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Mathematical Modeling of Rotary Electro Chemical Discharge Machining (RECDM)

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***Abstract*—Rotary Electrochemical Discharge Machining (RECDM) is an important manufacture technology in machining difficult-to-cut materials and to shape complicated contours and profiles with high material removal rate. This paper presents the physical and mathematical models for determination of the performance characteristics of RECDM. The results of experimental verification is also included.**

***Index Terms*—dissolution, discharges, removal rate, energy-consumption**

I. Introduction

The technological improvement of machining processes can be achieved by combining different physical/chemical actions on the materials being treated. In particular a mechanical action, which is used in conventional material removal processes, can be combined with respective interactions applied in unconventional manufacturing processes such as electrical discharge machining (EDM), electrochemical machining (ECM), and laser beam machining (LBM).

The reasons for developing a cross machining processes are to make use of the combined or mutually enhanced advantages, and to avoid or reduce some adverse effects the constituent processes produce when they are individually applied [1- 6].

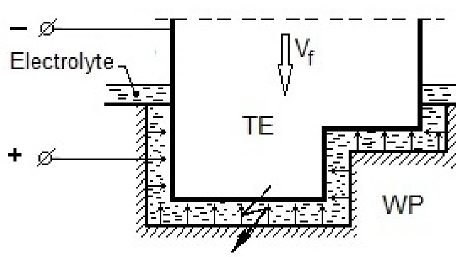

Figure 1.Scheme of ECDM/ECAM

In Figure 1 is shown schematic diagram of Electrochemical Discharge Machining (ECDM) or Electrochemical Arc machining (ECAM).

Electrochemical Discharge Machining (ECDM) using pulse voltage and Electrochemical Arc Machining (ECAM) using constant or pulse voltage are the combined methods of machining involving ECM and EDM. Electrochemical Machining (ECM) is characterized by high surface integrity, improved surface finish, high machining rate, and the absence of tool-electrode wear. But as compared with EDM, it has low accuracy of reproduction of the tool electrode shape into the workpiece. Electrical Discharge Machining (EDM) can provide a high surface finish and high machining accuracy only with a low productivity. An increase in the EDM rate result in a significantly higher roughness and deeper damaged surface layer. However, a reduction in surface roughness leads to increased of the tool electrode wear. A combination of ECM with EDM in one process ECDM/ECAM in an electrolyte solution (such as $NaNO_3$, NaCl), has shown to contain the benefits of both processes, provided that the parameters of the combined process are properly selected.

The process yields rates of material removal that can be as much as five to fifty times greater than ECM and EDM respectively [1-4]. Applications in which a combined form of both processes has been used in hole drilling, die sinking [3, 4], and cutting [6]. The ECDM/ECAM/ are especially effective when dealing with materials with tensile strength higher than 1500 N/mm^2 and heat resistance alloys. Machining capacity of the order 10^4 mm^3/min, accuracy of 0.02-0.2 mm and surface roughness of Ra = 1.25-2.5 μm are obtained.

