Fuzzy Adaptive Control for a Class of Non-Affine Systems via Time Scale Separation

Daoxiang Gao, Dunmin Lu and Zengqi Sun

***Abstract*—A fuzzy adaptive control method is proposed for a class of non-affine nonlinear systems. By combing implicit function theorem and time scale separation, the control input is derived from the solution of a fast dynamical equation. Stability analysis shows that the proposed approach can guarantee the boundedness of the tracking error semi-globally, which can be made arbitrarily small by choosing appropriate design parameters. Tracking performance is illustrated by simulation results.**

***Index Terms*—fuzzy control; time scale separation; singular perturbation; non-affine system.**

I. Introduction

Tremendous researches have been made in recent years in the area of controller design for nonlinear systems. Many remarkable results and new design tools, for instance, back-stepping design, fuzzy and neural networks adaptive control methods, were facilitated by advances in geometric nonlinear control theory, in particular, feedback linearization method [1], by which the nonlinear system is transformed into a linear one, then linear control design methods can be applied to acquire the desired performance. Most of these researches are devoted to the control problem of nonlinear systems in affine form, which are characterized by the control input appearing linearly in the system state equation. For non-affine systems, the implicit function theorem [2] is commonly used to demonstrate the existence of the optimal solution for the control input, but it does not provide a way to construct such controller to achieve control objective.

Because it is difficult to invert the non-affine nonlinearities to obtain the inverting control input, fuzzy logic systems (FLs) or neural networks (NNs) are used to approximate the desired feedback control input. In [3∼7], under some assumptions on the original system, several direct/ indirect adaptive controllers based on NNs or FLs are proposed to deal with the control problem of non-affine system. These approaches use the adaptive controller to approximate the optimal control input directly with a parameter adaptation law designed by the Lyapunov theorem. These direct adaptive control methods are further extended to the output feedback adaptive control in [8∼11]. The main feature of the previous approach is that the uncertainty to be approximated by an adaptive signal contains the adaptive signal itself as a part of uncertainty, which leads to a fixed-point problem. Thus, it needs to involve more restriction on both the input magnitude and the input change. The indirect adaptive control method is concentrated on transforming the original system non-affine in control input to a new system in which the new control input variable appears in an affine form. In [12], FLs are used to approximate the plant model and the control input can be solved by inverting the fuzzy model in an affine form. In [13], the authors use Taylor series expansion to transform the original non-affine system into the affine-like one, then the well-developed adaptive control scheme for affine nonlinear system can be used directly to the non-affine one. However, the indirect adaptive approach has the drawback of the controller singularity problem. In [14], a dynamic feedback adaptive control method is presented by differentiating the original nonlinear state equation once so that the resulting augmented state equation appears linear in the new state variable—the derivative of the control input, which can be used as new control variable. Recently, in [15], [16], the authors propose a control method based on singular perturbation theory by the combination of time scale separation and dynamic inversion. The control input is derived from a solution of fast dynamical equation and is shown to stabilize the original non-affine system asymptotically by using Tikihonov theorem [17] directly. This result is extended to deal with the control problem of the pure-feedback systems in [18].

In this paper, we develop a fuzzy adaptive controller for non-affine systems by using time scale separation. Unlike [15], [16], the Lyapunov theory is used to show the system stability instead of using Tikhonov theorem directly. The error system dynamics are constructed to facilitate the stability analysis by combing the implicit function theorem and the mean value theorem. It is noted that, from the Lyapunov stability analysis, the tracking error can be made arbitrarily small by choosing appropriate controller parameters. we introduce the generalized fuzzy hyperbolic tangent model (GFHM) [19] to be the fuzzy basis function. The properties of the hyperbolic tangent function are exploited to design the adaptive law to guarantee the existence of the solution for control input.

This paper is organized as follows. Section II presents a class of non-affine systems that will be considered and some assumptions to facilitate the controller design. In section III, a brief description of GFHM is presented and fuzzy adaptive controller is designed for the unknown non-affine systems Finally, An illustrative example and some conclusion remarks are given in sections IV and V.

II. **Problem Formulation**

Consider the following non-affine nonlinear system

$$\begin{aligned} \dot{x}_i &= x_{i+1} \qquad i = 1, \cdots, n-1 \\ \dot{x}_n &= f(x, u) \\ y &= x_1 \end{aligned} \tag{1}$$

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where $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$, is the state vector of the system which is assumed available for measurement, $u \in R$ is the scalar control input, $y \in R$ is the system output and the function $f : R^{n+1} \to R$ is a smooth nonlinear function.

