for the Ω joint and $P y_P z_P$ for the P joint), it is equivalent to two scalar equations. Next step is to rewrite (3) into the scalar form:

$$\sqrt{T_1^2 + T_2^2} = \mu\sqrt{N_1^2 + N_2^2}\,. \tag{7}$$

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It is necessary to take into account that the **T** and **N** vectors are always perpendicular, so their scalar product is always equal to zero. This condition, (3) and (6) previously rewritten into the scalar form are together a system of four equations. Each reaction force component (radial and tangent) can be computed from the equation system:

$$\begin{cases} \left|\mathbf{T}^{(k)}\right| = \mu\left|\mathbf{N}^{(k)}\right| \\ \mathbf{R}^{(k)} = \mathbf{T}^{(k)} + \mathbf{N}^{(k)} \\ \mathbf{T}^{(k)} \cdot \mathbf{N}^{(k)} = 0 \end{cases}. \tag{8}$$

Moment of friction force (static or kinetic) acting on cylindrical surfaces of journal and bearing can be obtained using absolute value of friction force:

$$M_k^{(T1)} = \frac{d_k}{2}\left|\mathbf{T}^{(k)}\right|, \tag{9}$$

where d_k is a diameter of k-th hinge joint.

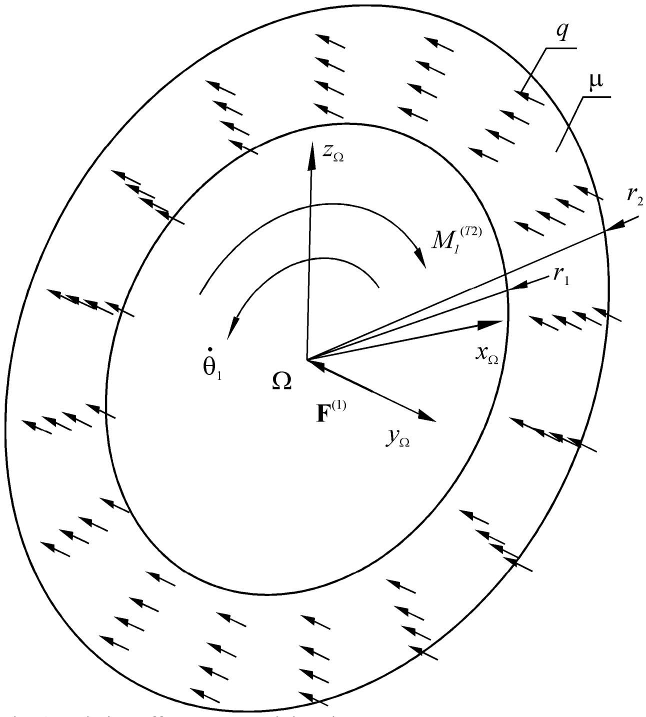

Fig. 4. Friction effects on a retaining ring.

In this work, moments of friction forces acting on side (retaining) surfaces are taken into account, as shown on Fig. 4.

If one considers that the side surface has a shape of ring, the moment of friction force can be obtained using a following formula:

$$M_k^{(T2)} = \frac{2}{3}\mu\left|\mathbf{F}^{(k)}\right|\frac{r_1^2 + r_1 r_2 + r_1^2}{r_1 + r_2}, \tag{10}$$

where $\mathbf{F}^{(k)}$ is an axial force in k-th joint, r_1 and r_2 are internal and external radii of the ring, respectively. $\mathbf{F}^{(k)}$ force acts along proper direction for each joint. For Ω joint $\mathbf{F}^{(k)}$ force acts along y_Ω axis, for P joint - along x_P axis. One can notice that at every step of time in each joint only one of two retaining surfaces is under pressure. It means that the moment of side friction acts only on one side of each joint.

Total moments of friction forces are sums:

$$M_k^{(T)} = \left(M_k^{(T1)} + M_k^{(T2)}\right)\operatorname{sgn}\left(\dot{\theta}_k\right). \tag{11}$$

Values of computational moments can be obtained with following scheme:

$$M_k = \begin{cases} -M_k^{(G)} & \left|\dot{\theta}_k\right| < \varepsilon \ \wedge \ \left|M_k^{(G)}\right| < \left|M_k^{(T)}\right| \\ -M_k^{(T)} \quad for & \left|\dot{\theta}_k\right| < \varepsilon \ \wedge \ \left|M_k^{(G)}\right| > \left|M_k^{(T)}\right| \\ -M_k^{(T)} & \left|\dot{\theta}_k\right| > \varepsilon \end{cases}, \tag{12}$$

where $M_k^{(G)}$ is k-th moment of conservative forces acts in k-th joint.