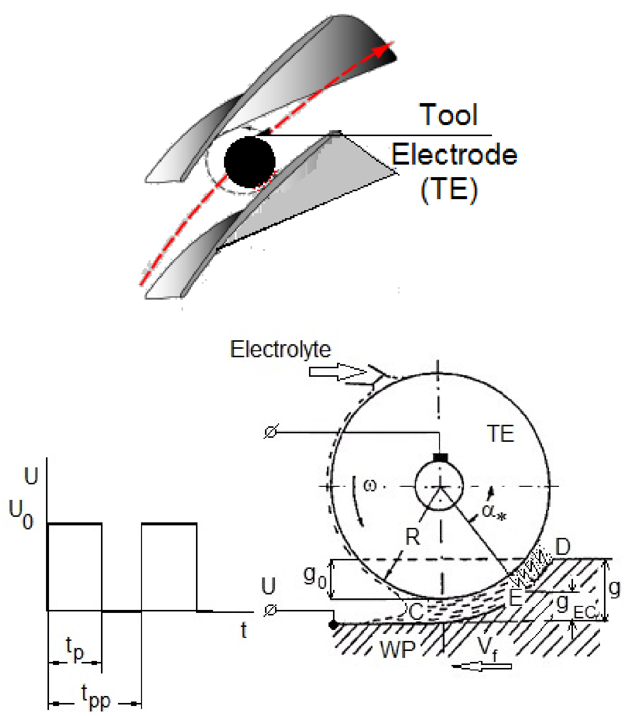

Figure 2. Schematic diagram of Rotary Electrochemical Discharge Machining

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One of the effective means of improving machining accuracy and smoothing is to impart a rotating movement to the universal or profile–shaped electrode. The kinematics of the machining is analogous to milling and grinding. The scheme of rotary ECDM (RECDM) is shown in Figure 2. The rotating tool-electrode (TE) is connected to the negative polarity, while the workpiece (WP) is connected to the positive polarity of a pulse generator or continuous power supply of direct current. There is a gap between the electrodes through which electrolyte flow. During machining, the rotating tool-electrode is set at the depth of cut g_0, while the workpiece (or tool) moves with a feed rate V_f (Figure 2).

In RECDM process, the machining region is established in two zones: EC with electrochemical dissolution and electrical discharge zone ED. Electrical discharges in ED zone can be created by electrical breakdown of the vapor-gas layer as well as by instantaneous short-circuits between the electrodes. In both cases, the deciding factor is the appearance of vapor-gas layers resulting mainly by heating of the electrolyte to the boiling point. In reality, the dimensions of both zones EC and ED fluctuate. It is so because, during the electrical discharges, the hydrodynamic conditions in these zones are changing, and also, local gaps are periodically changing with time. Due to the effect of electrical erosions on the machining surface, a large number of craters are formed in ED zone. These surface irregularities are subject of electrochemical smoothing by dissolution in the EC zone and roughness is reduced. Therefore better quality of machined surface and simultaneously larger MRR in RECDM than in ECM or EDM are obtained.

II. Mathematical modeling

The purpose of presented mathematical modeling and computer simulation is determination of characteristic of the process, namely the output data considering linear current density *I*, material removal rate *MRR*, specific energy consumption *e* as a function of chosen input data, such as setting depth of cut g_0, working voltage *U*, angular velocity ω, feed rate V_f and duty factor $\sigma = t_p/t_{pp}$ (where t_p , t_{pp} are pulse-on time and pulse period, respectively).

The unknown magnitudes of *e* and *I* depend particularly upon the dimensions of the zones EC and ED. The model is based on the assumption that the dimensions of the both zones are statistically invariable with respect time. The border between zones is conditioned by the boiling of electrolyte, because the effect of hydrogen generation on development of the vapor-gas layer is not significant in typical RECDM conditions. It is necessary to notice that due to changes in temperature across the gap size, the maximum temperature is higher than average temperature in given section [7, 8]. Owing to these reasons, electrical discharge occurs at a certain average temperature T_*, lower than the boiling point. As theoretical and experimental investigations have shown, the value of T_* changes slightly as machining parameters are altered and can be assumed as T_*=335 [7]. Assume further that the distribution of gap size and physic-chemical conditions change periodically after the transient phase of machining, i.e., we have a quasi-steady state of the RECDM.

In determining the distribution of the gap size, ***S***, for quasi-steady state, the following assumptions are made:

- Processes in both zones are independent and the occurrence of electrical discharges does not have significant effect upon the process of anodic dissolution and vice versa,
- In electrochemical zone EC, the distribution of electrical potential along the normal to the tool electrode is linear and local current density, ***i***, can be approximated as following:

$$i = \kappa \frac{U - E}{S} \qquad (1)$$

  where $\boldsymbol{\kappa}$ is electrolyte electrical conductivity, and ***E*** is total overpotential of electrodes,
- There is Couette - Poiseuille laminar flow in thin layer between curvilinear electrodes,
- Flow parameters have been determined previously through the distribution of gap size in the EC zone and rotation of the tool electrode.

In the quasi-steady state of process, the local gap is changing during pulse on-time due to anodic dissolution and the forced relative movement of electrodes with constant velocity V_f

Whereas considering machining, the distribution of gap size (*S=AC*) is conveniently described in polar coordinates as shown in Figure 3.

The velocity of gap changes $\vec{V}_S = \frac{\partial S}{\partial t} \cdot \vec{n}_C$ is the component in the direction $\vec{n}_C$ of the resultant velocity $\vec{V}_\Sigma$, which is summation of dissolution velocity $\vec{V}_n$ and relative velocity of electrode feed rate $\vec{V}_f$.

