Linear Quadratic Regulation of a Rotating Shaft being Subject to Gyroscopic Effect

Rudolf Sebastian Schittenhelm, Matthias Borsdorf, Bernd Riemann, Stephan Rinderknecht

Abstract— **A linear quadratic regulator is designed for a rotor system on the basis of a finite element model. The rotor is subject to gyroscopic effect and is actively supported by means of piezoelectric actuators installed at one of its two bearings. As a result of the first aspect, its dynamic behavior varies with rotational frequency of the rotor. This aspect is challenging for linear time invariant control techniques since it results in a demand for high robustness. Furthermore, if controller and observer are calculated using a model at a specific design frequency, the separation principle does not hold. In this article a proposal for combined Linear Quadratic Regulator and Kalman Filter design on the basis of physical considerations is given. The space of design parameters is reduced to a manageable number of 3. It is shown that it is possible to design a single controller which guarantees stability and good performance in the whole operating range, which includes two resonance frequencies. The controller is implemented at the rig and experimental results are presented.**

Index Terms— **active vibration control, linear quadratic regulator, piezoelectric actuator, rotor**

I. Introduction

ROTOR vibrations are a limiting factor for high speed applications such as aircraft engines or high speed cutting machines. This aspect gains in importance since the demand for lightweight construction leads to lighter machines being prone to excitation by dynamic forces. In many cases, attenuation of vibration amplitudes can be achieved by passive means, i.e. balancing, damping elements like squeeze film dampers (SFD), or targeted manipulation of eigenfrequencies and eigenvectors. If those methods reach their limits, active means are an attractive alternative.

Possible actuators and semi-active components for use in vibration control of rotating machinery are active SFD [1], electrorheological dampers [2], active magnetic bearings and electromagnetic actuators [3], [4], [5], [6], [7], [8] as well as piezo-actuated bearings [9], [10], [11]. Advantages of piezoelectric stack actuators are low weight accompanied by high forces in a broad frequency range. Furthermore, even though there is hysteresis present in piezoelectric material, nonlinearity is small in comparison to other actuator types and a linear approximation of the piezoelectric effect is sufficient for controller design.

The most important component of any active system is the controller. For vibration control, feedforward as well as feedback methods are suitable.

Feedforward approaches are typically adaptive because of the inevitable model mismatch. The algorithm most frequently used for adaption in the context of noise and vibration control is the Filtered x Least Mean Squares (FxLMS) algorithm. In [4], [10] the viability of the FxLMS algorithm is investigated in the context of rotating machinery. The extent of vibration attenuation seems promising. Also, no model of the frequency response functions (FRFs) from the disturbances to the outputs is required for implementation. However, the convergence behavior is difficult to predict under fluctuating operating conditions. This is especially true for systems having time dependent dynamics, such as rotor systems being subject to gyroscopic moments during a run up, since the algorithm requires a model of the so-called secondary path from the actuators to the sensors. If there is a phase mismatch in the secondary path greater than 90°, the algorithm will diverge [12]. This is critical for weakly damped mechanical systems, since there are very steep phase jumps of 180° at resonance as well as anti-resonance frequencies. Thus, just small mismatch of resonance or anti-resonance frequencies in the model leads to large mismatch in secondary path phase in a region around these frequencies.

Feedback methods for vibration control, from the authors' point of view, can be divided into three categories:

The first one is built up by classical controllers such as PID-control, Integral Force Feedback, Positive Position Feedback and similar techniques, see [13]. They are used as a reference in many articles and can be implemented without a model of the plant. However, their performance is often far below optimal values due to a lack of closed solutions for calculation of optimal controller gains of multiple input multiple output systems.

The basic idea of frequency domain approaches, e.g. controllers designed via μ-Synthesis or H∞-optimal controllers, is a more intuitive performance specification by means of frequency dependent weighting functions for the FRFs of the system. This kind of problem description in the frequency domain is convenient for mechanical engineers involved in structural dynamics, since they often think in FRFs. By inclusion of model inaccuracy in the control design process, it is possible to achieve robust stability and even robust performance in the case of μ-Synthesis. As a drawback, the weighting functions offer infinite degrees of freedom as they can be shaped arbitrarily and as a result usually have to be defined in an iterative manner. Furthermore, in contrast to state space controllers, the order

Manuscript received June 23, 2012; revised August 09, 2012. This work was supported by the Deutsche Forschungsgemeinschaft.