The following assumptions are made for system (1).

Assumption 1. The map $f : R^{n+1} \to R$, is C^1, $f(0, 0) = 0$.

Assumption 2. The inequality $g(x, u) = \partial f(x, u)/\partial u \neq 0$ holds for $(x, u) \in R^{n+1}$. It is implied that $g(x, u)$ is either positive or negative and is bounded away from zero for $(x, u) \in R^{n+1}$. Without losing the generality, we assume that there exist $\overline{g} > \underline{g} > 0$, such that $\overline{g} > g(x, u) > \underline{g} > 0$. This assumption is made for the controllability of the system (1) because $g(x, u)$ can be viewed as the control gain of the system (1).

Assumption 3. The given desired trajectory $y_d(t)$ and its derivatives up to $(n + 1)$-th are bounded.

The aim of this paper is to find u for system (1) such that the system output $y(t)$ tracks a desired trajectory $y_d(t)$ while keeping all the signals of the closed-loop system bounded.

III. Fuzzy Adaptive Controller Design

In this section, the GFHM-based fuzzy logic system is used to approximate the unknown function and the adaptive law is designed to ensure the existence of the control input.

A. Description of GFHM-Based Fuzzy Logic System

The fuzzy logic system performs a mapping from an input vector $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in U \subseteq R^n$ to a scalar output $y \in R$. In this paper, we use GFHM as fuzzy basis function, which can be characterized by a set of if-then rule as follows [19],

If χ_1 is $F^l_{\chi_1}$ and χ_2 is $F^l_{\chi_2}$ and $\cdots$ and χ_ω is $F^l_{\chi_\omega}$,

Then $y^l = \sum_{m=1}^{\omega} c^l_{F_{\chi_m}}$ where, $\chi = [\chi_1, \cdots, \chi_\omega]^{\mathrm{T}}$ is the generalized input vector, derived from the transformation of input vector $x = [x_1, \cdots, x_n]^{\mathrm{T}}$,

$$
\begin{array}{rrcl}
 & \chi_1 & = & x_1 - d_{11} \\
 & \chi_{r_1+1} & = & x_2 - d_{21} \\
 & & \vdots & \\
 & \chi_{r_1+\cdots+r_{n-1}+1} & = & x_n - d_{n1} \\
\cdots & \chi_{r_1} & = & x_1 - d_{1r_1} \\
\cdots & \chi_{r_1+r_2} & = & x_2 - d_{2r_2} \\
 & & \vdots & \\
\cdots & \chi_{r_1+\cdots+r_n} & = & x_n - d_{nr_n}
\end{array}
$$

$\omega = \left(\sum_{j=1}^{n} r_j\right)$ is the total number of the generalized input variables of the fuzzy logic system, r_j are the numbers of the generalized input variables derived from transforming x_j, $(j = 1, \cdots, n)$. d_{ji} $(j = 1, \cdots, n,\ i = 1, \cdots, r_j)$ is the tranformation offset for x_j. F_{χ_m} is the fuzzy sets with respect to χ_m, $m = 1, \cdots, \omega$, which includes only two linguistic expressions, i.e., Positive (P_{χ_m}) and Negative (N_{χ_m}), with respect to which $c_{P_{\chi_m}}$ and $c_{N_{\chi_m}}$ are consequent parameters.. The membership function of the fuzzy sets P_{χ_m} and N_{χ_m} for the generalized input variables are depicted as

$$
\begin{array}{rcl}
\mu_{P_{\chi_m}} & = & \exp\left[-(\chi_m - \overline{k}_m)^2/2\right] \\
\mu_{N_{\chi_m}} & = & \exp\left[-(\chi_m + \overline{k}_m)^2/2\right]
\end{array} \tag{2}
$$

where, $\overline{k}_m$, $m = 1 \cdots, \omega$, is a constant offset. According to these fuzzy rules, the fuzzy logic system with singleton fuzzifier, product inference engine and center-average defuzzifier, is in the following form,

$$y \;=\; \theta^{\mathrm{T}} W(x) \tag{3}$$

where $\theta = [C_0, C_1]^{\mathrm{T}}$, $C_0 = \sum_{m=1}^{\omega} \frac{c_{P_{\chi_m}} + c_{N_{\chi_m}}}{2}$, $C_1 = \left[\frac{c_{P_{\chi_1}} - c_{N_{\chi_1}}}{2}, \cdots, \frac{c_{P_{\chi_\omega}} - c_{N_{\chi_\omega}}}{2}\right]$, $W(x) = [1, \tanh(\overline{k}_1\chi_1), \cdots, \tanh(\overline{k}_\omega\chi_\omega)]^{\mathrm{T}}$.

