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GeDEA-II: A Novel Evolutionary Algorithm for Multi-Objective Optimization Problems Based on the Simplex Crossover and The Shrink Mutation

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***Abstract*—The key issue for an efficient and reliable multi-objective evolutionary algorithm is the ability to converge to the True Pareto Front with the least number of objective function evaluations, while covering it as much as possible. To this purpose, in a previous paper performance comparisons showed that the Genetic Diversity Evolutionary Algorithm (GeDEA) was at the same level of the best state-of-the-art MOEAs due to it intrinsic ability to properly conjugate exploitation of current non-dominated solutions and the exploration of the search space. In this paper, an improved version, namely the GeDEA-II, is proposed which features a novel crossover operator, the Simplex-Crossover, and a novel mutation operator, the Shrink-Mutation.**

GeDEM operator was left unchanged and completed using the non-dominated-sorting based on crowding distance. The comparison among GeDEA-II and GeDEA, as well as with three other modern elitist methods, on different extremely multidimensional test problems, clearly indicates that the performance of GeDEA-II is, at least in these cases, superior. In addition, authors aimed at putting in evidence the very good performance of GeDEA-II even in extremely multidimensional landscapes. To do this, four test problems were considered, and the GeDEA-II performance tested as the number of decision variables was increased. In particular, ZDT test functions featured a number of decision variables ranging from the original proposed number up to 1000, whereas on $DTLZ$ the decision variables were increased up to 100 times the original proposed number. Results obtained contribute to demonstrate further the GeDEA-II breakthrough performance.

***Index Terms*—Evolutionary algorithms, Simplex Crossover, Shrink Mutation, Pareto optimality, multi objective optimization, Empirical - Comparison.**

I. Introduction

In the past, a number of powerful Multi-Objective Evolutionary Algorithms (MOEAs) were proposed, e.g. NSGA-II [1], SPEA-II [2] and IBEA [3]. GeDEA [4] algorithm, which was designed around the genetic diversity preservation mechanism called GeDEM, proved to be able to compete and, in some cases, to outperform, the aforementioned MOEAs as far as speed of convergence and covering uniformity of the Pareto Front are concerned. However, the common drawback of all of the previously mentioned multi-objective evolutionary algorithms concerns the huge amount of objective function evaluations (or number of generations) required to reach and sufficiently cover the Pareto Front.

To try to overcome this common weakness, during the last decade several authors started hybridizing evolutionary algorithms (EAs) with local search (LS) operators.

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Several examples can be found in literature (some recent works are presented in [5] and [6]).

In spite of the different frameworks, in all the previously mentioned works, the local search, based on the Simplex algorithm, and the global exploration based on the Evolutionary algorithm, are performed separately, in a sequential manner, that is, a point of the search space is calculated via either the first or the latter.

In the authors' opinion, the previously mentioned examples of hybridization with local search often degrade the global search ability of MOEAs. Moreover, local search based on the Nelder and Mead requires additional and several functions evaluations.

In this paper, GeDEA-II is presented, aiming at reducing the potential weaknesses of its predecessor and competitors, while retaining its very good performance, that is, a good balance between exploration and exploitation. In particular, we propose a different approach to combine the Evolutionary algorithm-based global search and the Simplex theory, since global exploration and local search are intimately related and performed simultaneously, in such a way that they take advantage from each other. In details, we introduce a novel crossover operator, which we called the "Simplex-crossover" and which will be described hereafter; following this, the individuals created by the proposed algorithm via the Simplex-based crossover undergo mutation in a subsequently step, using another new typer of operator, which we called the "Shrink-Mutation", so as to promote global search capabilities of the algorithm. Moreover, important modifications have been brought about to the original Simplex theory, in order to enhance further the local search capabilities without penalizing the exploration of the search space.