IV. Exemplary Results of Numerical Computation

The exemplary results of numerical calculations have been obtained for the system parameters shown in Tables I, II and III.

TABLE I
EXEMPLARY COMPUTATION – PROPERTIES OF BODIES

Parameter	Value(s)		
	Body I	Body II	Body III
Dimensions $x \times y \times z$ [m]	0.1×0.15×1	0.15×0.1×0.9	3×0.4×0.6
Density ρ [kg·m^{-3}]	7700	7700	900
Mass m [kg]	115.5	103.95	648

TABLE II
EXEMPLARY COMPUTATION – PROPERTIES OF JOINTS

Parameter	Value(s)	
	Ω	P
Static friction coefficient μ_S [-]	0.2	0.3
Kinetic friction coefficient μ_K [-]	0.1	0.2
Radius of journal (internal radius of retaining ring) [m]	0.05	0.04
External radius of retaining ring [m]	0.06	0.05
Interval of static friction [rad·s^{-1}]	10^{-6}	10^{-6}

TABLE III
EXEMPLARY COMPUTATION – INITIAL POSITION OF THE CRANE

Parameter	Value(s)
X_Ω [m]	2.898
Y_Ω [m]	0
Z_Ω [m]	5.298
Initial value of φ [rad]	0
Initial value of Φ [rad]	0

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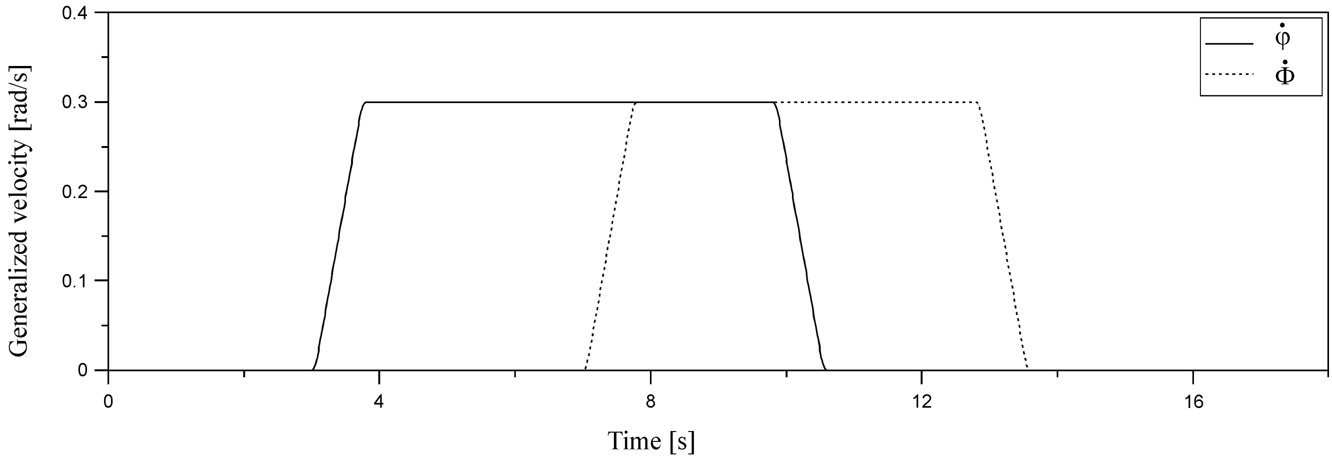

Fig. 5. The exemplary computation – kinematical forcings used in the simulation.

The considered system consists of three rigid bodies with dimensions and mass parameters as given in Table I. First two bodies are made of steel, the third (a load) is made of wood. The presented results of calculations include the initial equilibrium state, the motion of the load carrying system according to the working cycle represented by so-called kinematic forcings and further free motion as the result of previously forced motion.

A working cycle which has been chosen for the exemplary computation is characterized by the changes of time derivatives of generalized coordinates φ and Φ versus time as shown in Fig. 5. The chosen kinematical forcings include the rotation of crane boom starting in 3rd second and the rotation of the gripper with load starting in 7th second. Properties of joints (friction coefficients, dimensions of joints, etc.) are listed in Table II. In Table III, the initial position of Ω point (end of the crane boom) in global frame and initial values of angles φ and Φ are presented.

The solution of the initial value problem is obtained numerically, using the Runge-Kutta method of the fourth-order and the performed computational program. In the program the variable integration step has been used and its maximum value was equal to 0.001 second.

