Hydrodynamic Analysis of Earthquake Excited Dam-Reservoirs with Sloping Face

E. Demirel

***Abstract*—A 2D computational model is used for the hydrodynamic analyses of earthquake excited dam-reservoirs. The model is based on the finite volume solution of the Navier-Stokes equations considering compressibility effects due to sudden change in pressure field during earthquake. Volume of Fluid Method (VOF) is used to track the nonlinear free-surface waves in the reservoir. A non-reflecting boundary condition is applied at the outflow boundary to prevent the pressure and free-surface waves reflecting from the boundary back into the domain. Numerical results are compared with the existing semi-analytical solutions developed for dams with vertical and sloping upstream faces to demonstrate the capability of the computer code for the realistic simulation of earthquake excited dam-reservoirs.**

***Index Terms*—Hydrodynamic pressure, earthquake, dam-reservoir, free-surface, VOF.**

I. INTRODUCTION

A dam designer needs an accurate computational model and a computer code in hydrodynamic analysis of earthquake excited dam-reservoirs. The computer code should consider the compressibility effects due to sudden change in pressure field during earthquake and free-surface waves in reservoirs with respect to possibility of wave overtopping on the dam crest.

Westergaard first analyzed the earthquake response of dam-reservoirs [1]. He proposed an analytical expression for hydrodynamic pressures on dam face neglecting compressibility effects. Chopra suggested analytical formulas for hydrodynamic response of dam-reservoirs considering compressibility effects during harmonic and arbitrary ground motions [2]. Hung and Wang analyzed earthquake excited dam-reservoir system solving dimensionless Navier-Stokes equations and pressure equation by finite difference method and they used kinematic boundary condition to track the free surface [3]. Kinematic boundary condition can track only non-breaking waves, so the free-surface tracking algorithm should predict breaking surface waves for the possibility of wave breaking during arbitrary ground motion. They concluded that viscous effects are negligible in hydrodynamic analysis of dam–reservoirs. Chen investigated the hydrodynamic pressures in dam-reservoir with sloping bottom and water rising in the reservoir using the same mathematical model [4]. He reported the maximum wave run-up on a vertical dam-face as functions of dimensionless displacements for various reservoir bottom slopes.

In this study, a computational model and a computer code were proposed and validated. It can be concluded that the model can compute the hydrodynamic pressures accurately on the vertical and inclined dam-faces. The proposed computer code *NASSLARD* (Navier-Stokes Solver for Large Domains) can predict the maximum wave run-up accurately on a sloping dam-face during earthquake considering the possible wave breaking.

II. GOVERNING EQUATIONS

The momentum equations for two dimensional flows in a vertical plane (Fig.1), integrated over a control volume are written as

$$\frac{\partial}{\partial t}\int_{CV} u d\forall + \int_{CS} u\vec{V}.d\vec{A} = -\frac{1}{\rho}\int_{CV}\frac{\partial p}{\partial x}d\forall + \vartheta\int_{CS}\vec{\nabla}u.d\vec{A} - \int_{CV} a_x\, d\forall \quad (1)$$

$$\frac{\partial}{\partial t}\int_{CV} w d\forall + \int_{CS} w\vec{V}.d\vec{A} = -\frac{1}{\rho}\int_{CV}\frac{\partial p}{\partial z}d\forall + \vartheta\int_{CS}\vec{\nabla}w.d\vec{A} - \int_{CV} g\, d\forall \quad (2)$$

where x and z are coordinate axes in horizontal and vertical directions respectively, a_x is horizontal ground acceleration, u and w are velocity components, $\vec{V}$ is velocity vector relative to the moving ground, p is pressure, t is time, g is gravitational acceleration, ν is kinematic viscosity, ρ is fluid density, $\vec{\nabla}$ is the del operator, CV indicates control volume, CS indicates control surface and $d\vec{A}$ is the area element normal to the control surface pointing out of the control volume. Horizontal ground acceleration is included to represent earthquake excitations.