The main differences of GeDEA-II in comparison with GeDEA regard its new Simplex-Crossover operator, and its new Shrink-Mutation operator. The diversity preserving mechanism, the Genetic Diversity Evaluation Method (GeDEM) already used in the GeDEA release, was retained in GeDEA-II and left unchanged due to its superior performance compared to other types of mechanisms.

II. Genetic Diversity Evolutionary Algorithm (GeDEA)

The Genetic Diversity Evolutionary Algorithm II (GeDEA-II), is a framework that is strictly designed around GeDEM [4] to exalt its characteristics. To briefly introduce the GeDEM principle, it is worth underlining that the multi-objective optimization process has two objectives, which are themselves conflicting: the convergence to the

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Pareto-optimal set and the maintenance of genetic diversity within the population. The basic idea of GeDEM is to actually use these objectives during the evaluation phase and to rank the solutions with respect to them, emphasizing the non-dominated solutions as well as the most genetically different. To this purpose, GeDEM computes the actual ranks of the solutions maximizing (i) the ranks scored with respect to the objectives of the original MOOP, the non-dominated solutions having the highest rank, and (ii) the values assigned to each individual as a measure of its genetic diversity, calculated according to the chosen distance metric, i.e. the (normalized) Euclidean distance in the objective functions space.

III. Genetic Diversity Evolutionary Algorithm-II (GeDEA-II)

GeDEA proved to be an efficient algorithm, able to explore widely the search space, while exploiting the relationships among the solutions. In order to enhance GeDEA algorithm performance further, several main features were added to the previous GeDEA version, yet retaining its constitutive framework. The main innovation is the novel crossover operator, namely the Simplex-crossover, which substitutes the previous Uniform crossover. A novel mutation operator was also developed, namely the Shrink-mutation, which allows exploring more effectively the design space. The remaining steps characterizing GeDEA algorithm, in particular the GeDEM, were left unchanged. The latter was integrated with the Non-Dominating sorting procedure based on the crowding distance, developed and thoroughly described in [1].

A. The SIMPLEX Crossover

In [7], authors proposed a simplex crossover (SPX), a new multi-parent recombination operator for real-coded GAs. The experimental results with test functions used in their studies showed SPX well performed on functions having multimodality and/or epistasis. However, the authors did not consider the application of the SPX to multiobjective problems. Moreover, they did not consider the possibility to take into account the fitness of the objective function/s as the driving force of the simplex. Therefore, we decided to integrate in the GeDEA-II the simplex crossover with these and further new distinctive features. Unlike the Simplex-crossover presented in [7], in GeDEA-II only two parents are required to form a new child. These two parents are selected according to the selection procedure from the previous population, and combined following the guidelines of the simplex algorithm. Let assume ***p1***, ***p2*** being the two parent vectors, characterized by different, multiple fitness values, the child vector ***Child*** is formed according to the reflection move described in [8]:

$$\mathbf{Child} := (1 + Refl) \cdot \mathbf{M} - Refl \cdot \mathbf{p_2} \quad (1)$$

where ***Child*** is the new formed child and *Refl* is the reflection coefficient. It is assumed that ***p1*** is the best fitness individual among the two chosen to form the ***Child***, whereas ***p2*** the worst one. *Refl* coefficient is set equal to a random number ($refl \in [0, 1]$), unlike the elemental Simplex theory, which assumes a value equal to 1 for the *Refl* coefficient. This choice allows to create a child every time distant in a random manner from the parents, hence to explore more deeply the design space. Since the Simplex algorithm is itself a single-objective optimizer, a strategy was implemented to adapt it to a multi-objective algorithm: the objective function considered to form the new child is chosen randomly in order to enhance the design space exploration of the crossover, required in highly dimensional objective spaces. This new crossover operator was expected to combine both exploration and exploitation characteristics. In fact, the new formed child explores a design space region opposite to that covered by the worst parent, that means it explores a region potentially not covered so far. In the early stages of the evolution, this means that child moves away from regions covered from bad parents, while exploring new promising ones. In addition, the characteristics of the good parents are deeply exploited to accelerate the evolution process. During evolution, GeDEA-II makes use exclusively of the Simplex Crossover until three-quarters of the generations has been reached. After that, Simplex Crossover is used alternatively with the Simulated Binary Crossover (SBX) (described for the first time in [9]) with a switching probability of 50 percent.

