

# Using Kohonen-SOM & K-Means Clustering Techniques to Analyze QoS Parameters of RSVP

Hartinder Singh Johal, Balraj Singh, Harwant Singh, Amandeep Nagpal and Harjot Singh Virdi

**Abstract**— Promise of actually realizing and delivering the unanimously committed quality of service (QoS) is a big challenge, given the explosive growth in usage of real-time based multimedia applications online. The queuing strategies do critically affect the bandwidth allocation of packets while making critical selection for packets to be dropped. This also have an effect on latency. Resource Reservation Protocol (RSVP) in concurrence with weighted fair queuing (WFQ) attempts to accomplish bandwidth reservation with intention to predetermine and guarantee of improved QoS. At peak traffic with impending congestion, this scheme fails to cap different QoS parameters within premeditated limits. We propose an innovative solution by combining priority queuing scheme with RSVP instead of WFQ. Using Kohonen's Self Organizing Map (SOM) & K-Means clustering techniques as analysis tool, we come across that this strategy allows us to sustain a better quality of service for extremely high precedence real-time video traffic. We are further able to demonstrate that the delegated QoS by this strategy is immune to different firewall implementation schemes, across the network.

**Index Terms**—Self Organizing Map, K-Means, weighted fair queuing, resource reservation protocol, principal component analysis.

## I. INTRODUCTION

THE expectations from real-time based video applications have increased exponentially over the course of time. Different attributes like ftp, http, upload & download response time play significant part in success of these online applications. Application performance has on the large lagged to keep pace with user expectations.

This all transcend down to QoS parameters. Discrete assessment and evaluation of application response time, media access delay, latency, jitter; packet loss, retransmission, throughput, congestion & queuing delay and

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network utilization generate considerable impact on the ultimately delivered experience. Efficient resource allocation is the key if we deem to make an estimate about how the buffering of packets must happen while awaiting transmission. The queuing strategies do critically affect the bandwidth allocation of packets while making critical selection for packets to be dropped [1]. RSVP strategy when used in conjunction with different queuing schemes allows multimedia based real-time data transfer to identify & set aside the resources required for making realistic assumptions about latency accumulations [2]. RSVP works in tandem with WFQ by assigning weights to every transfer queue which in turn quantifies the proportion of bandwidth for those queues. We start with assessing the effect of realizing different queuing profiles on utilization, throughput, queuing & end to end delay for voice and video. Subsequent to this we apply queuing disciplines with RSVP & record observations about different QoS parameters. We use Kohonen's Self Organizing Map (SOM) to visualize the effects of different queuing strategies mutually exclusively & with RSVP [3]. Visualization is used to better predict & analyze the traffic patterns generated by discretely capturing the nonlinear attributes of the traffic flow. Along with this physical meaning of clusters is explored with reflection on evolving vectors of traffic flow [4]. Subsequently we perform comparison of the results with Principal Component Analysis (PCA) algorithm & with K-Means [5]. Eventually we realize the amalgamation of SOM with Hierarchical Agglomerative Clustering (HAC) to further justify the results.

## II. EXPERIMENTAL SETUP

We use network structure in fig 1, for mutually exclusive application of WFQ, PQ, PQ with RSVP & WFQ with RSVP. The scenario consists of extending different voice; video & ftp applications to end users. We apply WFQ, PQ, PQ with RSVP & WFQ with RSVP one after another & assess what affect it has on these applications concurrent to network utilization and throughput. We employ token bucket rate as measurement for flow. In this method once the choice about diverse resources to be reserved has been achieved the protocol invokes different modules for setup of reservation. We record the dataset encompassing observations about average end to end delay for video conferencing & voice, average point to point queuing delay, throughput & utilization. For implementing WFQ based RSVP with firewall & PQ based RSVP with firewall we use net map in fig 2, for generating corresponding observations.