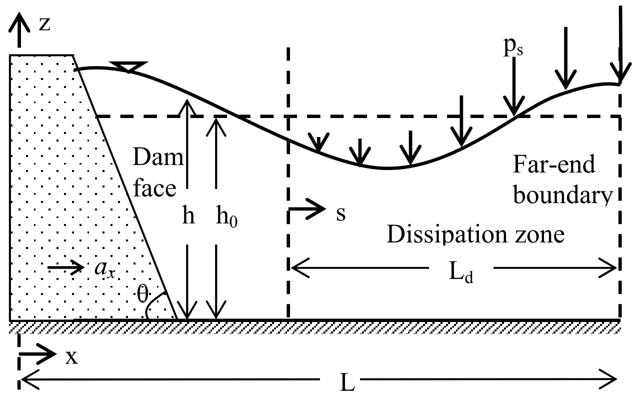

Fig. 1. Definition sketch of dam-reservoir system subjected to earthquake

E.D. Eskisehir Osmangazi University, Department of Civil Engineering, Eskisehir, 26480, Turkey, (e-mail:edemirel@ogu.edu.tr)

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The continuity equation is simplified for weakly compressible fluid by dropping the spatial variations of density since they are negligible in dam-reservoir hydrodynamics. An equation of state is applied to represent the density variations with pressure through the definition of acoustic velocity. The time variation of density is linked to pressure variations and the continuity is recast to be solved for the pressure field:

$$-\frac{1}{\rho a^2}\frac{\partial}{\partial t}\int_{CV} p d\forall = \int_{CS}\vec{V}.d\vec{A} \tag{3}$$

in which *a* (=1,438 m/s) = acoustic velocity in water.

III. NUMERICAL SOLUTION

The computational domain and the grid system are assumed to move with the ground. The integral equations are discretized on a staggered grid arrangement.

$$u_{i,j}^{n+1} = F_{i,j}^n + \Delta t\{[(p_{i,j}^{n+1} - p_{i+1,j}^{n+1})/\rho]/\Delta x_{i+1/2}\} \tag{4}$$

$$w_{i,j}^{n+1} = G_{i,j}^n + \Delta t\{[(p_{i,j}^{n+1} - p_{i,j+1}^{n+1})/\rho]/\Delta z_{j+1/2} - g\} \tag{5}$$

$$F_{i,j}^n = u_{i,j}^n + \Delta t\{[\vartheta(Difu)_{i,j}^n - (Conu)_{i,j}^n]/(\Delta x_{i+1/2}\Delta z_j)\} \tag{6}$$

$$G_{i,j}^n = w_{i,j}^n + \Delta t\{[\vartheta(Difw)_{i,j}^n - (Conw)_{i,j}^n]/(\Delta x_i\Delta z_{j+1/2})\} \tag{7}$$

$$(u_{i,j}^{n+1} - u_{i-1,j}^{n+1})/\Delta x_i + (w_{i,j}^{n+1} - w_{i,j-1}^{n+1})/\Delta z_j \tag{8}$$

where Δt is the time step, Δx and Δz are mesh sizes, *Dif* and *Con* represent diffusive and convective fluxes. To utilize the advantage of staggered grid system, convenient control volumes are selected for each equation. First order derivatives in diffusive fluxes are discretized using second order polynomial approximation on a variable mesh. Convective fluxes are evaluated by first order upwind (FOU). Pressure solution is obtained from the Poisson equation for pressure. Discretized form of the Poisson equation for pressure is obtained by substituting Eqs. 4 and 5 into 8

$$[(p_{i,j}^{n+1} - p_{i+1,j}^{n+1})/\Delta x_{i+1/2} - (p_{i-1,j}^{n+1} - p_{i,j}^{n+1})/\Delta x_{i-1/2}]/\Delta x_i + [(p_{i,j}^{n+1} - p_{i,j+1}^{n+1})/\Delta z_{j+1/2} - (p_{i,j-1}^{n+1} - p_{i,j}^{n+1})/\Delta z_{j-1/2}]/\Delta z_j = -\left[\frac{F_{i,j}^n - F_{i-1,j}^n}{\Delta x_i} + \frac{G_{i,j}^n - G_{i,j-1}^n}{\Delta z_j}\right]\frac{\rho}{a^2\Delta t} \tag{9}$$