It has been proven that GFHM-based fuzzy logic system (3) can approximate any continuous function over a compact set $D \subset R^n$ to arbitrary accuracy [19]

$$f(x) = \theta^{*\mathrm{T}} W(x) + \zeta(x) \tag{4}$$

where θ^* is the optimal weight parameter, and $\zeta(x)$ is the approximate error. For simplicity, $\zeta(x)$ is denoted by ζ. We assume that there exist optimal weight parameters such that, $|\zeta| \leq \zeta_M$ with constant $\zeta_M > 0$ for all $x \in D$. Moreover, θ^* is bounded by $||\theta^*|| \leq \theta_M$ on the compact set D, where $\theta_M > 0$ is a constant. Let θ be the estimate of θ^*, and the weight parameter estimation error be $\tilde{\theta} = \theta - \theta^*$.

Remark 1. A property of GFHM is used in this paper to facilitate our controller design, i.e., $0 \leq |\tanh(Z)| \leq 1, 0 \leq |\partial \tanh(Z)/\partial Z| = |1 - \tanh^2(Z)| \leq 1, \forall Z \in R$.

B. GFHM-Based Fuzzy Controller Design

In this section, we develop a fuzzy controller by use of implicit theorem [2] and singular perturbation theory [17] for the case where the plant model (1) is assumed to be unknown.

Let $e = x_1 - y_d$ and the corresponding tracking error vector is $\mathbf{e} = [e, \dot{e}, \cdots, e^{(n-1)}]^{\mathrm{T}}$. We define the filtered tracking error as

$$\xi = [\mathbf{k}^{\mathrm{T}}\ 1]\mathbf{e} \tag{5}$$

where $\mathbf{k} = [k_1, \cdots, k_{n-1}]^{\mathrm{T}}$ is determined so that $s^{n-1} + k_{n-1}s^{(n-2)} + \cdots + k_1$ is Hurwitz, i.e., $\mathbf{e} \to 0$ as $\xi \to 0$. From (1), the following error dynamic is immediate

$$e^{(n)} = f(x, u) - y_d^{(n)} \tag{6}$$

Adding and subtracting bu, we can rewrite (6) as

$$e^{(n)} = f(x, u) - bu + bu - y_d^{(n)} \tag{7}$$

where $b > 0$ is a design constant, which will be specified later. The term bu is used to ensure the existence of the control input for the tracking problem. The fuzzy logic system is used to approximate the unknown function $f_b(x, u) = f(x, u) - bu$

$$f_b(x, u) = \theta^{*\mathrm{T}} W(x, u) + \zeta \tag{8}$$

then, we have

$$e^{(n)} = \theta^{\mathrm{T}} W(x, u) - \tilde{\theta}^{\mathrm{T}} W(x, u) + \zeta + bu - y_d^{(n)} \tag{9}$$

where $\theta = [\theta_x^{\mathrm{T}}, \theta_u^{\mathrm{T}}]^{\mathrm{T}} \in R^\omega$, $W(x, u) = [W(x)^{\mathrm{T}}, W(u)^{\mathrm{T}}]^{\mathrm{T}} \in R^\omega$, $\omega = \left(\sum_{i=1}^{n} r_i + r_u\right)$ is the total number of the generalized input variables of the fuzzy system. r_i, $i = 1, \cdots, n$ and r_u are the numbers of the generalized inputs by transforming x_i and u. θ_x^{T} and θ_u^{T} are the estimation parameters with respect to $W(x)$ and $W(u)$. ζ is the approximate error. Let $v = K\xi + [0\ \mathbf{k}^{\mathrm{T}}]\mathbf{e} - y_d^{(n)}, K > 0$, then the error dynamic is as the following

$$\dot{\xi} = -K\xi + [\theta^{\mathrm{T}} W(x,u) + bu + v - \tilde{\theta}^{\mathrm{T}} W(x,u) + \zeta] \quad (10)$$

