Application of Self-Tuning Control System for Solution of Fault Tolerance Problem

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Abstract **- Self-tuning control can provide desirable behaviour of a process even though the process parameters are unknown or may vary with time. Conventional self-tuning control requires that the speed of adaptation must be more rapid than that of the parameter changes. However, in practice, problems do arise when this is not the case. For example, when fault occurs in a process, the parameters may change very dramatically. A new approach based on simultaneous identification and adaptation of unknown parameters is suggested for compensation of rapidly changing parameters. High dynamic precision self-tuning control can be used for the solution of a fault tolerance problem in complex and multivariable processes and systems.**

Index Terms **- Self-tuning control, identification, fault tolerance, Singular Value Decomposition.**

I. DETERMINATION OF A MATHEMATICAL MODEL OF A PROCESS

A mathematical model of a process on a stationary regime can be found from the sequence of Markov parameters using the classical Ho algorithm [1]. The Markov parameters can be obtained from input – output relationships or more directly as an impulse response of the system. It is well known that according to the theorem of Kronecker the rank of the Hankel matrix constructed from the Markov parameters is equal to the order of the system from which the parameters are obtained. Therefore, by consistently increasing the dimension of the Hankel matrix Γ until

$$rank\ \Gamma_r = rank\ \Gamma_{r+1}$$

the order of the system can be obtained as equal to r. However, in practical implementation, this rank – order relationship may not give accurate results due to several factors: sensitivity of the numerical rank calculation and bias of the rank if information about the process is corrupted by noise. This problem can be avoided using singular value decomposition (SVD) of the Hankel matrix:

$$\Gamma = USV^T \tag{1}$$

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where

$$U^T U = V^T V = I\ ,$$
$$S = diag(\sigma_1, \sigma_2, ..., \sigma_l, \sigma_{l+1}, ...\sigma_n)\ .$$

Here U and V are orthogonal matrices. The diagonal elements of the matrix S (the singular values) in (1) are arranged in the following order $\sigma_1 > \sigma_2 > ... > \sigma_n > 0$. Applying the property of SVD to reflect the order of a system through the smallest singular value, the order of the system can be determined with the tolerance required. From practical point of view a reduced order model is more preferable. Taking into account that the best approximation in the Hankel norm sense is within a distance of σ_{l+1}, the model of order l can be found. However, a relevant matrix built from Markov parameters of this reduced order model should also be of the Hankel matrix. But it is not an easy matter to find such a Hankel matrix for the reduced order process. A simpler solution, although theoretically not the best, can be found from the least squares approximation of the original Hankel matrix [2], [3] and [4]. The discrete time state space realisation of the process can be determined from the relationship between Markov parameters and representation of the Hankel matrix through relevant controllability and observability matrices of the process:

$$\Gamma = \begin{bmatrix} C_d \\ C_d A_d \\ C_d A_d^2 \\ . \\ . \end{bmatrix} \begin{bmatrix} A_d & A_d B_d & A_d^2 B_d & . & . \end{bmatrix} = \Omega \mathrm{E} \tag{2}$$

where

A_d is the system matrix,
B_d is the control matrix,
C_d is the output matrix,
Ω is the observability matrix,
E is the controllability matrix.

II. THE SELF-TUNING CONTROL SYSTEM

Consider a continuous time single input – single output second order plant (a process) given in the following canonical state space realisation form:

$$\begin{aligned} \dot{x} &= A_c x + B_c u \\ y &= C_c x \end{aligned} \tag{3}$$

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where

$$A_c = \begin{bmatrix} 0 & 1 \\ a_{1p} & a_{2p} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_c = \begin{bmatrix} c_{1p} & c_{2p} \end{bmatrix},$$

u is the control signal,
y is the output of the plant.

Assume that at the time t parameters a_{1p} and a_{2p} change dramatically due to a fault in the system, but parameters c_{1p} and c_{2p} remain constant. The mathematical model of plant (3) can be represented in the following form:

$$\ddot{x}_p = (\bar{a}_{2p} + \Delta a_{2p}(t))\dot{x}_p + (\bar{a}_{1p} + \Delta a_{1p}(t))x_p + u$$
$$y_p = \bar{c}_{2p}\dot{x}_p + \bar{c}_{1p}x_p$$

where
$a_{1p} = \bar{a}_{1p} + \Delta a_{1p}(t)$,
$a_{2p} = \bar{a}_{2p} + \Delta a_{2p}(t)$,
$\bar{a}_{1p}, \bar{a}_{2p}, \bar{c}_{1p}, \bar{c}_{2p}$ are the nominal parameters (constant) of the plant,
$\Delta a_{1p}(t)$, $\Delta a_{2p}(t)$ are the biases of the plant parameters (variable) from their nominal values,
x_p is the plant state,
y_p is the plant output.