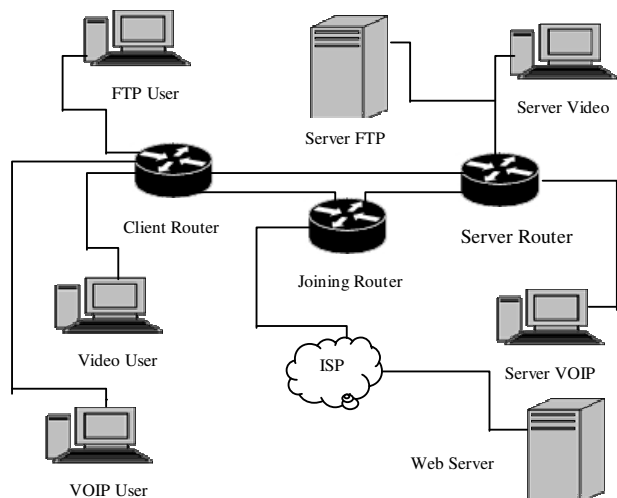


Fig 1. Net-map for Queuing Profiles Application with RSVP.  
We record data for following attributes:  
PQ: PQ Profile without RSVP & Firewall.  
WFQ: WFQ Profile without RSVP & Firewall  
PQ/RSP: PQ profile with RSVP  
WFQ/RSP: WFQ profile with RSVP  
PQ/RSP/FW: PQ/RSVP with Firewall  
WFQ/RSP/FW: WFQ/RSVP with Firewall

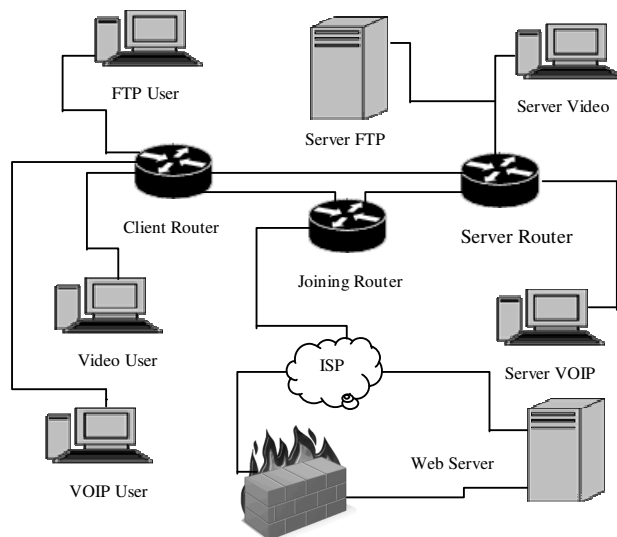


Fig 2. Net-map for Queuing Profiles Application with Firewall.

### III. METHODOLOGY

For analysis an artificial neural network based self organizing map is used to generate a primitive level of discrete representation of different QoS parameters from training set [6]. It helps us in grouping data into clusters which can be further introspected for eventual analysis [7]. When achieving clustering with Kohonen-SOM the precondition is that at least one input attribute in data set  $(V_1, V_2, V_3, \dots, V_n)$  must have continuous distribution. On successful realization of this technique, a new attribute is added to the data set which encompasses the cluster information. Best fit unit is arrived by

$$d = \sqrt{\sum_{i=1}^n (V_i - W_i)^2}$$

SOM generates results of standard comparable to K-Means

[9]. Similar for K-Means at least one input attribute must have continuous distribution & it involves portioning  $n$  recordings  $(x_1, x_2, x_3, \dots, x_n)$  each being  $d$ -dimensional vector into different  $k$  sets  $(S_1, S_2, S_3, \dots, S_k)$  for  $k \leq n$ . The objective is to minimize the within cluster sum of squares (WSS). Subsequently we make efforts to perform comparison analysis with Principal Component Analysis (PCA) technique [10]. To get hold of best cluster size we use Hierarchical Agglomerative Clustering (HCA) to further justify our findings.

### IV. RESULTS

We generate Univariate statistics for average end to end delay for video conferencing, voice communication, point to point queuing delay, point to point throughput & point to point utilization. For all datasets there are 6 descriptors with 41 instances. All the variables are in continuous distribution & hence qualify for operation of Kohonen-SOM algorithm. After computing continuous statistics we encounter that the dataset is free from constants & all variables are precisely defined on common scale.