Free surface position is captured using the VOF (Volume of Fluid) method which was originally developed by Hirt and Nichols [5]. In the VOF method, a function *F* is introduced

$$\frac{\partial F}{\partial t} + \vec{\nabla}.(\vec{V}F) = 0 \tag{10}$$

where *F* is a flow variable with values between zero and one . In particular, *F* = 1 corresponds to full cell, *F* = 0 to an empty cell, and 0 < *F* < 1 to a surface cell. In the VOF/PLIC (Piecewise Linear Interface Reconstruction) algorithm an interface line is constructed in free surface cells using the gradient of *F* function. In the reconstruction of the interface, normal direction to the interface is calculated by

$$\vec{n} = \frac{\vec{\nabla}F}{|\vec{\nabla}F|} \tag{11}$$

The momentum equations and the pressure Poisson equation are solved by sequential iterations. The detailed description of the computational algorithm is given in [6]. A computer code named *NASSLARD* is developed to perform the computations [6], [7].

IV. BOUNDARY CONDITIONS

At the dam-face and reservoir bottom, normal velocities are set to zero and no-slip boundary conditions are used for the tangential velocities. At the free-surface, an extrapolation procedure proposed by Miyata is used for velocities [8]. Free-surface velocities are extrapolated from the neighboring momentum velocities in this method. Pressure on the free-surface is computed from the free-surface stress conditions given by Griebel et al. [9] At the far-end of the reservoir, a combination of Sommerfeld non-reflecting boundary condition with a dissipation zone method is applied to minimize the wave reflection. The velocity components along the open-end boundary are computed from the Sommerfeld non-reflecting boundary condition which is modified to include ground accelerations:

$$\frac{1}{St}\left(a_\phi + \frac{\partial\phi}{\partial t}\right) + c\frac{\partial\phi}{\partial x} = 0 \tag{12}$$

where ϕ stands for the velocity components *u* and *w*, *c* is the wave velocity, *a* represent the ground accelerations in the velocity directions. In addition to Sommerfeld boundary condition, a dissipation zone is included to dissipate the wave energy as the end-boundary is approached (see Fig. 1). Non-atmospheric pressures are applied on the free surface to damp the surface waves similar to approach described by Westhuis [10]. A linearly increasing surface pressure is applied to produce a gradual damping on the waves over the length of the dissipation zone.

V. RESULTS

A. Hydrodynamic Forces on a Vertical Dam-Face During Arbitrary Ground Motion

Hydrodynamic forces on a vertical dam face are investigated in order to validate the computational model and computer code *NASSLARD*. Chopra presented an analytical expression for the variation of hydrodynamic pressures on a vertical dam face during arbitrary ground motion [2]:

$$p^x(0,z,t) = \frac{4\rho a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}cos\lambda_n z\int_0^t a_x(\tau)J_0\{\lambda_n c(t-\tau)\}d\tau \tag{13}$$

where J_0=the Bessel function of the first kind of zero. The determination of hydrodynamic response of a vertical dam face to prescribe earthquake motion involves the numerical evaluation of Eq. (13). Mathematica software was used to evaluate the above expression numerically [11].

Fig.2(a) shows the time history of Duzce Earthquake record (1999) with the peak ground acceleration 0.307g. The time variation of total hydrodynamic forces on dam face is shown in Fig. 2(b) for 182.88 m (600 ft) deep reservoir.

It's clear form this figure that the hydrodynamic forces can be calculated by *NASSLARD* accurately considering compressibility effects for reservoir subjected to arbitrary ground motion.