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ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

of frequency domain controllers usually exceeds the model order. The order of H∞-optimal controllers is given by the model order plus the order of the weighting functions [14] while the order of µ-controllers is usually even higher. This results in the need for a subsequent controller reduction if there are limits regarding computational effort of the controller. Controller reduction can reduce the closed loop performance significantly and sometimes even leads to an unstable closed loop system. In rotordynamics, the frequency domain controllers are often used in the context of active magnetic bearings, see [5], [6] for example.

The third category contains the state space controllers. Besides the classic examples, the Linear Quadratic Regulator (LQR) and pole placement controllers, which mainly aim at the alteration of system dynamics, there have emerged other techniques to design state feedback controllers which e.g. consider nonlinearities in the system [15] or affect disturbance rejection behavior by using Linear Matrix Inequalities (LMI) for the definition of requirements, see [16] for an example in the field of rotordynamics. However, implementation of these techniques often fails because of mathematical obstacles in e.g. the solving process of LMIs. Furthermore, the problem description is far more complex as in the case of the classic state space controllers, which offer the user relatively simple implementation and a good understanding of the alteration of the dynamic characteristics of the system by the controller. They are thus frequently applied to control problems throughout literature. For instance, the LQR has been successfully applied to rotor systems excited by unbalance, see [7], [8], and [11]. Results show that the controller is suitable for the problem. However, even though dependence of the system dynamics on the rotational frequency due to gyroscopic effects is mentioned in some of these articles, [7] and [11], the problem of model mismatch is not addressed in detail in the context of controller design.

In this article, a LQR and a Kalman Filter, resulting in a Linear Quadratic Gaussian (LQG) controller [17], are applied to a rotor system. The choice of these types of controller and observer is due to their ease of implementation and the advantage of the design parameters being physically interpretable. Furthermore, the LQR is robust to model inaccuracies and Kalman Filters can be designed to be insensitive to measurement or actuator noise [17].

II. System Description and Modeling

A. Test Rig

In this paper, a controller is applied to the rotor system shown in Fig. 1. The flexible rotor consists of a shaft of 9 mm diameter and 320 mm length which has two discs mounted on it. The active bearing, consisting of a bearing cup, two piezoelectric actuators with a maximum stroke of 60 µm and two springs to apply pre-stress to the actuators is shown in detail in Fig. 2. Besides the active bearing, which is located between the discs, there is a passive bearing at the other end of the shaft. The displacements of the discs into the radial direction are measured by four eddy current sensors. The resulting sensor signals are low pass filtered by means of a first order analogue filter in order to avoid aliasing effects during in digitalization process of the measurement data. The rotor is accelerated by means of a 250 Watt DC motor. The maximum rotational speed of the rotor is 160 Hz. It has two unbalance induced resonance frequencies in its operating range at approximately 47 and 108 Hz. A dSpace real time system (DS1104) is used for data acquisition and controller implementation.

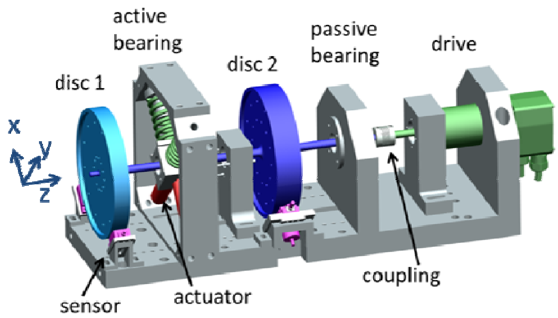

Fig. 1: Test rig

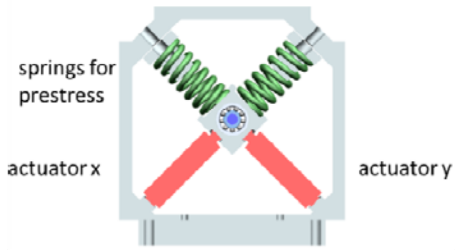

Fig. 2: Active bearing

B. Modeling

A model of the rig is derived by means of finite element (FE) method on the basis of Timoshenko beam theory. A FE model is used rather than an identified one, in order to achieve a model in which the states are physically interpretable, i.e. assignability to vibration modes in this case. The bearings are modeled by discrete springs and piezoelectric elements. The piezoelectric effect is approximated by linear equations, even though there exists some hysteresis in reality [18]. The results in this article show, that this simplification is permissible. Damping is introduced by means of viscous damping and a uniform damping ratio of 0.9% for all modes. The model is reduced by means of modal reduction to an order of 16 in state space. The governing equations read:

$$\begin{aligned} \dot{x} &= A_\Omega x + B_\Omega u + E_\Omega d \\ y &= C_\Omega x + D_\Omega u \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ are the system states, $u \in \mathbb{R}^{n_u}$ the control inputs, i.e. the voltages applied to the actuators, $d \in \mathbb{R}^{n_d}$ the disturbances and $y \in \mathbb{R}^{n_y}$ the sensor signals. $A_\Omega, B_\Omega, C_\Omega, D_\Omega, E_\Omega$ are system matrices with appropriate dimensions. The states of (1) are arranged with ascending eigenfrequencies. Due to gyroscopic effects, the system

ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

dynamics are dependent on the rotational frequency Ω, which is indicated by the respective subscript in (1). After reduction, the model is augmented in order to capture the effects of filters, amplifiers and sampling. The low pass filters and the amplifiers are described by first and second order models respectively and sampling is introduced to the model by a second order padé approximation. For controller design purposes in this article, it is equivalent to add the filters to the model at the inputs rather than on the physically correct outputs. Thus, effects of filters, amplifiers and sampling are described by a single 2 x 2 model of order 10,

$$\begin{aligned} \dot{x}_P &= A_\mathrm{P} x_P + B_\mathrm{P} u_{in} \\ u &= C_\mathrm{P} x_P \end{aligned} \tag{2}$$

and included into the overall model as shown below:

$$\begin{aligned} \dot{x}_S &:= \begin{bmatrix} \dot{x} \\ \dot{x}_P \end{bmatrix} = \begin{bmatrix} A_\Omega & B_\Omega C_\mathrm{P} \\ 0 & A_\mathrm{P} \end{bmatrix} \begin{bmatrix} x \\ x_P \end{bmatrix} + \\ &+ \begin{bmatrix} E_\Omega & 0 \\ 0 & B_\mathrm{P} \end{bmatrix} \begin{bmatrix} d \\ u_{in} \end{bmatrix} = \\ &=: A_{S,\Omega} x_S + E_{S,\Omega} d + B_S u_{in} \\ y &= [C_\Omega \quad D_\Omega C_P] x_S =: C_S x_S \end{aligned} \tag{3}$$

The model matches reality with quite high accuracy, see Fig. 3, where the simulated and the identified FRFs from the actuator pointing into the x direction to the sensor pointing into the x direction at disc 1 (referred to as sensor 1x in the following) are shown for the case of nonrotating rotor ($\Omega = 0$).

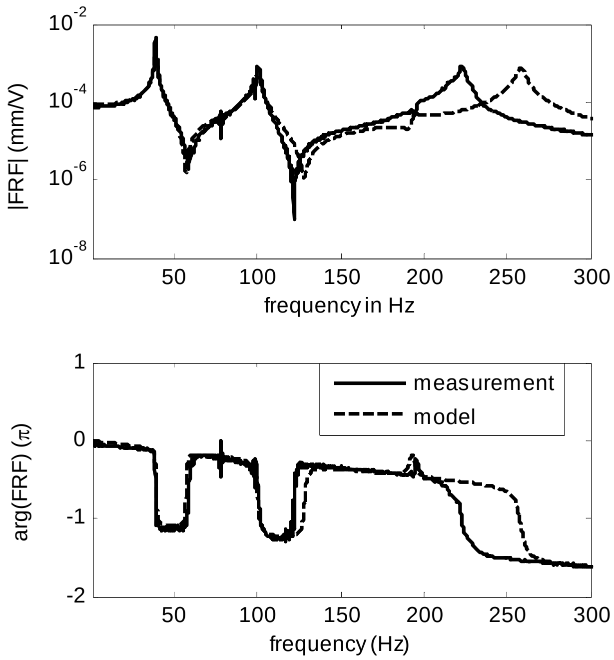

Fig. 3: FRF of plant and model from actuator x to sensor 1x

The first 2 modes are replicated with high accuracy, in the frequency region of higher order modes there is some deviation of the simulated from the measured data. Also, effects of the low pass filters, amplifiers and sampling are visible in the model as well as the measurement when inspecting the phase of the transfer function. The model could be further improved in the upper frequency range, but experiments show that accuracy is sufficient for controller implementation.