We design the following fast dynamic system to obtain the control input u,

$$\epsilon \dot{u} = -\theta^{\mathrm{T}} W(x,u) - bu - v \quad (11)$$

If we choose $\epsilon = 0$, (11) is reduced to an algebraic equation

$$0 = -\theta^{\mathrm{T}} W(x,u^*) - bu^* - v \quad (12)$$

Where u^* is the desired control input of u. The existence of u^* is demonstrated by implicit theorem [2]. Because ξ, $[0\ \ \mathbf{k}^{\mathrm{T}}]\mathbf{e}$ and $y_d^{(n)}$ are not the functions of u, we have

$$\begin{aligned} \hat{g}(x,u) &= \partial[\theta^{\mathrm{T}} W(x,u) + v + bu]/\partial u \\ &= \theta_u^{\mathrm{T}} \partial W(u)/\partial u + b \end{aligned} \quad (13)$$

In order to guarantee the existence of the solution for the control input in (11) and (12), it should be ensured that $\hat{g}(x,u) \neq 0$. To be consistent with the assumption 2 for original system (1), let $\hat{g}(x,u) > 0$ and $b > 0$. We should design an adaptive law and select an appropriate constant b_1 such that $\hat{g}(x,u) > b_1 > 0$ is bounded away from zero. From remark 1, the following inequality holds,

$$\begin{aligned} |\hat{g}(x,u)| &\geq |b| - ||\theta_u^{\mathrm{T}}||\ ||\partial W(u)/\partial u|| \\ &\geq b - ||\theta_u^{\mathrm{T}}|| \\ &> b_1 \end{aligned} \quad (14)$$

Thus, the adaptive law should ensure $||\theta_u^{\mathrm{T}}|| < b - b_1$. We can get $b_1 < \hat{g}(x,u) < 2b - b_1$. Let $\hat{f}_b(x,u) = \theta^{\mathrm{T}} W(x,u) + bu$. Using mean value theorem, there exists $0 < \lambda < 1$, such that

$$\hat{f}_b(x,u) = \hat{f}_b(x,u^*) + \hat{g}_\lambda (u - u^*) \quad (15)$$

where $\hat{g}_\lambda = \hat{g}_\lambda(x, u_\lambda)$, $u_\lambda = \lambda u + (1-\lambda)u^*$. Note that $2b - b_1 > \hat{g}_\lambda > b_1 > 0$.

Let $\eta = u - u^*$. From (10)–(12) and (15), we obtain the following error dynamics (a general form of singular perturbed system [17]).

$$\left\{ \begin{aligned} \dot{\xi} &= -K\xi + \hat{g}_\lambda \eta - \tilde{\theta}^{\mathrm{T}} W(x,u) + \zeta \\ \epsilon\dot{\eta} &= -\hat{g}_\lambda \eta - \epsilon \dot{u}^* \end{aligned} \right. \quad (16)$$

The adaptive law is

$$\begin{aligned} \dot{\theta}_x &= \Gamma_1 [W(x)\xi - \delta_1 \theta_x] \\ \dot{\theta}_u &= \begin{cases} \Gamma_2 W(u)\xi & \text{if } ||\theta_u|| < b - b_1 \\ & \text{or } ||\theta_u|| = b - b_1 \\ & \text{and } \xi\theta_u^{\mathrm{T}} W(u) \leq 0 \\ \Gamma_2 W(u)\xi - \Gamma_2 \dfrac{\xi\theta_u^{\mathrm{T}} W(u)}{||\theta_u||^2}\theta_u & \text{if } ||\theta_u|| = b - b_1 \\ & \text{and } \xi\theta_u^{\mathrm{T}} W(u) > 0 \end{cases} \end{aligned} \quad (17)$$

where Γ_1 and Γ_2 are positive defined diagonal matrices, $\delta_1 > 0$ is a constant.

Consider the Lyapunov function,

$$V = 1/2 \left(\xi^2 + \eta^2 + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta} \right) \quad (18)$$

where $\Gamma = \mathrm{diag}(\Gamma_1, \Gamma_2)$, we have the following theorem.