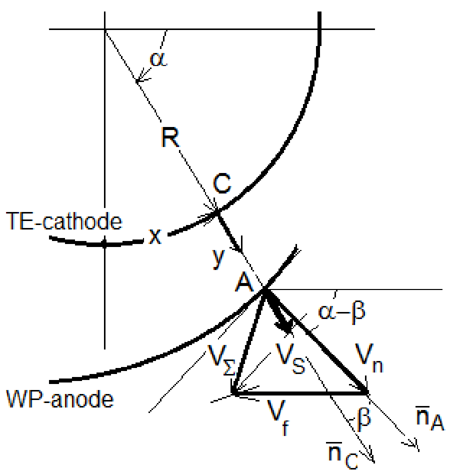

Figure 3. Scheme for modeling of electrochemical shaping

Therefore, Figure 3 shows that

$$V_S = \frac{\partial S}{\partial t} = \frac{1}{\cos\beta}\left[V_n - V_f \cdot \cos(\alpha - \beta)\right] \qquad (2)$$

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where β is the angle between normal $\vec{n}_C$ to the tool-cathode and normal $\vec{n}_A$ to workpiece-anode (Figure 2), which can be expressed by:

$$\beta = \arctan \frac{1}{R + S(\alpha, t)} \frac{\partial S}{\partial \alpha} \quad (3)$$

The dissolution velocity, V_n, on the surface of workpiece is

$$V_n = K_V i_a \quad (4)$$

where K_V is the electrochemical machinabilty coefficient, which is defined as the volume of material dissolved per unit electrical charge, and i_a is the current density on anode-workpiece surface.

At the instant of break of dissolution during pulse off-time, velocity $V_n = 0$, and at that time the gap size change can be expressed by the equation:

$$\frac{\partial S}{\partial t} = -\frac{V_f \cdot \cos(\alpha - \beta)}{\cos \beta} \quad (5)$$

Substituting equation (3) and (4) into equation (2), and regarding to equation (5), the system of equations describing the change in the gap size in any pulse period t_{pp}, takes the following forms (6):

$$\frac{\partial S}{\partial t} = \frac{1}{\cos \beta} \left[\kappa \cdot K_V \frac{U - E}{S} - V_f \cdot \cos(\alpha - \beta) \right]$$

at $0 \le t \le t_p$

$$\frac{\partial S}{\partial t} = -\frac{V_f \cdot \cos(\alpha - \beta)}{\cos \beta} \text{ at } t_p \le t \le t_{pp}$$

$$\beta = \arctan \frac{1}{R + S(\alpha, t)} \frac{\partial S}{\partial \alpha}$$

For the quasi-steady state, the distribution of gap size at the end of pulse period $S(\alpha, t_{pp})$ is equal to the distribution of gap size at the beginning of pulse denoted as $S(\alpha, 0) = S_0(\alpha)$.

For practical conditions of RECDM, we can conclude that the range of gap changes and range of angle β changing during pulse on-time and off-time are considerably small and not significant. After integrating the system of equations and transformations of equation (6), this fact allows us to obtain:

$$\frac{t_p}{t_{pp}} \frac{\kappa \cdot K_V}{S_0(\alpha)} \frac{1}{t_p} \int_0^{t_p} (U - E) dt = V_f \cos(\alpha - \beta) \quad (9)$$

Introducing the average value of voltage pulse

$$\overline{U} = \frac{1}{t_p} \int_0^{t_p} (U - E) dt,$$

the pulse duty cycle $\sigma = t_p / t_{pp}$, and the equilibrium gap $S_f = \kappa \cdot K_V \frac{\overline{U}}{V_f}$, we obtain,

$$\cos(\alpha - \beta) = \sigma \frac{S_f}{S_0(\alpha)}$$

After considering equation (3) and omitting the subscript, 0, the distribution of gap size in quasi-steady state is described by the following equation:

$$\frac{\partial S}{\partial \alpha} = (R + S) \cdot \tan(\alpha - \arccos \frac{\sigma \cdot S_f}{S}) \quad (10)$$

Equation (10) is similar to the equation for distribution of the gap size in The ECM-RE process [8]. To solve the equation, it is necessary to known the initial boundary condition. In the first approximation and on the basis of the model of ideal ECM process (κ = const, K_V = const and E = const), the condition takes the form: $S(\alpha = 0) = \sigma S_f$. For solution with higher accuracy, it is necessary to use the method of successive approximation described in ref [7, 8].