The excitation for the simulations in this article is derived by means of analysis of measurement data at the critical speeds by a procedure similar to influence coefficient balancing method [19]. Assuming that there are unbalances present at the discs only, the response at sensor 1x, represented by its complex amplitudes $\mathrm{y}_{1\mathrm{x}}(\Omega)$, to the unbalance excitation is given by:

$$\mathrm{y}_{1\mathrm{x}}(\Omega) = H(\Omega)\Omega^2 U \tag{4}$$

where $U = [U_1, U_2]$ represents the unbalances at the discs. For identification of these unbalances, complex amplitudes $\mathrm{y}_{1\mathrm{x}}(\Omega_{\mathrm{R1}})$ and $\mathrm{y}_{1\mathrm{x}}(\Omega_{\mathrm{R2}})$ at the unbalance induced resonances Ω_{R1} and Ω_{R2} are obtained by evaluation of the measurement signals at these frequencies by means of a digital implementation of the wattmeter measuring principle [20]. The desired approximation of the equivalent unbalances at the discs is given by

$$U = \begin{bmatrix} H(\Omega_{\mathrm{R1}})\Omega_{R1}^2 \\ H(\Omega_{\mathrm{R2}})\Omega_{R2}^2 \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{y}_{1\mathrm{x}}(\Omega_{\mathrm{R1}}) \\ \mathrm{y}_{1\mathrm{x}}(\Omega_{\mathrm{R2}}) \end{bmatrix} \tag{5}$$

Because of residual mismatch between model and plant regarding resonance frequencies, see Fig. 4, the resonance frequencies of the model and not the rotational frequencies of the measurements are substituted into (5).

C. Model mismatch

The model as discussed so far just incorporates basic rotordynamics, the gyroscopic effect and unbalance excitation. However, in real rotor systems there are additional effects to be considered. Fig. 4 shows the spectrogram of sensor 1x for a run up to a rotational frequency of 115 Hz.

Besides the power spectral density of sensor 1x, there are straight lines in the figure corresponding to vibration at $\frac{1}{3}\Omega$, $\frac{2}{3}\Omega$, Ω and 2Ω, often referred to as engine orders. Also the first four eigenfrequencies of the model are included in the figure, which are altered significantly by the presence of gyroscopic moments and are thus a function of rotational frequency. Excitation with rotational frequency Ω corresponds to unbalance excitation and is most relevant for rotor systems. It is observed, that only modes of even order, referred to as forward whirl modes, are excited by unbalance in contrast the backward whirl modes of uneven order. Excitation at 2Ω results from the rotor not being fully isotropic in the plane perpendicular to the rotor axis, leading to excitation of the rotor by gravitational forces [19]. Increased vibration amplitudes at $\frac{1}{3}\Omega$ and $\frac{2}{3}\Omega$ for $\Omega/2\pi \approx$

ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

80 Hz result from nonlinearities in the system, presumably clearances and nonlinear characteristics of ball bearings [21]. For controller synthesis, just unbalance excitation and a linear model are considered, since it is assumed that other effects are of minor interest for rotor behavior as well as controller design.