B. The Shrink Mutation

As far as mutation is concerned, a new Shrink-mutation operator is introduced in the GeDEA-II.

In the literature, this kind of mutation strategy is referred to as *Gaussian mutation* [10], and conventional implementations of Evolutionary Programming (EP) and Evolution Strategies (ES) for continuous parameter optimization using Gaussian mutations to generate offspring are presented in [11] and [12], respectively.

In general, mutation operator specifies how the genetic algorithm makes small random changes in the individuals in the population to create mutation children. Mutation provides genetic diversity and enables the genetic algorithm to search a broader space. Unlike the previous version of mutation featuring GeDEA algorithm, where some bits of the offspring were randomly mutated with a probability p_{mut}, here the mutation operator adds a random number taken from a Gaussian distribution with mean equal to the original value of each decision variable characterizing the entry parent vector. The shrinking schedule employed is:

$$Shrink_i := Shrink_{i-1} \cdot \left(1 - \frac{ignr}{ngnr}\right) \quad (2)$$

where $Shrink_i$ is a vector representing the current mutation range allowed for that particular design variable, *ignr* represents the current generation and *ngnr* the total number of generations. The shape of the shrinking curve was decided after several experimental tests. The fact that the variation is zero at the last generation is also a key feature of this mutation operator. Being conceived in this manner, the mutation allows to deeply explore the design space during the first part of the optimization, while exploiting the non-dominated solutions during the last generations. Once the current variation range has been calculated, one decision variable of a selected child is randomly selected, and mutated according to the following formula:

$$Child_{mut} := Child_{cross} + [Shrink_i] \quad (3)$$

Unlike crossover operator, which generates all the offspring, mutation is applied only on a selected part of the offspring. Before starting offspring mutation, offspring population is randomly shuffled to prevent locality effects. After that, a pre-established percentage (fixed to 40% for all of the test problems) of the individuals are selected for mutation. The initial Shrink factor is set equal to the whole variation range of the design variables. This mutation operator was found to be powerful especially in multi-objective problems requiring a huge exploration of the design space.

IV. Comparison with Other Multiobjective Evolutionary Algorithms

In order to judge the performance of the GeDEA-II, a comparison with other different state-of-the-art multi-objective EAs was performed. SPEA-2 [2], NSGA-II [1] and IBEA [3] were chosen as competitors, and their performance against GeDEA-II was measured on two test problems featuring the characteristics that may cause difficulties in converging to the Pareto-optimal front and in maintaining diversity within the population [13]: discrete Pareto fronts, and biased search spaces. In addition, their performance was tested also on two more recent and more challenging benchmark test functions chosen among the scalable Test Problems presented in [14]. The four test functions, the methodology and the metric of performance used in the comparison are briefly recalled in the following for easy reference.

A. Test Functions

Here only four test problems are presented due to layout constraints. The original version of ZDT_3 and ZDT_6 presented in [15] featured 30 and 10 decision variables, respectively. Here we propose them with 100 decision variables. As regards $DTLZ_3$, the number of variables suggested in [14] is 12. Here we propose it with 22 decision variables, respectively. As regards $DTLZ_7$, we increased the number of decision variables from the original one equal to 22, up to 100.