The solution results of the initial value problem are represented by the generalized coordinates (Fig. 6) versus time. For the considered case the trajectory of the crane boom end (the Ω point) and trajectories of centers of masses of each body in the global frame have been determined. The *OXY* trajectory projection is shown on Fig. 7 and the *OXZ* projection is shown on Fig. 8.

The total energy of the system has also been computed and its changes versus time for the considered case are shown in Fig. 9. The computation of the total energy of the system using the motion parameters, obtained as the initial problem solution, enables one to follow the changes of this value. This is helpful to recognize the correctness of the obtained numerical results.

V. Conclusion

In this work, the problems of modeling and analyzing of dynamic effects of crane's load carrying system have been considered. The formulation and solution of the initial value problem of dynamics of load carrying system have been presented. Friction effects have been taken into consideration with different static and kinetic friction coefficients.

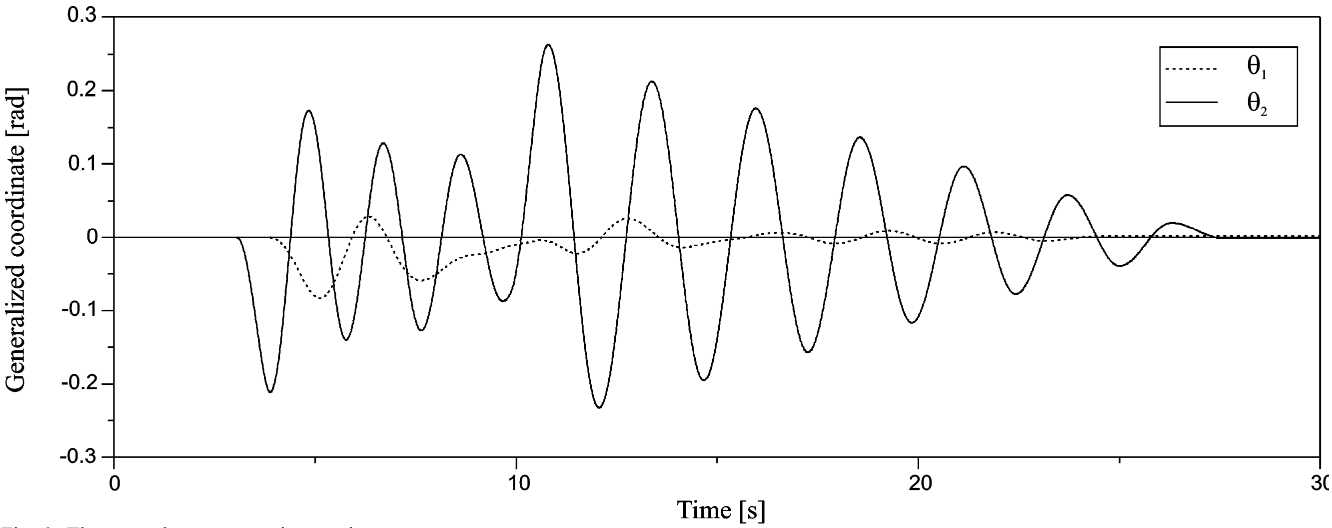

Fig. 6. The exemplary computation results.