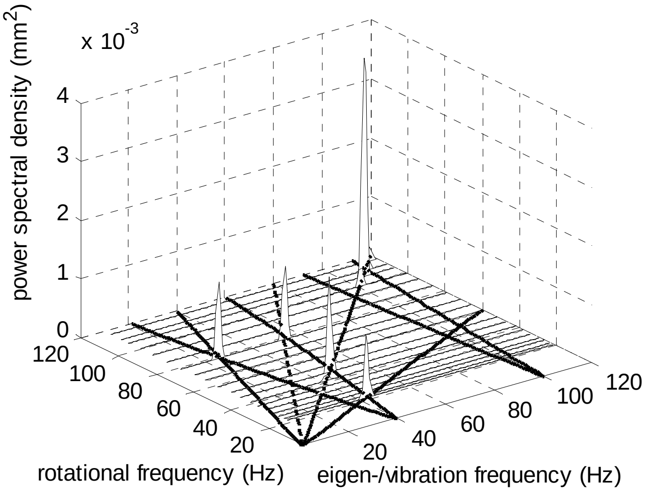

Fig. 4: Spectrogram of a run up, power spectral density normalized to measurement duration and sampling frequency, sensor 1x

III. Controller design

The main hindrance for controller implementation is the dependence of the rotor matrices in (3) on the rotational frequency. In this section it will be shown that it is possible to design a single controller which leads to stable behavior and good performance in the whole rotational frequency range considered. A consequence of the fluctuation of the system characteristics, despite the demand for high robustness, is that the separation principle does not hold if a simple constant observer is applied. There are possibilities to design observers for systems with additive uncertainties in the system matrices [15], however, the conditions for existence are very restrictive. In this article, the coupled problem of controller and observer design is treated, and a proposal to reduce the problem to 3 design parameters is given.

A linear quadratic regulator (LQR) minimizing the cost functional

$$J_1 = \int_0^\infty (x_s^T Q x_s + u_{in}^T R u_{in})\, dt \qquad (6)$$

for $x_s(0) \neq 0$ and $d \equiv 0$, is applied to the system. One may raise the question how the LQR fits to the problem of rotating shafts excited by unbalance. By proper weighting of the modes of the system it is achieved, that the damping ratios ζ_i of modes of interest are increased, leading to reduced vibration amplitudes at resonance.

The weighting matrices for the LQR are chosen to be of diagonal shape. Due to symmetry of the rotor and the exclusive consideration of unbalance excitation, one can choose $R = I_2$ without losing relevant controllers, where $I_k \in \mathbb{R}^{kxk}$ denotes the identity matrix. Q is chosen to be of the shape

$$Q = diag(q_3 I_2, q_1 I_2, q_3 I_2, q_2 I_2, q_3 I_8, 0_{10}) \qquad (7)$$

such that the modes which are excited by unbalance are assigned the weights q_1 (for the first forward whirl mode) and q_2 (for the second forward whirl mode) respectively. Other rotor modes are weighted by q_3 and the modes corresponding to the model of filters, amplifiers and sampling are not weighted. In order to further reduce the number of parameters, and to achieve good vibration attenuation in both of the unbalance induced resonances, the respective weighting factors are set to $q_1 = q_2 = 10^{q_C}$. q_C is one of the 3 design parameters to be determined. Simulations indicate that the resulting controllers possess poor robustness if the factor q_3 is too small compared to q_1 and q_2, such that it is set to $q_3 = 10^{q_C - 1}$. In this way the degree of stability [13] is increased for these modes.

Since the system states are not measureable in the case of the rig, an observer has to be implemented for the system (3). The observer feedback matrix, denoted by L is also computed by the LQR design method, leading to a so-called Kalman filter being insensitive to measurement and actuator noise if designed properly. If one assumes noise of equal level at all sensors and also at all actuators respectively, it is sufficient to choose the weights for the observer as shown below:

$$R_O = r_B I_4, Q_O = B_\Omega {B_\Omega}^T \qquad (8)$$

where r_B describes the ratio of measurement and actuator noise levels assumed. This is true because just the ratio of assumed actuator and sensor noise levels affects the observer matrix L but not the absolute values. Even though the value r_B is not assigned on the basis of noise levels at the rig in this article, this physical consideration helps to reduce the number of free parameters to be determined in observer design significantly.

As stated above, the differential equations describing the system are dependent on rotational frequency Ω of the rotor. As a result, if a linear time invariant controller is to be applied to the system, it has to be robust against changes in rotational frequency. Furthermore, separation principle does not hold and thus controller and observer design is a coupled problem. The approach in this article is to design a single pair of controller and observer at some specific design frequency Ω_D, leading to a robustly stable system for $\Omega \in [0; \Omega_{max}]$ and good attenuation of unbalance induced vibration. A remarkable aspect of the LQR, designed at Ω_D, in combination with the rotor system under consideration is that despite the fluctuation of eigenvalues as well as eigenvectors of the system, the weights q_1 and q_2 still target at the forward whirl modes for the whole frequency range considered.