Theorem 1: Consider the system (1) regulated by the control law in (11). Suppose that the Lyapunov function (18) is bounded by a given positive constant p for all initial conditions, and that the estimation parameters are updated according to (17). Then, all the closed loop signals are semi-globally uniformly bounded, and the tracking errorr is attracted to a neighborhood of the origin, whose size can be adjusted by control parameters.

proof: Firstly, we should give the upper bound of $\dot{u}^*$. Differentiate the right hand side of (12) and after some simple manipulations, we have

$$\begin{aligned} \dot{u}^* &= -\left[b + \theta_u^{\mathrm{T}} \frac{\partial W(u^*)}{\partial u^*} \right]^{-1} \left[\theta_x^{\mathrm{T}} \frac{\partial W(x)(\dot{x}_1, \cdots, \dot{x}_n)^{\mathrm{T}}}{\partial(x_1, \cdots, x_n)} \right. \\ & \left. \quad + K\dot{\xi} + [0\ \mathbf{k}^{\mathrm{T}}]\dot{\mathbf{e}} - y_d^{(n+1)} \right] \end{aligned} \quad (19)$$

By induction, there exists a continuous function $B(y_d, \cdots, y_d^{(n+1)}, \tilde{\theta}, \xi, \eta)$, such that,

$$\dot{u}^* = B(y_d, \cdots, y_d^{(n+1)}, \tilde{\theta}, \xi, \eta) \quad (20)$$

Define the compact sets, $D_0 := \{y_d, \dot{y}_d, \cdots,\ y_d^{(n+1)} | y_d^2 + \dot{y}_d^2 \cdots + \left(y_d^{(n+1)}\right)^2 \leq B_0\}$, $D_1 := \left\{\xi^2 + \eta^2 + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta} \leq 2p\right\}$ for $p > 0$. Clearly, $D_0 \times D_1$ is compact in $R^{n+3+\omega}$, where ω is the dimension of $\tilde{\theta}$. Therefore, B has a maximum M on $D_0 \times D_1$.

The derivative of the Lyapunov function is

$$\begin{aligned} \dot{V} &= \xi\dot{\xi} + \eta\dot{\eta} + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \dot{\theta} \\ &= -K\xi^2 + g_\lambda \xi\eta - \tilde{\theta}^{\mathrm{T}} W(x,u)\xi + \xi\zeta - \epsilon^{-1} g_\lambda \eta^2 \\ & \quad -\eta\dot{u}^* + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \dot{\theta} \end{aligned} \quad (21)$$

Let $\underline{\beta} \leq \hat{g}_\lambda \leq \overline{\beta}$, where $\overline{\beta} = 2b - b_1$, $\underline{\beta} = b_1$.

Using the facts,

$$\begin{aligned} \xi^2 + (1/4)\eta^2 &\geq \xi\eta \\ \xi^2 + (1/4)\zeta^2 &\geq \xi\zeta \end{aligned}$$

we have

$$\begin{aligned} \dot{V} &\leq (-K + 1 + \overline{\beta})\xi^2 + (1/4)\overline{\beta}\eta^2 + (1/4)\zeta^2 \\ & \quad -\epsilon^{-1}\underline{\beta}\eta^2 + |\eta B| + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \dot{\theta} - \tilde{\theta}^{\mathrm{T}} W(x,u)\xi \end{aligned} \quad (22)$$

Using (17), we can deduce the following inequality,

$$\tilde{\theta}_u^{\mathrm{T}} (\Gamma_2^{-1} \dot{\theta}_u - \xi W(u)) \quad \leq \quad 0$$

Substitute the adaptive law (17) into (22),

$$\begin{aligned} \dot{V} &\leq (-K + 1 + \overline{\beta})\xi^2 + (1/4)\overline{\beta}\eta^2 + (1/4\zeta^2) \\ & \quad -\epsilon^{-1}\underline{\beta}\eta^2 + |\eta M| - \delta_1 \tilde{\theta}_x^{\mathrm{T}} \theta_x \end{aligned} \quad (23)$$

Choose $K = 1 + \overline{\beta} + \gamma_0$ and $\epsilon^{-1} = (1/\underline{\beta})[(1/4)\overline{\beta} + M^2/(2\varrho) + \gamma_0]$, where γ_0 and ϱ are positive constants. Using $2\tilde{\theta}_x^{\mathrm{T}} \hat{\theta}_x \geq ||\tilde{\theta}_x||^2 - ||\theta_x^*||^2$, we obtain,

$$\begin{aligned} \dot{V} &\leq \left[-\gamma_0(\xi^2 + \eta^2) + \varrho/2 + (1/4)\zeta^2 \right] \\ & \quad -(1/2)\delta \left(||\tilde{\theta}_x||^2 - ||\theta_x^*||^2 \right) \\ &\leq \left[-\gamma_0(\xi^2 + \eta^2) - \frac{\delta}{2\lambda_{\max}(\Gamma^{-1})} \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta} \right] \\ & \quad + \varrho/2 + (1/4)\zeta^2 + (\delta/2)||\theta^*||^2 \end{aligned} \quad (24)$$