A desirable behaviour of the plant can be determined by the following reference model:

$$\begin{aligned} \ddot{x}_m &= a_{2m}\dot{x}_m + a_{1m}x_m + g \\ y_m &= c_{2m}\dot{x}_m + c_{1m}x_m \end{aligned}, \qquad (4)$$

where
g is the input signal,
a_{1m}, a_{2m}, c_{1m}, c_{2m} are parameters of the model.

In order to compensate for the plant parameters' biases, a controller can be used. The closed loop system with the controller is represented in the following form:

$$\begin{aligned} \ddot{x}_p &= (\bar{a}_{2p} + \Delta a_{2p}(t))\dot{x}_p + (\bar{a}_{1p} + \Delta a_{1p}(t))x_p \\ &\quad + (\bar{k}_2 + \Delta k_2(t))\dot{x}_p + (\bar{k}_1 + \Delta k_1(t))x_p + g \end{aligned} \qquad (5)$$

where
$\bar{k}_1$, $\bar{k}_2$ are the constant parameters of the controller,
$\Delta k_1(t)$, $\Delta k_2(t)$ are the adjustable parameters of the controller.

The desirable quality of the process behaviour can be obtained from the following relationships:

$$\bar{k}_1 + \bar{a}_{1p} = a_{1m}$$
$$\bar{k}_2 + \bar{a}_{2p} = a_{2m}.$$

According to equations (4) and (5), the error equation is obtained as follows:

$$\ddot{e} = a_{2m}\dot{e} + a_{1m}e + z_2\dot{x}_p + z_1x_p, \qquad (6)$$

where:
$e = x_m - x_p$,
$z_1 = \Delta a_{1p}(t) + \Delta k_1(t)$,
$z_2 = \Delta a_{2p}(t) + \Delta k_2(t)$.

It can be seen from equation (6) that in order to achieve the desirable error $e \to 0$, it is necessary to provide the following conditions:

$$z_1 \equiv 0, \quad z_2 \equiv 0. \qquad (7)$$

The conditions (7) can be achieved by adjusting parameters $\Delta k_1(t)$ and $\Delta k_2(t)$ according to the following laws [5]:

$$\Delta\dot{k}_1(t) = \sigma x_p \qquad (8)$$
$$\Delta\dot{k}_2(t) = \sigma\dot{x}_p,$$

where $\sigma = Pe$.

The positive definite symmetric matrix P can be obtained from the solution of the relevant Lyapunov equation. The main problem associated with algorithms (8) is that all self-tuning contours are linked through the dynamics of the plant. The consequence is that high interaction of each contour with others will occur. This further results in poor dynamic compensation of plant parameters' biases Δa_{ip} i $=1,2,\ldots,m$, where m is a number of self-tuning contours. The idea of decoupling self-tuning contours from plant dynamics, based on simultaneous identification and adaptation, is suggested for the solution of this problem with fault tolerance. This could considerably improve performance of the overall system, especially for high dimension and multivariable plants and processes.

It can be shown [6], [7] that the self-tuning contours will be decoupled from the plant dynamics if σ can be formed such that:

$$\sigma^* = \ddot{e} - a_{2m}\dot{e} - a_{1m}e.$$

In this case the following relationship can be obtained:

$$\sigma^* = (\Delta a_{2p}(t) + \Delta k_2(t))\dot{x}_p + (\Delta a_{1p}(t) + \Delta k_1(t))x_p. \qquad (9)$$

In order to solve equation (9) with two variable parameters, the following approach is suggested: Multiply both parts of equation (9) by state variables x_p and $\dot{x}_p$ and integrate the resultant equations on the time interval (t_1, t_2), where: $t_2 = t_1 + \Delta t$. Taking the initial conditions as $t_1 = 0$, $\Delta k_i = 0$, $(i = 1,2)$ the following equations are obtained:

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$$\int_{t_1}^{t_1+\Delta t} \sigma^* x_p dt = \Delta a_{2p} \int_{t_1}^{t_1+\Delta t} \dot{x}_p x_p dt + \Delta a_{1p} \int_{t_1}^{t_1+\Delta t} x_p^2 dt$$

$$\int_{t_1}^{t_1+\Delta t} \sigma^* \dot{x}_p dt = \Delta a_{2p} \int_{t_1}^{t_1+\Delta t} \dot{x}_p^2 dt + \Delta a_{1p} \int_{t_1}^{t_1+\Delta t} x_p \dot{x}_p dt . \qquad (10)$$

Introduce the following notations:

$$\int_{t_1}^{t_1+\Delta t} \sigma^* x_p dt = c_1 , \qquad \int_{t_1}^{t_1+\Delta t} \sigma^* \dot{x}_p dt = c_2 ,$$

$$\int_{t_1}^{t_1+\Delta t} x_p^2 dt = l_{11} , \qquad \int_{t_1}^{t_1+\Delta t} x_p \dot{x}_p dt = l_{21} ,$$

$$\int_{t_1}^{t_1+\Delta t} \dot{x}_p x_p dt = l_{12} , \qquad \int_{t_1}^{t_1+\Delta t} \dot{x}_p^2 dt = l_{22} . \qquad (11)$$

According to notations (11), equations (10) can now be written in the form:

$$\begin{aligned} c_1 &= \Delta a_{1p} l_{11} + \Delta a_{2p} l_{12} \\ c_2 &= \Delta a_{1p} l_{21} + \Delta a_{2p} l_{22} \end{aligned} . \qquad (12)$$

From the solution of equations (12) the bias of the plant parameters Δa_{ip}, $(i = 1,2)$ can be determined. The controller can be adjusted according to the estimated parameter bias as:

$$\Delta k_i = -\Delta a_{ip} .$$

Therefore, conditions (7) are satisfied, which in turn means that the behaviour of system (5) follows the desirable trajectories of model (4), even in the presence of dramatic plant parameters changes.

For the solution of equations (12) one needs to take into account of the hypothesis of quasi-stationarity of the process, where the interval time Δt is selected such that the biases of parameters Δa_{ip} must be constant at this interval. However, the interval Δt should be sufficiently large in order to accumulate a larger quantity of variables x_p and $\dot{x}_p$ for the solution of the equations.

III. THE NUMERICAL RESULTS

The Hankel matrix Γ, constructed from the Markov parameters (obtained from the experiment, see APPENDIX), is as follows:

$$\Gamma = \begin{bmatrix} 6.5000000e\text{-}02 & 1.4550000e\text{-}01 & 1.6442500e\text{-}01 \\ 1.4550000e\text{-}01 & 1.6442500e\text{-}01 & 1.5056000e\text{-}01 \\ 1.6442500e\text{-}01 & 1.5056000e\text{-}01 & 1.2447038e\text{-}01 \end{bmatrix} . \qquad (13)$$

Applying the singular value decomposition procedure (1) on the Hankel matrix (13), it is found that

$$U = \begin{bmatrix} 5.1633320e\text{-}01 & 8.1190203e\text{-}01 & 2.7242453e\text{-}01 \\ 6.2194166e\text{-}01 & -1.3682059e\text{-}01 & -7.7101797e\text{-}01 \\ 5.8871776e\text{-}01 & -5.6753434e\text{-}01 & 5.7560070e\text{-}01 \end{bmatrix}$$

$$V = \begin{bmatrix} 5.1633320e\text{-}01 & -8.1190203e\text{-}01 & 2.7242453e\text{-}01 \\ 6.2194166e\text{-}01 & 1.3682059e\text{-}01 & -7.7101797e\text{-}01 \\ 5.8871776e\text{-}01 & 5.6753434e\text{-}01 & 5.7560070e\text{-}01 \end{bmatrix}$$

$$S = \begin{bmatrix} 4.2773559e\text{-}01 & 0.0000000e+00 & 0.0000000e+00 \\ 0.0000000e+00 & 7.4455532e\text{-}02 & 0.0000000e+00 \\ 0.0000000e+00 & 0.0000000e+00 & 6.1531296e\text{-}04 \end{bmatrix} \qquad (14)$$

Using relations (1), (2) and (14) the discrete time state space realisation of the reduced order system is obtained as follows:

$$A_d = \begin{bmatrix} 9.7950468e\text{-}01 & -3.4211654e\text{-}01 \\ 3.4211654e\text{-}01 & 3.4867831e\text{-}01 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 3.3767560e\text{-}01 \\ -2.2160613e\text{-}01 \end{bmatrix}$$

$$C_d = \begin{bmatrix} 3.3767560e\text{-}01 & 2.2160613e\text{-}01 \end{bmatrix} \qquad (15)$$

The behaviour of the full order model and the reduced order model is given in Figure 1. It can be seen in Fig. 1 and APENDIX that the Markov parameters of the reduced order model are a close approximation to the Markov parameters of the original system.