By above considerations, the space of design parameters is reduced to 3, i.e. q_C, r_B, and Ω_D have to be found. In order

ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

to do so, simple brute force optimization is found to lead to a controller showing good performance, not exceeding the maximum permissible control input and being robustly stable against variations in $\Omega \in [0; \Omega_{max}]$.

The cost functional used for optimization is

$$J_2 = \frac{1}{\Omega_{max}} \int_0^{\Omega_{max}} \sum_{i=3,4,7,8} |x_i|^2 \, d\Omega \qquad (9)$$

in order to focus on the modes excited by unbalance within the operating range, i.e. the first two forward whirl modes. J_2 is minimized over a subset of the predefined set of design parameters in which robust stability for $\Omega \in [0; \Omega_{max}]$ is guaranteed and the maximum control effort is not exceeded. The resulting optimal design parameters are used as a starting point for manual fine tuning as explained below.

The controller resulting from the optimization does not account for measurement noise. In order to avoid high noise amplification, q_C is reduced, starting from the optimal value to q_C^{noise}, just before the point where control performance is diminished significantly. The obvious approach to reduce noise amplification, i.e. increase of r_B, leads to significant loss in performance, cannot compensate for high controller gains entirely and is thus infeasible without reduction of controller gains.

Furthermore it is observed in simulations, that vibration attenuation at the first unbalance induced resonance is much lower than at the second one. To overcome this problem, the respective weights are set to $q_1 = 10^{q_C^{noise}+1}$ and $q_2 = 10^{q_C^{noise}-1}$. If q_1 and q_2 are futher altered from the optimal value, q_C^{noise}, the system is no longer robustly stable for $\Omega \in [0; \Omega_{max}]$.

The resulting controller is checked for stability in the presence of spillover effects caused by the not modeled rotor modes. From a mathematical point of view, stability of the system is not proven by inspection of the system for all $\Omega \in [0; \Omega_{max}]$ because Ω is time dependent during a run up. Nonetheless it is clear from experience, that the system will be stable if it is stable for $\Omega \in [0; \Omega_{max}], \Omega = const.$ and the system dynamics change slowly. This statement is given on a theoretical basis in [22]. Because of above considerations, the proof of stability for the linear time invariant case is skipped.

IV. Simulation

Simulation results for displacement amplitudes of sensor 1x with and without control are shown in Fig. 5 for steady state operation.

It is observed, that good vibration attenuation is achieved at both resonance frequencies within the operating range. The system is stable for all rotational frequencies under consideration. The required control inputs, i.e. the voltages applied to the actuators, are well below the maximum load of 500 V. It may be assumed, that voltages will also remain below design limit in the experiment despite modeling errors and simplifications made. It is also observed, that the control inputs are equal due to symmetry of the system, which has already been taken advantage of in the choice of R in section III.

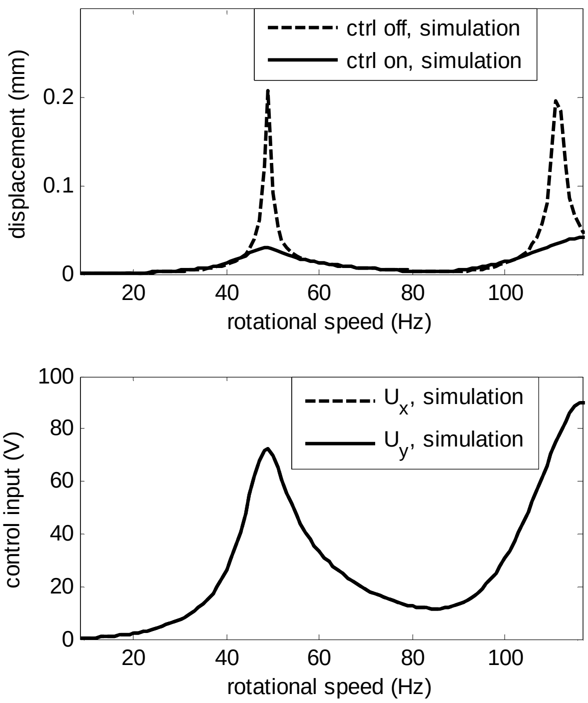

Fig. 5: Simulation results, amplitudes at sensor 1x and control effort

V. Experiment

The controller is validated in an experimental setup. In order to do so, the rotor is accelerated to a maximum rotational speed of 115 Hz within about 5 minutes, i.e. virtually at steady state operation, with and without control. The results are shown in Fig. 6, where the envelopes of the displacement signals of sensor 1x with and without control as well as the control inputs are shown.