Let $e = (1/4)\zeta^2 + (\delta/2)||\theta^*||^2$, because $|\zeta| < \zeta_M$ and $||\theta^*|| < \theta_M$, then we get $e \leq (1/4)\zeta_M^2 + (\delta/2)||\theta_M||^2 = e_M$. Choose $\gamma = \min\left(\gamma_0, \delta/\left[2\lambda_{\max}(\Gamma^{-1})\right]\right)$,

$$\dot{V} \leq -2\gamma V + (\varrho/2 + e_M) \tag{25}$$

Let $\gamma > (\varrho/2 + e_M)/(2p)$, then $\dot{V} < 0$ on $V(t) = p$. Let $L = (\varrho/2 + e_M)/(2\gamma)$ and for all $t \geq 0$, the solution of inequality (24) is

$$0 \leq V(t) \leq L + [V(0) - L]\exp(-2\gamma t) \tag{26}$$

It means that $V(t)$ is eventually bounded by L. Thus, ξ, η are uniformly bounded. By choosing appropriate value K, ϵ, the quantity L can be made arbitrarily small. Because ξ is bounded, from (5), it follows that $\mathbf{e} = [e, \dot{e}, \cdots, e^{(n-1)}]^{\mathrm{T}}$ is bounded. Then, all the signals in the closed loop system are bounded. It is clear that increasing the values of K, reducing the value of $\lambda_{\max}(\Gamma^{-1})$ and ϵ, i.e., increasing the value of γ will result in a better tracking performance, but lead to a high gain control scheme. Decreasing δ_1 will help to reduce e, however, a very small δ_1 may not be enough to prevent the fuzzy weight estimates from drifting to very large values.

IV. Numerical Simulation

In this section, the following example are considered to illustrate the proposed control method.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^3 + x_1 x_2 + u^3/3 + 0.2(1 + x_2^2)u \\ y &= x_1 \end{aligned} \tag{27}$$

where, x_1 and x_2 are state variables, u is control input, y is the output. Clearly, system (27) is in non-affine form (1) and satisfies assumption 1 and 2. The control objective is to find a control input u such that all the signals in the closed-loop system remain bounded and the output y can track the desired trajectory $y_d = 0.75[\sin(t) + \cos(0.5t)]$.

For the unknown system, we design the GFHM-based fuzzy adaptive controller in the following procedure.

We use $[x_1, x_2, u]^{\mathrm{T}}$ as the input vector of the fuzzy logic system. The generalized input vector χ can be obtained by transforming the input variables x_1, x_2, u. In simulation, every input variable is transformed for three times. For x_1 and x_2, the transformation offsets are $d = [-0.5, 0, 0.5]$ and the constant offset is $\overline{k} = 0.8$. For u, the transformation offsets are $d = [-1.5, 0, 1.5]$ and the constant offset is $\overline{k} = 2$. Two membership functions depicted as (2) are chosen for each generalized input variable (see e.g., Fig.1 and Fig.2). Choose the following design parameters, $k = 2$, $\xi = \dot{e} + ke$, $K = 5$, $\epsilon = 0.02$, $\Gamma_1 = \Gamma_2 = \mathrm{diag}\{10\}$, $\delta_1 = 0.1$, $b_1 = 0.2$. The fuzzy adaptive controller is

$$\epsilon\dot{u} = -\theta^{\mathrm{T}} W(x, u) - bu - K\xi - k\dot{e} + \dot{y}_d, \quad u = \int \dot{u}\mathrm{d}t \tag{28}$$

Remark 2: The choice of b is critical for the closed-loop stability and the tracking performance. On the one hand, too small value of b cannot guarantee the existence of solution for u in (11) because $b + \partial[\theta^{\mathrm{T}} W(x, u)]/\partial u = 0$ may occur and it leads to the controller singularity, on the other hand, the signal of $f(x, u)$ is submerged by the signal of bu if too large value of b is chosen (see (8)) and it may degrade the tracking performance because the actual dynamic of system (1) cannot be updated correctly by the fuzzy logic system through on-line learning. According to assumption 2, we choose $\underline{g} < b_1 < b < (\underline{g} + \overline{g})/2$, such that, $\overline{g} > \hat{g}(x, u) > \underline{g} > 0$ holds.