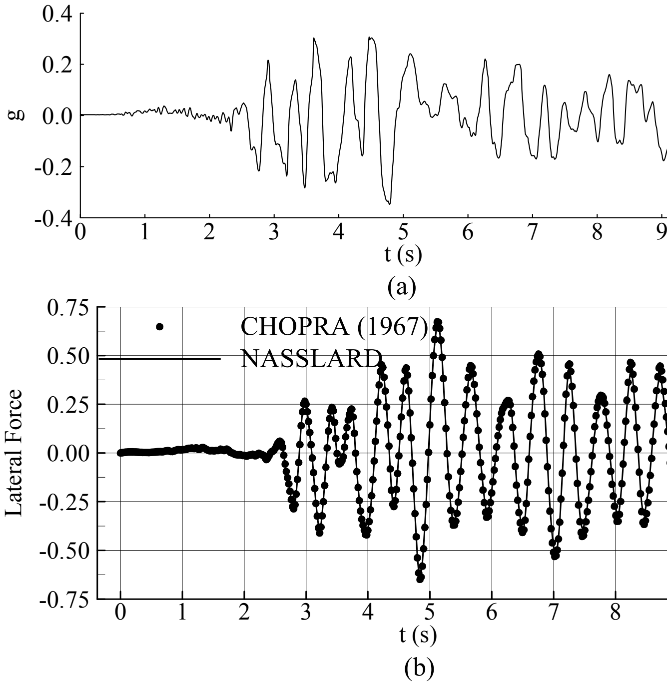

Fig. 2. a) Ground acceleration, Duzce Earthquake, Nov 12, 1999 b) total hydrodynamic force on dam face, reservoir depth 182. 88 m (600 ft)

B. Hydrodynamic Pressures on an Inclined Dam-Face

Hydrodynamic pressures on an inclined dam-face for inclination angles θ =45°, 75° and 90° were computed for a constant ground acceleration $a_x=\alpha g$ and compared with Chwang's analytical results [12] in Fig. 3. Compressibility effects are neglected in simulations adjusting the sound speed in water to a large value since Chwang's linear solutions are valid for only incompressible fluid. A "stair step" approach which defines the solid body as aligned with cell edges is used to adjust the interface position of sloping boundary on Cartesian grid.

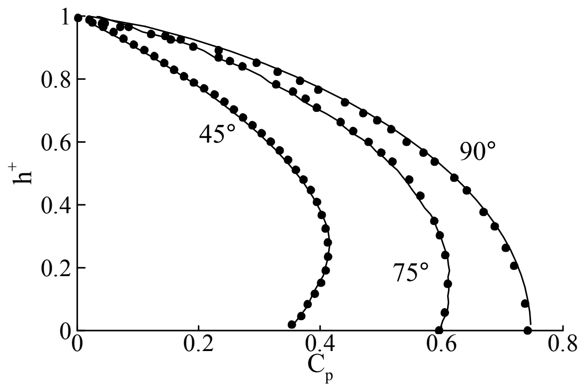

Fig. 3. Variation of hydrodynamic pressures with depth as function of inclination angle of dam-face with $\alpha = 0.4g$.

The parameters h^+ and C_p, dimensionless water depth and hydrodynamic pressure coefficient defined as

$$h^+ = \frac{h}{h_0} \tag{14}$$

$$Cp = \frac{p^*}{\rho\alpha g h_0} \tag{15}$$

where h_0 is initial water depth in reservoir and p^* is hydrodynamic pressure (excessive over hydrostatic pressure).

C. Surface Wave Run-up On a Vertical Dam-Face

The computed free-surface profiles were compared with Chwang's analytical results [13] in Fig. 3 for $\epsilon^2 = 0.005$ and $\epsilon^2 = 0.1$. The parameter ϵ^2 indicates instantaneous displacement of ground motion computed as

$$\epsilon^2 = \frac{1}{2}\alpha g t^2 \tag{16}$$

There is a discrepancy between the computed and analytical results at $\epsilon^2 = 0.1$ which may be due to neglected nonlinear convective and diffusive terms in Chwang's analytical solution [13].