B. Methodology

The methodology used in [15] is strictly followed. GeDEA and competitors are executed 30 times on each test function. There are different parameters associated with the various algorithms, some common to all and some specific to a particular one. In order to make a fair comparison among all the algorithms, most of these constants are kept the same. In GeDEA-II, GeDEA and in competitors algorithms, the population size is set to 100. In the following, the parameters of the competitors MOEA are reported following the terminology used in PISA implementation. The individual mutation probability is always 1 and the variable mutation probability is fixed at $1/n$, n being the number of the decision variables of the test problem considered. The individual recombination probability along with the variable recombination probability are set to 1. The variable swap probability is set to 0.5. $\eta_{mutation}$ is always set to 20 and $\eta_{recombination}$ is fixed to 15. For IBEA algorithm, tournament size is always set to 2, whereas additive epsilon is chosen as the indicator. Scaling factor $kappa$ is set to 0.05, and rho factor is fixed to 1.1. For all of the competitors, tournament size is given a value equal to 2. NSGA-II, SPEA-2 and IBEA are run with the PISA[1] implementation [16], with exactly the same parameters and variation operators. The maximum number of generations for test functions ZDT_6 is set to 30 for all the algorithms and to 40 for test function ZDT_3. For test function $DTLZ_7$ the number of generations is set to 100, whereas to 150 for $DTLZ_3$ test function. In Table I, the original number of generations characterizing test problems presented in [15] and [14], is compared to the ones used here. The number of generations was intentionally reduced in order to test the convergence properties of the investigated algorithms, and contribute to justify the different results reported here, when compared to those presented in the original papers [15], [14].

TABLE I
Original and proposed number of generations for the *ZDT* and *DTLZ* test problems.

	Number of generations	
	Original version problems	Proposed test problems
ZDT3	250	40
ZDT6	250	30
DTLZ3	500	150
DTLZ7	200	100

C. Metric of Performance

Different metrics can be defined to compare the performance of EAs with respect to the different goals of optimization itself [15]: how far is the resulting non-dominated set from the Pareto front, how uniform is the distribution of the solutions along the Pareto Approximation set/front, how wide is the Pareto Approximation set/front. For measuring the quality of the results, we have employed the Hypervolume approach, due to its construction simplicity and for the reason, which will be soon explained. The hypervolume approach measures how much of the objective space is dominated by a given nondominated set. Zitzler et al. [17] state it as the most appropriate scalar indicator since it combines both the distance of solutions (towards some utopian trade-off surface) and the spread of solutions. The Hypervolume[2] is defined as the area of coverage of PF_{known} with respect to the objective space for a two-objective MOP. In this work we use the version implemented by Fonseca et al. and presented in [19].

D. Results of Comparison

As in Zitzler et al. [15], Figures 1, 3 and 4 show an excerpt of the non-dominated fronts obtained by the EAs and the Pareto-optimal fronts (continuous curves). The points plotted are the non-dominated solutions extracted from the union set of the outcomes of the first five runs, the best and the worst one being discarded. The performance of GeDEA-II is also compared to that of the competitors according to the hypervolume metric as defined in [19]. The distribution of these values is shown using box plots in Figures 2 and 5. On each box, the central line represents the median, the edges of

[1]This software is available for public use at PISA website http://www.tik.ee.ethz.ch/pisa/

[2]The Hypervolume is a Pareto compliant indicator as stated in [18].

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the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually, with a Plus sign. Results are normalized with the best Hypervolume value coming from the union set of all of the runs, extended to all of the algorithms. For each test problem, the reference point is assumed equal for all of the algorithms, and equal to the maximum value for each objective function from the union of all of the output points.