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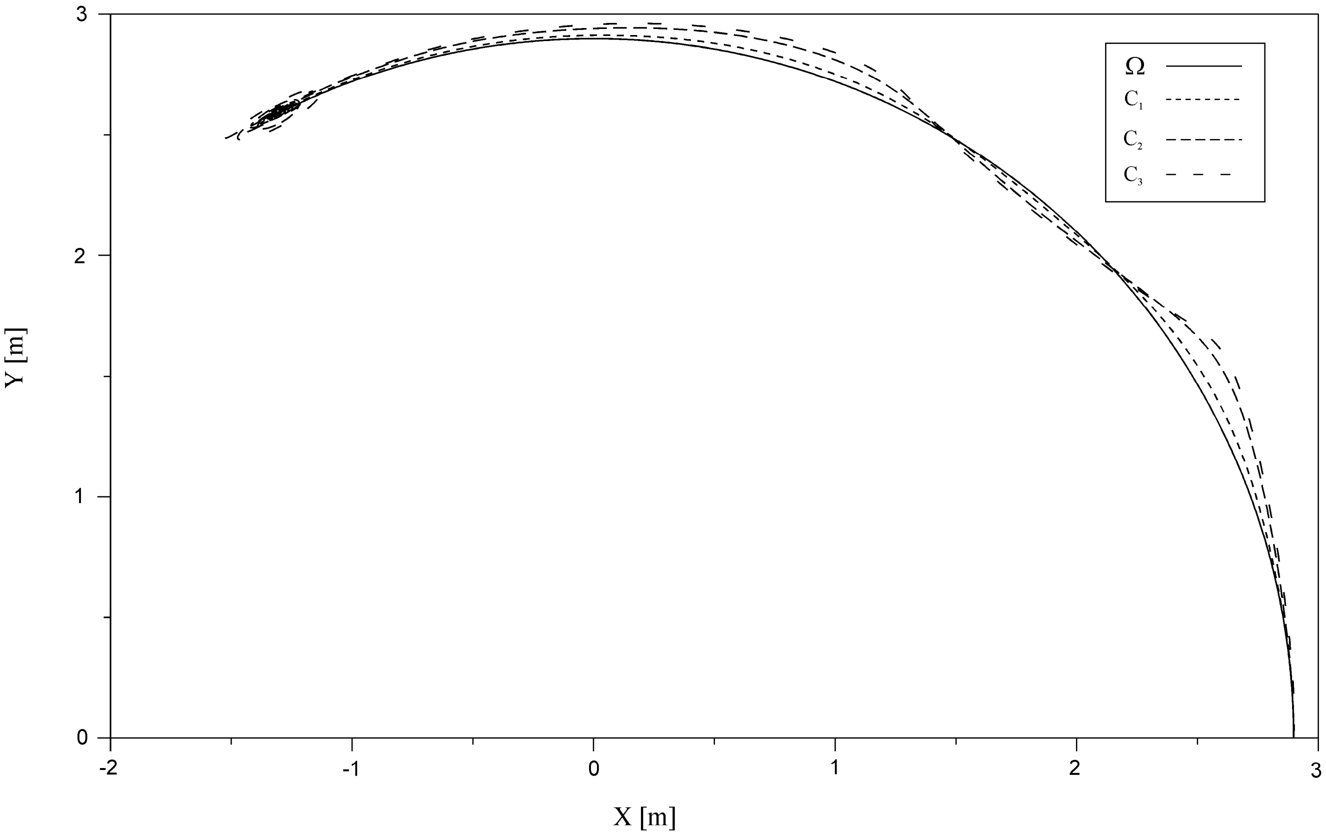

Fig. 7. The *OXY* projection of trajectories: of the mass centers C_1, C_2, C_3 and of the boom end Ω.

The worked out computational model enables one to analyze the dynamical behavior of the system of the chosen configuration during the machine working cycle. The solution is valid for the forced motion, according to the kinematic forcings, and for further free motion of the system, according to the initial conditions resulting from the previous motion.

The proposed model could be used to study the dynamics of other multibody systems equipped with revolute joints including the friction effects. The mathematical model could