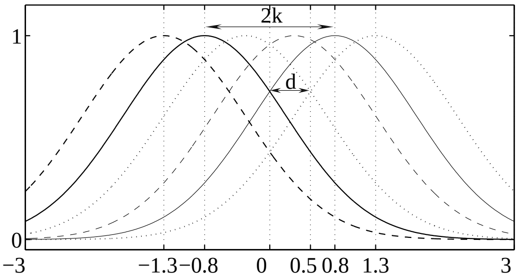

Fig. 1. Membership functions of the generalized input variables for x_1 (x_2). Dashed line: $x_1 - 0.5$; Solid line: x_1; Dotted line: $x_1 + 0.5$

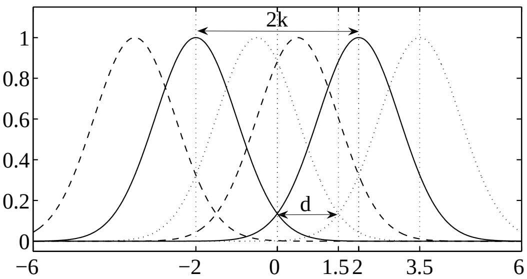

Fig. 2. Membership functions of the generalized input variables for u. Dashed line: $u - 1.5$; Solid line: u; Dotted line: $u + 1.5$

To show how the choice of b affects the control performance of the system, We choose $b = 1, 2$ for the simulation and Fig.3 – Fig.6 show the simulation results with fuzzy adaptive controller. In Fig.5, it can be seen that the controller with $b = 1$ has a better tracking performance and too large value $b = 2$ degrades the performance but still guarantees the system stability. Fig. 4 and Fig. 6 show the boundedness of control input and the fuzzy estimation parameters for different choices of b.

V. **Conclusion**

In this paper, time scale separation based fuzzy adaptive control method is developed for a class of non-affine nonlinear systems. The implicit function theorem is used to demonstrate the existence of the optimal control input for the non-affine system, which is approximated by the solution of a fast dynamical equation. The error system dynamics are constructed by combining implicit function theorem and the mean value theorem. Stability analysis based on Lyapunov theory shows that the developed control scheme achieves semi-globally uniform boundedness of all the signals in the closed-loop, and the bounded errors can be made arbitrarily small by choosing appropriate design parameters.

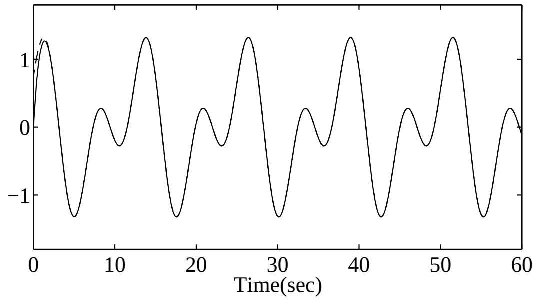

Fig. 3. Actual (solid) and desired (dotted) output ($b = 1$).

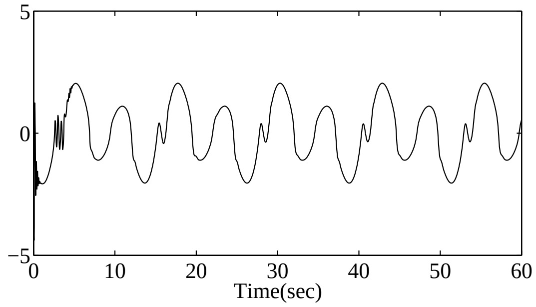

Fig. 4. Adaptive control input ($b = 1$).

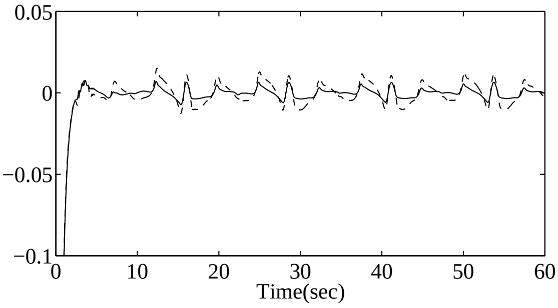

Fig. 5. Tracking errors (solid line: b=1; dotted line: b=2).

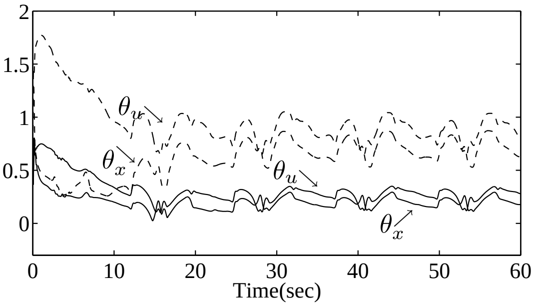

Fig. 6. L_2 norms of θ_x, θ_u. solid line: $b = 1$; Dotted line: $b = 2$.