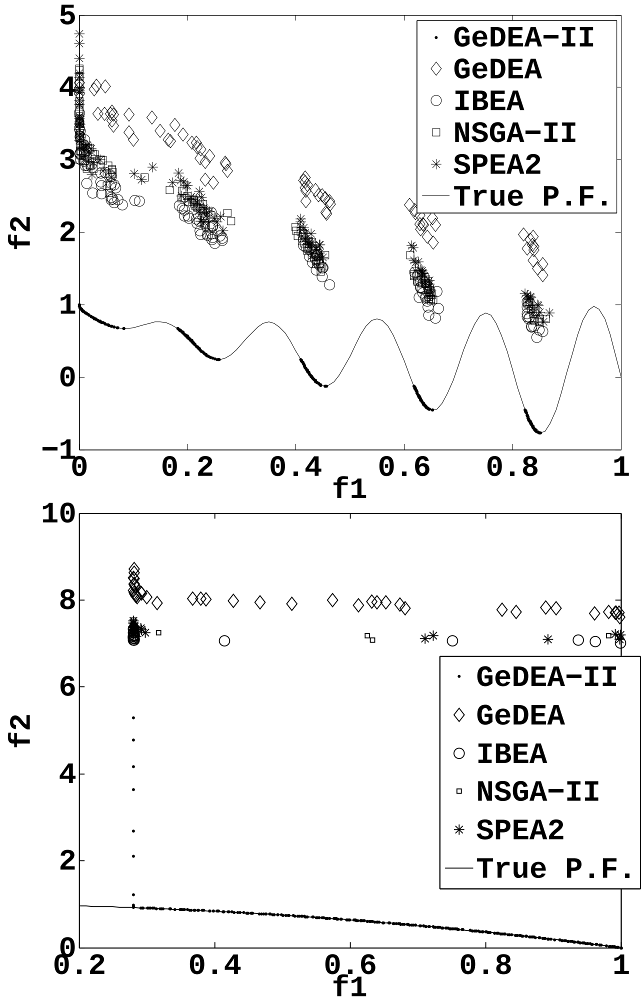

Fig. 1. Test functions ZDT_3 (AT THE TOP) and ZDT_6 (AT THE BOTTOM).

In general, the experimental results show that GeDEA-II is able to converge towards the True Pareto-optimal front and to develop a widely and well distributed non-dominated set of solutions. The comparison with the other three best-performing MOEAs according to the *Hypervolume* metric proves that the performance of GeDEA-II is somewhat superior. Considering the specific features of the two *ZDT* test functions, GeDEA-II shows similar performance both on multi-front and biased Pareto-optimal fronts. NSGA-II, SPEA-2 and IBEA seem instead to have more difficulties with discreteness (test function ZDT_3). The performance of GeDEA-II is particularly remarkable in the case of biased search space (test function ZDT_6) where it is also able to evolve a well-distributed non-dominated set. These results gain even more significance, since the number of decision variables was set to 100, unlike the original values of 30 (10 for the test function ZDT_6).

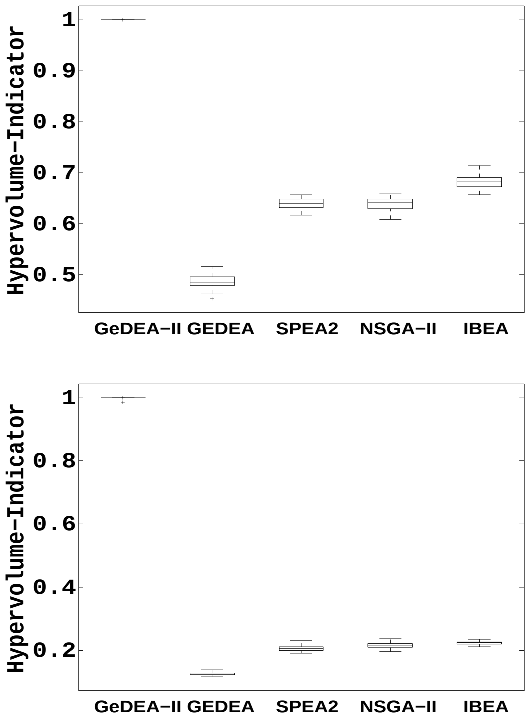

Fig. 2. Box plots based on the *Hypervolume* metric. Each square contains six box plots representing the distribution of *Hypervolume* values for the six algorithms. Results refer to the ZDT_3 (AT THE TOP) and ZDT_6 (AT THE BOTTOM) test functions.