Nominal parameters of the plant in the continuous time (3) are obtained from (15) as follows:

$$\bar{a}_{1p} = -3.1184 , \quad \bar{a}_{2p} = -3.0517 ,$$
$$\bar{c}_{1p} = -0.0318 , \quad \bar{c}_{2p} = 2.9132 .$$

Parameters of model (4) are chosen as $a_{1m} = \bar{a}_{1p}$, $a_{2m} = \bar{a}_{2p}$, $c_{1m} = \bar{c}_{1p}$, $c_{2m} = \bar{c}_{2p}$.

The performance of the high dynamic precision self-tuning control system are presented in Fig. 2 - 5.

Fig. 2. shows that the bias from the nominal parameter at time $t \geq 1$ sec. is $\Delta a_{1p} = 1$, ($\Delta a_{2p} = 0$). The adaptation is switched off.

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Fig. 3. shows the bias from the nominal parameter at $t \geq 1$ sec. with adaptation being switched on ($\Delta a_{1p} = 1$, $\Delta a_{2p} = 0$). It can be seen that the output of system y_p coincides with the model reference output y_m after $t \geq 4$ sec.

Fig. 4. shows that the bias from the nominal parameter at time $t \geq 1$ sec. is $\Delta a_{2p} = 1$, ($\Delta a_{1p} = 0$). The adaptation is switched off.

Fig. 5. shows the bias from the nominal parameter at $t \geq 1$ sec. with adaptation being switched on ($\Delta a_{2p} = 1$, $\Delta a_{1p} = 0$). It can be seen that the output of system y_p coincides with the model reference output y_m after $t \geq 9$ sec.

IV. CONCLUSIONS

The high dynamic precision self-tuning control system for the solution of a fault tolerance problem of a SISO process is suggested in this paper. The method, which is based on simultaneous identification and adaptation of unknown process parameters, provides decoupling of self-tuning contours from plant dynamics. The control system compensates the rapidly changing parameter when fault occurs in a process. The mathematical model of the process is formed from Markov parameters, which are obtained from the experiment as the process impulse response. The order of the model is determined using singular value decomposition of the relevant Hankel matrix. This allows one to obtain a robust reduced order model representation if the information about the process is corrupted by noise in industrial environment.

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APPENDIX

Markov parameters obtained from the experiment:	*Markov parameters of the reduced order model*:
0.0000000e+00	0.0000000e+00
6.5000000e-02	6.4934730e-02
1.4550000e-01	1.4578163e-01
1.6442500e-01	1.6384913e-01
1.5056000e-01	1.5077128e-01
1.2447038e-01	1.2511681e-01
9.7003263e-02	9.7037520e-02
7.2809279e-02	7.1509116e-02
5.3273657e-02	5.0478548e-02
2.7143404e-02	3.4252666e-02
1.9054881e-02	2.2345734e-02
1.3274250e-02	1.3971877e-02
9.1920232e-03	8.3100499e-03
6.3351771e-03	4.6301281e-03
4.3498142e-03	2.3388797e-03
2.9776238e-03	9.8319708e-04
2.0333343e-03	2.3330942e-04
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6.4072233e-04	-3.2618265e-04

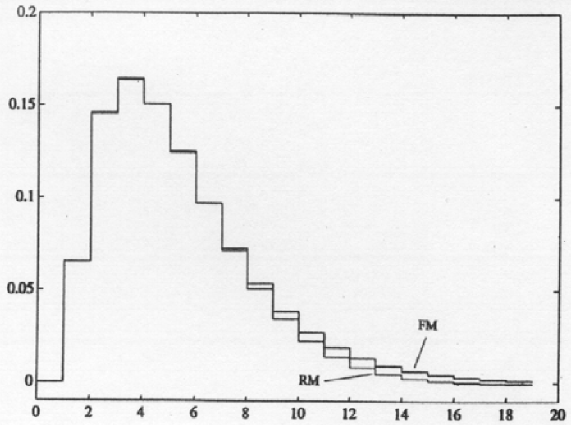

Fig.1. FM – the full order model (original system), RM – the reduced order model.