REFERENCES

[1] A. Isidori, *Nonlinear Control Systems*. Berlin, Germany: Springer-Verlag, 1989.

[2] T. M. Apostol, *Mathematical analysis*, Reading, Mass: Addison-Wesley, 1974.

[3] S. S. Ge, J. Zhang. Neural-network control of nonaffine nonlinear system with zero dynamics by state and output feedback. *IEEE Transactions on Neural Networks*, vol. 14, no. 4, 900-918, 2003.

[4] S. Labiod, T. M. Guerra, Adaptive fuzzy control of a class of SISO nonaffine nonlinear systems. *Fuzzy Sets and Systems*, vol. 158, no. 10, 1126-1137, 2007.

[5] J. H. Park, G. T. Park, S. H. Kim, C. J. Moon, Direct adaptive self-structuring fuzzy controller for nonaffine nonlinear system. *Fuzzy Sets and Systems*, vol. 153, no. 3, 429-445, 2005.

[6] B. J. Yang, A. J. Calise, Adaptive Control of a Class of Nonaffine Systems Using Neural Networks. *IEEE Transactions on Neural Networks*, vol. 18, no. 4, 1149-1159, 2007.

[7] S. Labiod, T. M. Guerra, Indirect adaptive fuzzy control for a class of nonaffine nonlinear systems with unknown control directions, *International Journal of Control, Automation, and Systems*, vol. 8, no. 4, 903-907, 2010.

[8] A.J. Calise, N. Hovakimyan, M. Idan, Adaptive output feedback control of nonlinear systems using neural networks. *Automatica*, vol. 37, no. 8, 1201-1211, 2001.

[9] N. Hovakimyan, F. Nardi, A. Calise, N. Kim, Adaptive output feedback control of uncertain nonlinear systems using single-hidden-layer neural networks. *IEEE Transactions on Neural Networks*, vol. 13, no. 6, 1420-1431, 2002.

[10] N. Hovakimyan, F. Nardi and A. J. Calise, A novel error observer-based adaptive output feedback approach for control of uncertain systems. *IEEE Transactions On Automatic Control*, vol. 47, no. 8, 1310-1314, 2002.

[11] J. H. Park, S. H. Kim, Direct adaptive output-feedback fuzzy controller for nonaffine nonlinear system. *IEE Proceedings: Control Theory and Applications*, vol. 151, no. 1, 65-72, 2004.

[12] P. S. Yoon, J. H. Park, G. T. Park, Adaptive fuzzy control of nonaffine nonlinear systems using Takagi-Sugeno fuzzy models. *IEEE International Conference on Fuzzy Systems*, Hawaii: IEEE Press, 642-645, 2002.

[13] J. Y. Park and G. T. Park. Robust adaptive fuzzy controller for non-affine nonlinear systems with dynamic rule activation. *International Journal of Robust Nonlinear Control*, 2003, 13(2): 117-139.

[14] J. D. Bošković, L. J. Chen and R. K. Mehra. Adaptive tracking control of a class of non-affine plants using dynamic feedback. *American Control Conference*, Arlington: IEEE Press, 2450-2455, 2001.

[15] N. Hovakimyan, E. Lavretsky and A. Sasane. Dynamic Inversion for Nonaffine-in-Control Systems via Time-Scale Separation: Part I. *American Control Conference*, Portland: IEEE Press, 3542-3547, 2005.

[16] E. Lavretsky and N. Hovakimyan, Adaptive Dynamic Inversion for Nonaffine-in-Control Systems via Time-Scale Separation: Part II. *American Control Conference*, Portland: IEEE Press, 3548-3553, 2005.

[17] H. K. Khalil, *Nonlinear Systems*. Upper Saddle River, NJ: Prentice-Hall, 1996.

[18] D. X. Gao, Z. Q. Sun, J. H. Liu, Dynamic inversion control for a class of pure-feedback systems. *Asian Journal of Control* vol. 14, Issue 2, 605-611, 2012.

[19] H. G. Zhang, Y. B. Quan, Modeling, Identification, and Control of a Class of Nonlinear Systems, *IEEE Transactions On Fuzzy Systems*, vol. 9, no. 2, 349-354, 2001.

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