The position of border between both zones EC and ED, is determined by temperature field in the electrolyte due to Joule heating, which internal sources intensity is equal:

$$q_J = \frac{i^2}{\kappa} \quad (11)$$

To simplify the calculations let us introduce a curvilinear coordinate system (x, y), connected with the tool-electrode in which a coordinate $\boldsymbol{x}$ lies on the given electrode and is measured from the inlet of the electrolyte and let axis $\boldsymbol{y}$ overlap its normal $\vec{n}_C$ (Figure 3). The heat transfer in the gap with respect to Joule's heat is described by:

$$\frac{\partial T}{\partial t} + w(x, y) \cdot \frac{\partial T}{\partial x} = a \frac{\partial^2 T}{\partial y^2} + \frac{i^2}{\rho_e \cdot C_p \cdot \kappa} \quad (13)$$

where: a - thermal diffusivity, ρ_e - density of electrolyte and C_p - specific heat of electrolyte.

The initial condition is $T(x, y, 0) = T_0$, and the boundary conditions are as follows: $T(x=0, t) = To$, $T(x, 0, t) = T_C$, $T(x, S, t) = T_A$, where: T_A and T_C are temperatures of the cathode and anode, respectively.

To complete the systems of Equations (11, 12 and 13)), the formulation the flow velocity must be included. On the basis of lubrication theory for the investigated hydrodynamic case, the distribution of flow velocity is expressed as follows [9]:

$$w = \omega \cdot R \left[1 - \frac{y}{S(x)} \right] \cdot \left[1 + 3 \left(\frac{x + L_0}{L} - 1 \right) \frac{y}{S(x)} \right] \quad (14)$$

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where ω is angular velocity of the tool electrode, L_0 is the length with reverse velocity in the Couette – Poiseuille flow, and $L = R \cdot \arccos \frac{g_0}{R}$.

The numerical solution has been based on the finite difference method.

On the basis of already calculated temperature distribution and condition $T(\alpha_* = x_* / R) = T_*$, it is possible determine the position of the border between zones EC and ED and their dimensions g_{EC} and g_{ED} (15):

$$g_{EC} = R \cdot (1 - \sin \alpha_*) + S(\pi / 2)$$
$$g_{ED} = g_0 - R \cdot (1 - \sin \alpha_*)$$

If at outlet of the gap $T_{out} < T_*$, only the electrochemical process occurs, i.e., RECDM changes to ECM using rotary electrodes. The parameters (for example, feed rate V_f^*), when RECDM is beginning can be determined from condition $T_{out} = T_*$.

We assumed a unit width of tool electrode, in this case total current is equal to linear current density, which is denoted by *I*. Taking into account the presence of electrochemical zones EC and electrical discharge zone ED for calculating *I*, it is necessary to determine the electrochemical component I_C and electrical discharge component I_D. For the quasi-steady state of pulse machining of curvilinear surface, we have relationship between machining feed rate V_f and average material removal rate during pulse on-time V_n as following:

$$V_n = \frac{1}{\sigma} V_f \cdot \cos \alpha \quad (16)$$

To determine the electrochemical component of linear density of current, we have

$$I_C = \int_{EC} i_{EC} R d\alpha$$

After considering equations (4) and (16) we obtain

$$I_C = \frac{V_f}{\sigma \cdot K_V} \int_{EC} R \cos \alpha \cdot d\alpha = \frac{V_f}{\sigma \cdot K_V} g_{EC}$$

Substituting g_{EC} from equation (15) we get:

$$I_C = \frac{V_f}{\sigma \cdot K_V} \left[R \cdot (1 - \sin \alpha_*) + S(\pi / 2) \right] \quad (17)$$

In electrical discharges ED-zone, the material removal rate under pulse on-time can be expressed as follows:

$$V_{ED} = K_D \cdot i \quad (18)$$

where *i* is average current density of a set of discharges. The value of the coefficient of electroerosive machinability K_D is known from experimental investigations.

Under the same considerations as those in the case of electrochemical current, we can derive the relationship for I_{ED}:

$$I_{ED} = \frac{V_f}{\sigma \cdot K_D} \left[g_0 - R \cdot (1 - \sin \alpha_*) \right] \quad (19)$$

Total linear current density is equal

$$I = \frac{V_f}{\sigma} \left(\frac{g_{EC}}{K_V} + \frac{g_{ED}}{K_D} \right)$$

or

$$I = \frac{V_f}{\sigma} \left(\frac{g}{K_D} + g_{EC} \left(\frac{1}{K_V} - \frac{1}{K_D} \right) \right) \quad (20)$$