As far as $DTLZ_3$ and $DTLZ_7$ test functions is concerned, GeDEA-II is able to reach the True Pareto Front, whereas the competitors remain trapped in the local Pareto Approximation Sets, as shown in Fig. 3 and 4.

Finally, box plots prove, in general, that the performance of GeDEA-II is superior to those of the competitors also as far as the repeatability of the results is concerned.

E. GeDEA-II Performance on Extremly Multidimensional Landscapes

In this section, authors aim at putting in evidence the outstanding performance of GeDEA-II even on high multidimensional environments. To do this, two test problems, chosen among those presented in Section IV-A are considered, and the GeDEA-II performance tested by changing every time the number of decision variables. Test functions chosen for this test are the ZDT_4 and $DTLZ_3$, that is, the most difficult to solve problems, as stated in [15] and [14].

In Table II, the number of variables and generations characterizing these tests are reported. In particular, ZDT_4 test function feature a maximum number of decision variables of 1000, whereas on $DTLZ_3$ test functions the maximum number of decision variables is increased up to 100 times the original proposed number [14].

To the best of the authors' knowledge, this is the first time a MOEA is tested on these test problems, with these number of decision variables. For each test problems, we performed 30 independent runs for each number of decision variables, and the boxplots were then built, following the guidelines already given in Section IV-D. Y-axes are scaled

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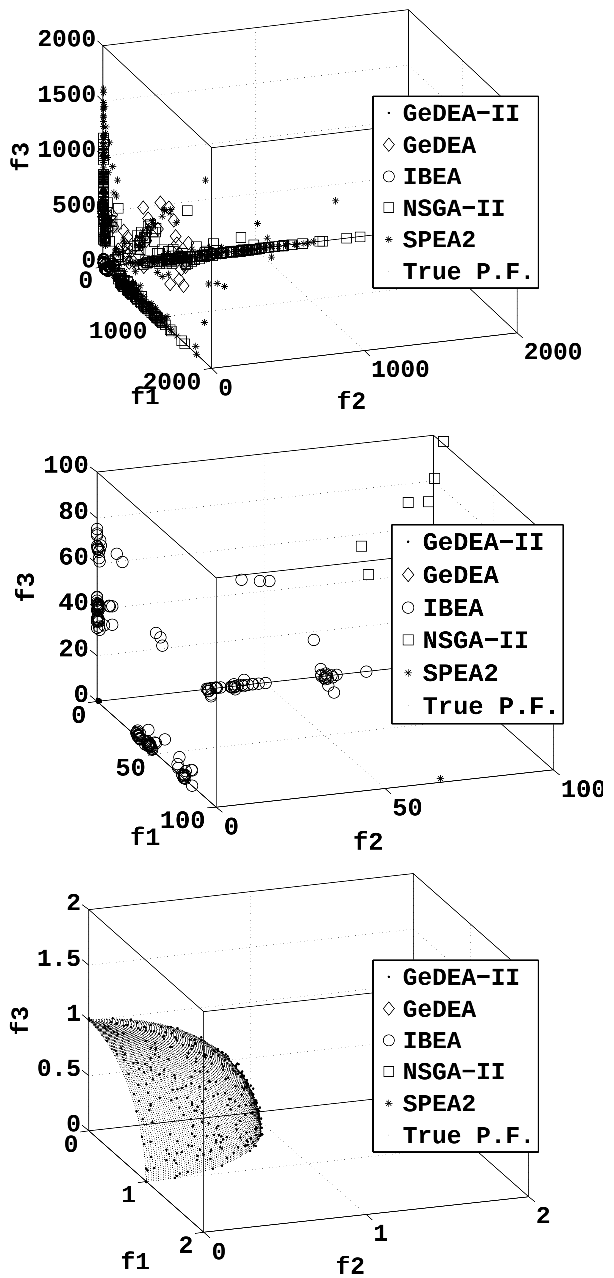

Fig. 3. Test function *DTLZ3*. From the left, Auto scale axes, Medium zoom and True Pareto Front region.