It is observed, that the model matches reality good in terms of eigenfrequencies and the unbalance response, i.e. amplitudes at resonance of the forward whirl modes. The effects not included in the simulation, i.e. resonance-like behavior at $\frac{1}{3}\Omega$ and $\frac{2}{3}\Omega$ as well as the weight induced resonance at 2Ω are visible in the envelope of the uncontrolled run at approximately 20 and 80 Hz as expected. As stated in section II, these excitations are of minor interest, since only moderate amplitudes are caused by them.

Control performance at the unbalance induced resonances matches the simulation results quite accurately and good performance is achieved. Prediction of control effort by the simulation is subject to uncertainties. The differences are assumed to be mainly caused by the linear approximation of the behavior of the actuators but also other simplifications during the modeling process. The controller attenuates vibration amplitudes at 2Ω efficiently. Excitation at $\frac{1}{3}\Omega$ and $\frac{2}{3}\Omega$ is literally eliminated with no significant control effort.

ISBN: 978-988-19252-4-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

It is assumed by the authors, that the bearing characteristics are altered in such a way by the controller, that the effect of nonlinearity in the bearings is reduced. Furthermore, since lower amplitudes occur in the controlled case and nonlinear effects like progressive spring characteristics are amplitude dependent, it may be assumed that the influence of nonlinearity is further reduced by the attenuation of vibration amplitudes.

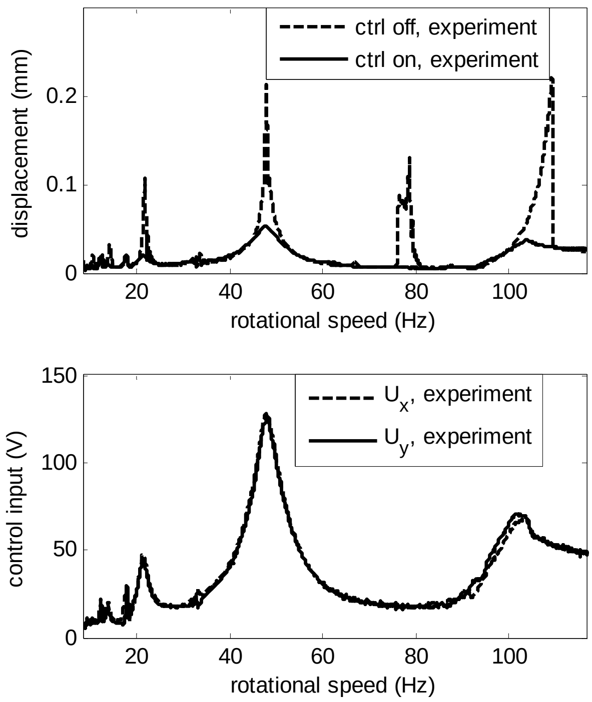

Fig. 6: Experimental results, amplitudes at sensor 1x and control effort

VI. Conclusion

A proposal based on physical considerations for combined design of an LQR and the respective observer for a rotor with gyroscopic effect has been given. The controller design procedure was discussed in detail in order to give the reader advice for controller design for this class of systems.

It has been shown, that it is possible to design a single controller leading to robust stability and good vibration attenuation in the whole operating range of the test rig under consideration. The controller was validated in simulation as well as experiment. It was shown that the controller is able to attenuate unbalance induced vibrations, as well as those caused by gravitational forces and nonlinearities in the system.

Acknowledgment

This paper is based on a research project in partnership with Rolls-Royce Deutschland Ltd & Co KG., supported by Deutsche Forschungsgemeinschaft within the framework of the graduate college 1344, „Instationäre Systemmodellierung von Flugtriebwerken".

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