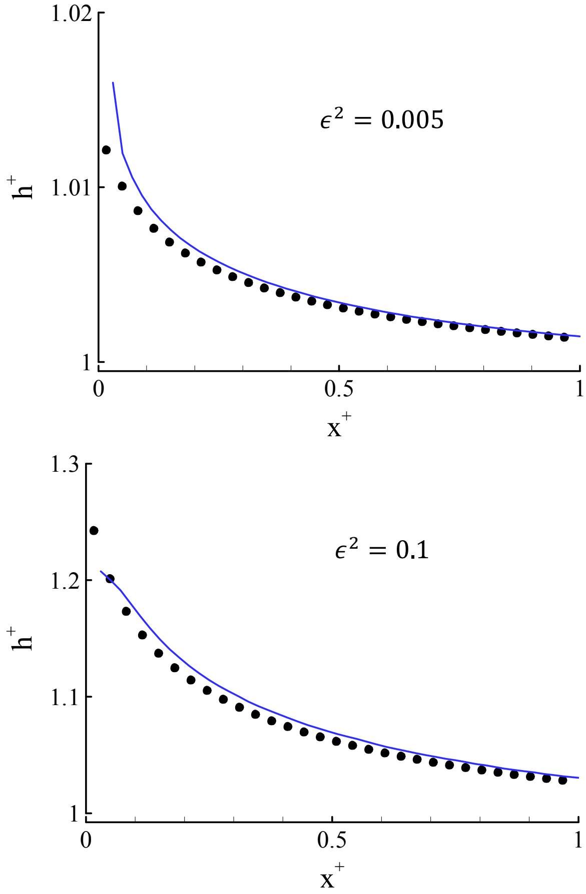

Fig. 4. Comparisons of free surface profiles with analytical results for $a_x = 0.4g$. Continuous lines show the present results, symbols show Chwang's (1983) analytical results

D. Surface Wave Run-up On an Inclined Dam-Face

It was shown that the present computational model can predict the hydrodynamic pressures on an inclined dam-face accurately at the previous section. Here, free-surface wave run-up on an inclined dam-face was investigated for inclination angle θ =45°. Fig. 5 shows the free-surface profiles corresponds to $\epsilon^2 = 0.3$ for vertical dam-face (a) and inclined dam-face (b) for inclination angle θ =45°. It can be seen from this figure that the maximum wave run-up on

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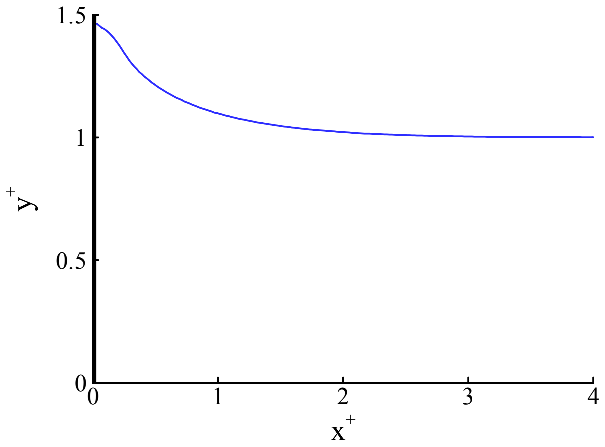
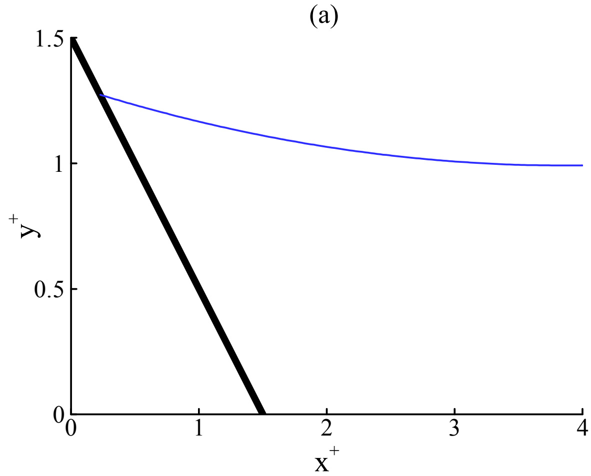
the inclined dam face is reduced to an amount of 87% of that on vertical dam-face.



(a)



(b)

Fig. 5. Free-surface profiles on dam-faces for $\epsilon^2 = 0.3$ a) vertical dam-face, b) inclined dam-face for $\theta = 45°$

VI. Conclusion

This work deals with numerical computation of hydrodynamic forces and breaking free-surface waves in dam-reservoirs subjected to constant and arbitrary ground motion. The numerical results agree fairly well with the existing analytical solutions. The maximum wave run-up occurs on a vertical dam-face in comparison to inclined dam-face, so the free-board design should be evaluated for a vertical case. Maximum surface wave run-up or especially run-down on an inclined dam-face must be considered in the landslide risk analysis.

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