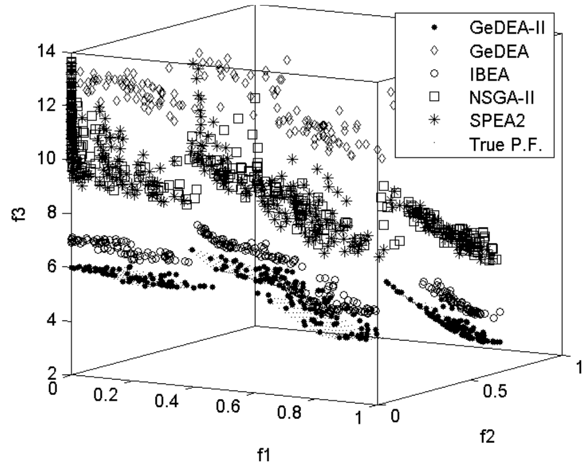

Fig. 4. Test function $DTLZ_7$.

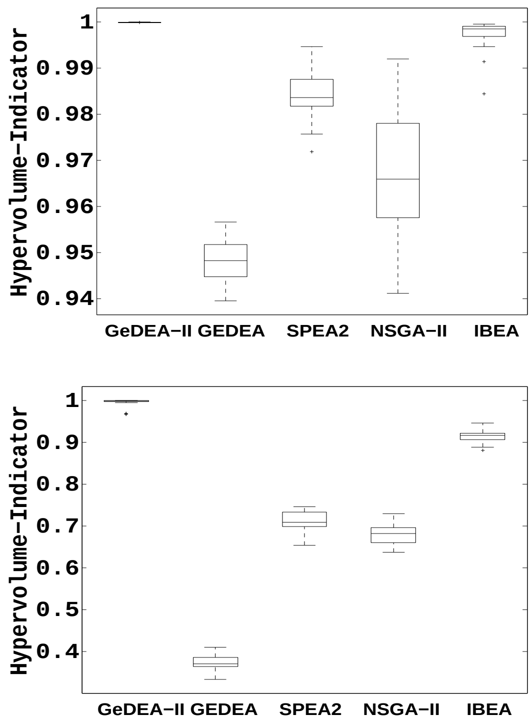

Fig. 5. Box plots based on the *Hypervolume* metric. Each square contains six box plots representing the distribution of *Hypervolume* values for the six algorithms. Results refer to the $DTLZ_1$ (AT THE TOP) and $DTLZ_7$ (AT THE BOTTOM) test functions.

TABLE II
MINIMUM AND MAXIMUM NUMBER OF DECISION VARIABLES FOR THE ZDT_4 AND $DTLZ_3$ TEST PROBLEMS.

	Number of generations	Minimum number of decision variables	Maximum number of decision variables
ZDT4	40	10	1000
DTLZ3	80	12	1200

in such a way the best run is given a value equal to 1. In Figure 6, the boxplots showing GeDEA-II performance are presented, as the decision variables are increased from the minimum value up to the maximum one. Results clearly states that GeDEA-II performance is high-level. In each test problem, performance is never lower than 99% of the maximum value, no matter how many the decision variables are. This clearly demonstrate GeDEA-II manages to evolve the initial population near to the True Pareto front, even when the number of decision variables is dramatically increased. Figure 7 shows in the objective space, the distribution of the final solutions obtained in the run with the lowest Hypervolume-value by the GeDEA-II for each test instance, for the maximum number of decision variables. It is evident that as regards the convergence to the True Pareto Front and spread of solutions, GeDEA-II performance is high level.

V. CONCLUSION

In this paper, we have presented GeDEA-II, an improved multi-objective evolutionary algorithm that employs novel variation operators compared to its predecessor GeDEA.