The specific energy consumption, which is defined as energy required to removing the unit volume of machining material can be determined with consideration of time period t_{pp} and for unit width of tool electrode. In this condition of ECDM-RE, the volume of material removed is $v = g \cdot V_f \cdot t_{pp}$, and the energy consumed is equal

$$E_m = \int_0^{t_{pp}} U(t) \cdot I(t) \cdot dt$$

Therefore, the specific energy consumption, *e*, is described by expression as following

$$e = \frac{1}{g \cdot V_f \cdot t_{pp}} \int_0^{t_{pp}} U(t) \cdot I(t) \cdot dt \quad (21)$$

For rectangular pulse

$$e = \sigma \frac{U \cdot I}{g \cdot V_f} \quad (22)$$

After substitution of equation (21), we have the following equation

$$e = U \left[\frac{1}{K_D} + \frac{g_{EC}}{g} \left(\frac{1}{K_V} - \frac{1}{K_D} \right) \right] \quad (23)$$

When $g_{EC} = g$, the process is purely electrochemical and $e(ECM) = U / K_V$, for large values of feed rate $g_{EC} / g << 1$, a contribution electrochemical dissolution in total energy consumption is negligible and $e \cong e(ED) = U / K_D$.

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For example, in typical RECDM parameters and for steel material: $K_V \cong 1.8$ mm^3/Amin, $K_D \cong 10$ mm^3/Amin, and U=22 V, we have $e\ (ECM)$=733 J/mm^3, and $e(ED)$=132 J/mm^3. On the basis of equation (23), we can conclude that energy consumption depends on the process parameters, which influence the value of ratio g_{EC} / g .

III. Computer simulation, results and analysis

The program simulation has been based on numerical solution of the system of equation (6), (12-14), (17, 19) and (20) with using of the finite differences method.The results will be discussed below for the cases in which the following input data are constant: R = 100 mm, κ = 0.01 A/Vmm, ρ_e = 1000 kg/m^3, C_p = 3700 J/kg K, K_V = 2 mm^3/Amin, K_D = 10 mm^3/Amin, T_0 = 293 K, T_* = 335 K. Machining parameters for the basic option of the simulation are: g_0 = 10 mm, V_f = 10 mm/min, U = 22 V, ω = 314 1/s and σ = 0.5. The range of variations of parameters are as follows: g_0 = 0.1 – 12.5 mm, V_f = 1-20 mm/min, U = 15 – 30 V, ω = 100 – 500 1/s and σ = 0.2 – 0.5. Effects of setting depth of machining g_0 are shown in Figures 4. For small values g_0, namely for g_0 < 1.8 mm, material removal takes place through anodic dissolution. *So,* $I = I_C$ is directly proportional to the real depth of $g = g_0 + S(\pi/2)$. The electrical discharge component of linear current density, I_d, increases linearly, whereas the electrochemical component I_C decreases. The increase in the depth of machining enlarges the length of narrowing channel, which cause a decrease of electrolyte flow rate through the gap and an increase in the temperature gradient. As a result, the electrical discharges take place closer to the inlet. The EC zone is reduced and is related to its depth g_{EC}. That is why I_C decreases and electrical discharge component I_D increases.

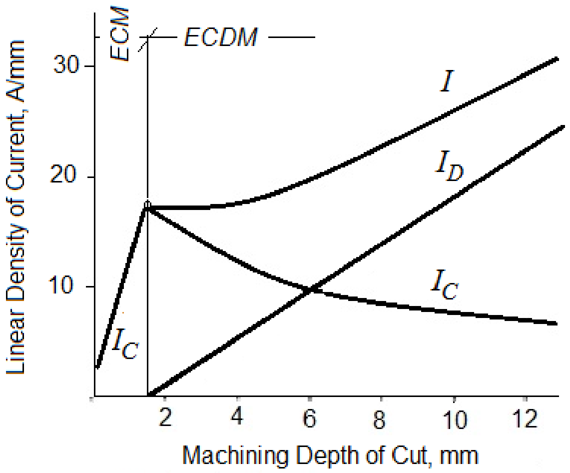

Figure 4. Effects of machining depth of cut and feed rate on linear current density

Figure 5 illustrate the effects of machining feed rate V_f. The qualitative characteristics for V_f are similar to those of g_0. The increase of V_f over some a critical value V_f^* ($V_f^* \cong 2.7$ mm/min, Figure 5) lead to increasing electrical discharge ED-zone as result of reducing gap size and flow rate and increasing temperature gradient along flow path. Therefore, results are similar as described effects of depth of machining.