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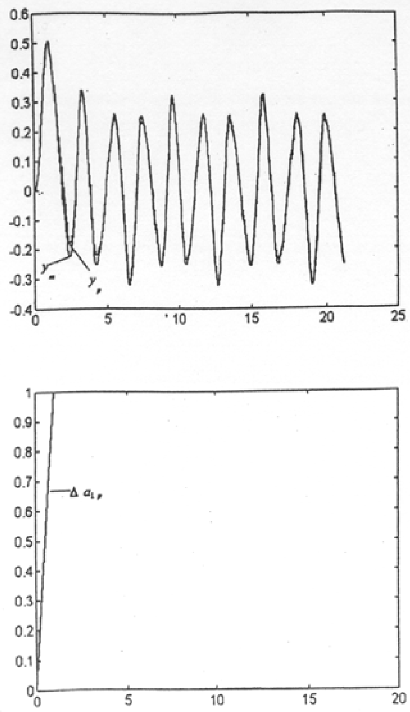

Fig. 2. Bias: $\Delta a_{1p} = 1$, $\Delta a_{2p} = 0$.
The adaptation is switched off.

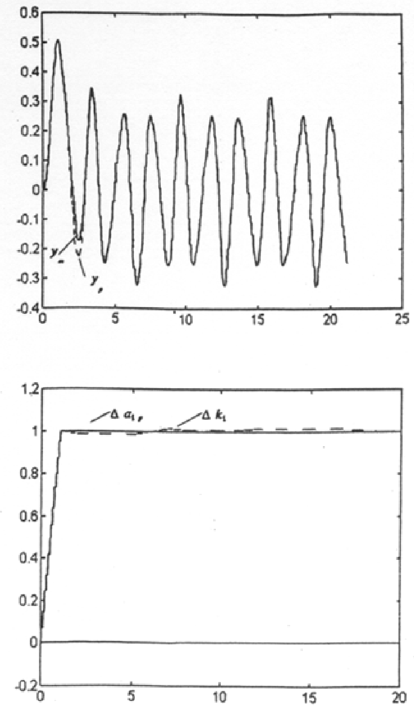

Fig. 3. Bias: $\Delta a_{1p} = 1$, $\Delta a_{2p} = 0$.
The adaptation is switched on.

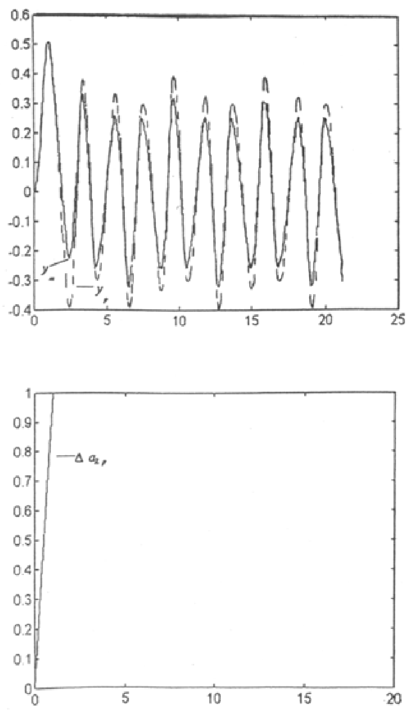

Fig. 4. Bias: $\Delta a_{1p} = 0$, $\Delta a_{2p} = 1$.
The adaptation is switched off.

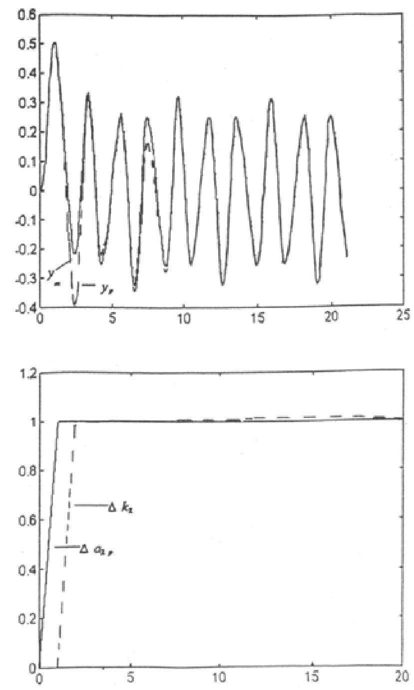

Fig. 5. Bias: $\Delta a_{1p} = 0$, $\Delta a_{2p} = 1$.
The adaptation is switched on.

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