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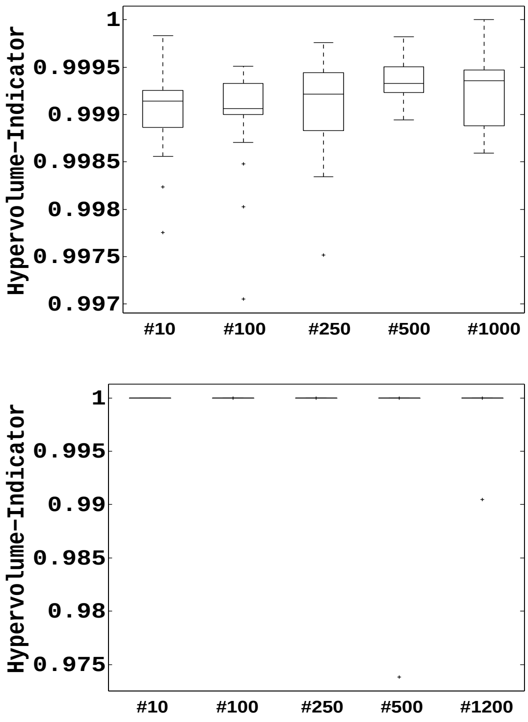

Fig. 6. Box plots based on the $Hypervolume$ metric. Each square contains five box plots representing the distribution of $Hypervolume$ values for the six number of decision variables.Results refer to the ZDT_4 (AT THE TOP) and $DTLZ_3$ (AT THE BOTTOM) test functions.

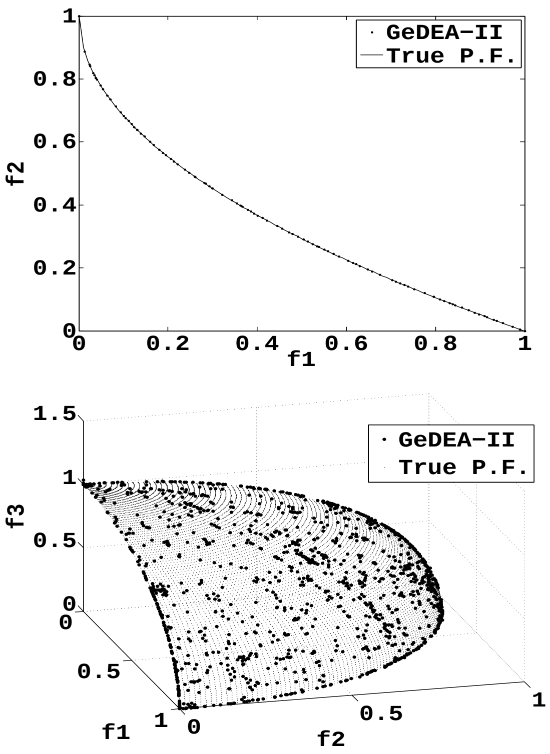

Fig. 7. Final Approximation Set reached by the GeDEA-II on test function ZDT_4 (AT THE TOP) and the non dominated solutions found by GeDEA-II on $DTLZ_3$ (AT THE BOTTOM), featuring 1200 decision variables.

Extensive numerical comparisons of GeDEA-II with GeDEA and with NSGAII, SPEA-2 and IBEA, three state-of-the-art recently proposed algorithms, have been carried out on various test problems. Moreover, optimization difficulties have been enhanced further, in order to test the robustness of the codes. The key results of the comparison show the excellent performance of the GeDEA-II, when compared to the competitors algorithm, in terms of both exploration and exploitation capabilities. Boxplots show that the reproducibility of results of GeDEA-II is high-level, when compared to that of the NSGAII, SPEA-2 and IBEA. In extremely high dimensional spaces, GeDEA-II clearly shows excellent performance. In addition to these characteristics, GeDEA-II performs these tasks with a reduced number of objective functions evaluations, a very useful feature when considering its application to real-world engineering problems.

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