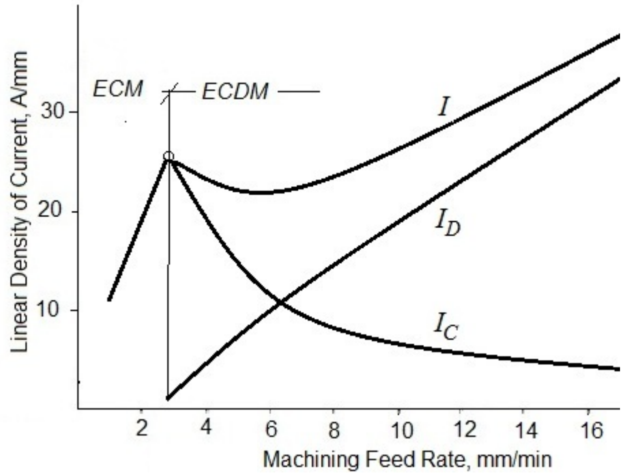

Figure 5 Effects of machining feed rate on linear current density

We have to take into account the fact that the application of pulse generator fundamentally changes the character of electrical discharges. For the continuous DC rectifier, the arc discharge lasts relatively longer, and diminishes due to the melting of the material and to the reaction of the hydrodynamic flow. Relatively deep craters are left on the machining surface. The application of pulse generators and shorter of pulse on-time, lead to reducing of crater dimensions and finally to improvement in the surface roughness. The decrement of machining maximum gap size, as a result of decrease in σ is also beneficial for smoothing process and accuracy of machining.

Selected results of experimental verifications are presented in Figures 6 and 7.

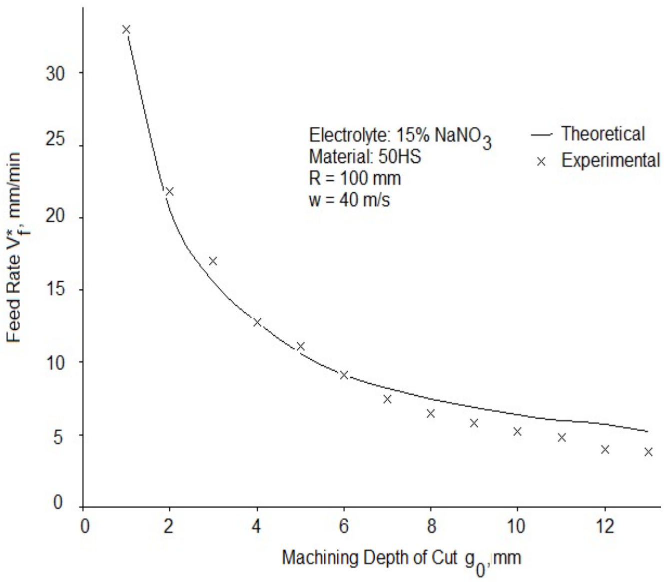

Figure 6. Critical feed rate vs. depth of machining

Figure 6 shows a good correlation between theoretical and experimental values of critical feed rate V_f^*, when RECDM is beginning.

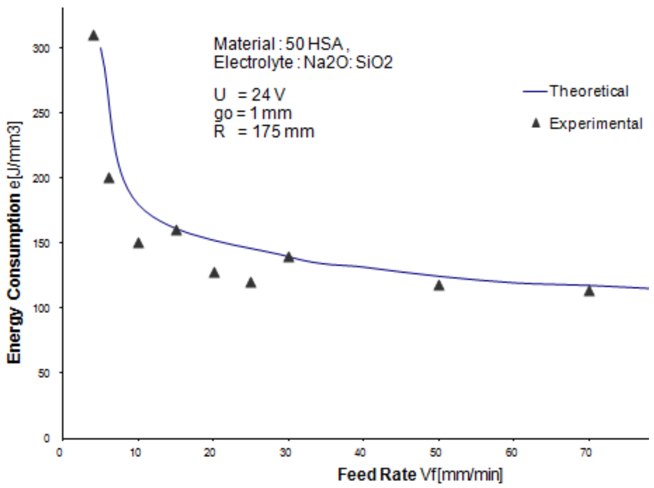

Figure 7. Specific energy consumption vs. feed rate

The effect of feed rate on energy consumption is shown in Figure 7.

Experimental validation of the presented mathematical model of RECDM process recommends the developed computer simulation program for practical applications.

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