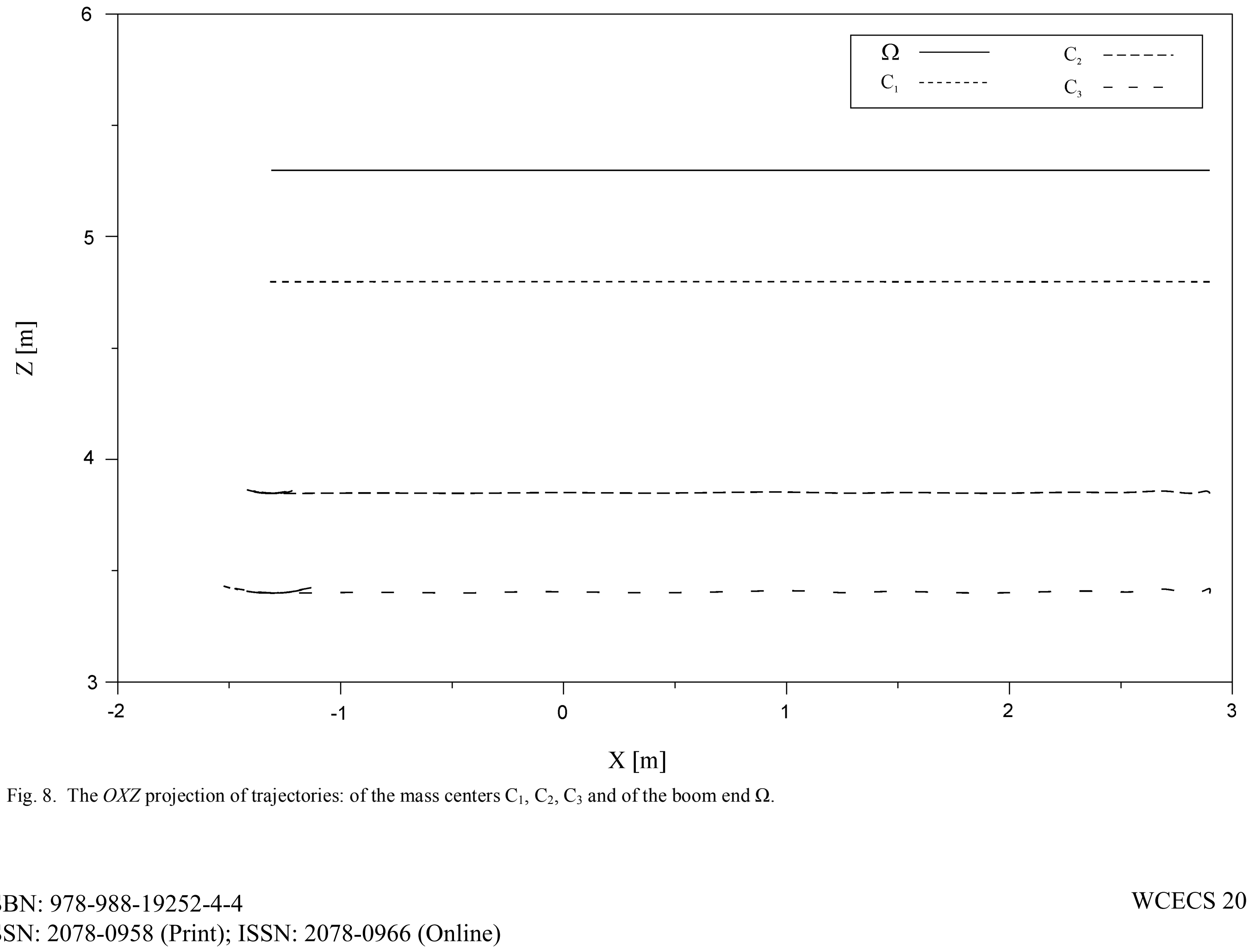

Fig. 8. The *OXZ* projection of trajectories: of the mass centers C_1, C_2, C_3 and of the boom end Ω.

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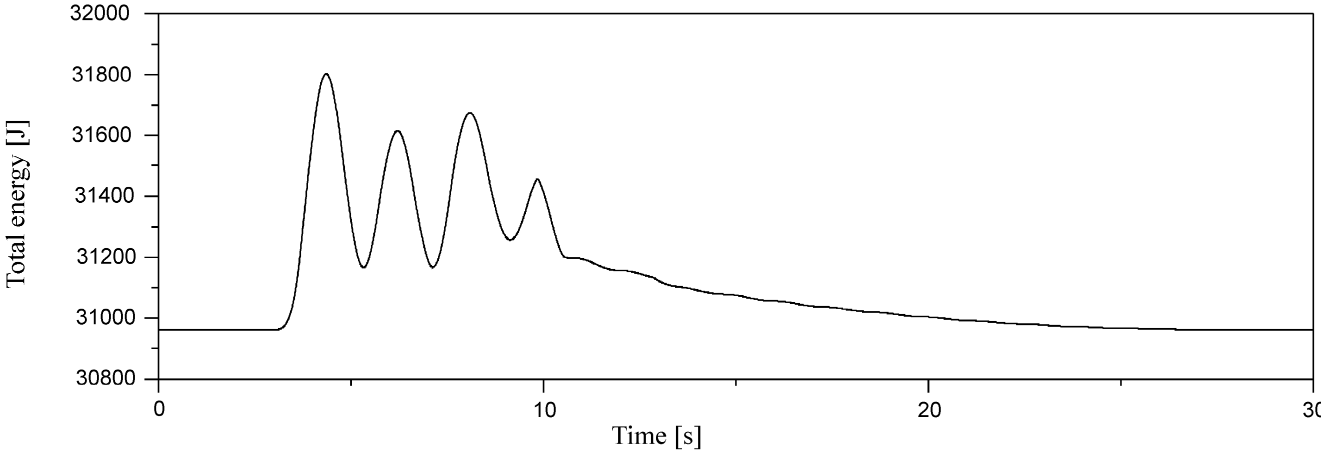

Fig. 9. The total energy of the system.

be extended by introducing joint clearances and deformation. The model could also be extended by introducing external influences.

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