TABLE I  
UNIVARIATE STATISTICS FOR AVERAGE END TO END DELAY (SEC) FOR VIDEO CONFERENCING

Attribute	Min	Max	Average	Std-dev	Std-dev/avg
PQ	0.146869	0.347665	0.3212	0.0416	0.1295
WFQ	0.149548	3.28567	1.6891	0.9544	0.5650
PQ/RSP	0.148653	0.640598	0.5624	0.1181	0.2099
WFQ/RSP	0.149548	3.28567	1.6891	0.9544	0.5650
PQ/RSP/FW	0.383232	16.3513	8.3012	4.8140	0.5799
WFQ/RSP/FW	0.149548	3.27347	1.6868	0.9502	0.5633

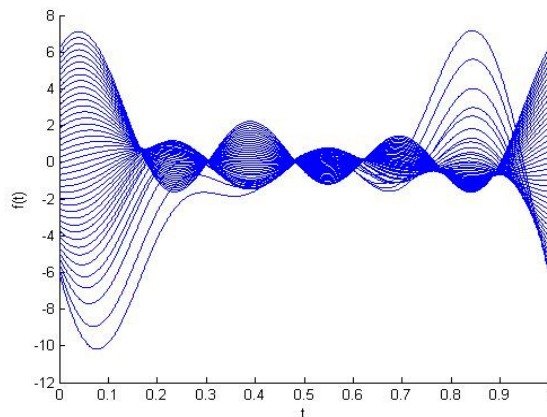


Fig 3. Standardized Plot for Average End to End Delay for Video.

TABLE II  
UNIVARIATE STATISTICS FOR AVERAGE END TO END DELAY (SEC) FOR VOICE COMMUNICATION

Attribute	Min	Max	Average	Std-dev	Std-dev/avg
PQ	0.00439167	0.00442628	0.0044	0.0000	0.0021
WFQ	0.00438368	0.00441336	0.0044	0.0000	0.0009
PQ/RSP	0.00438184	0.00443951	0.0044	0.0000	0.0033
WFQ/RSP	0.00438368	0.00441336	0.0044	0.0000	0.0009
PQ/RSP/FW	0.000696356	0.000696363	0.0007	0.0000	0.0000
WFQ/RVP/FW	0.00438368	0.00441336	0.0044	0.0000	0.0011

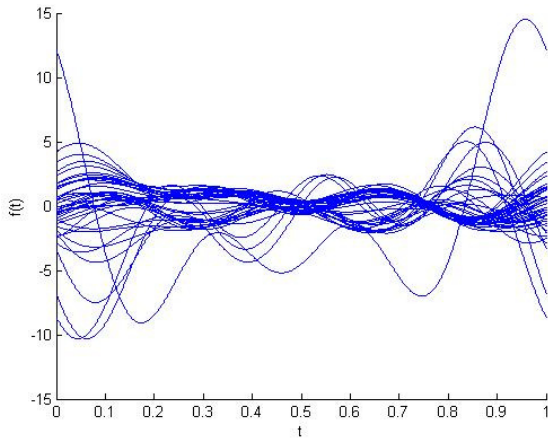


Fig 4. Standardized Plot for Average End to End Delay for Voice.

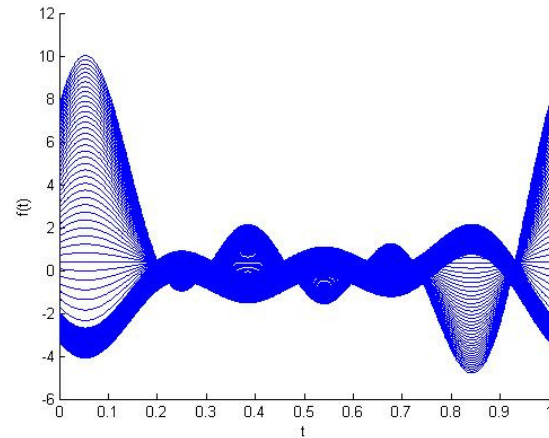


Fig 6. Standardized Plot for Average Point to Point Throughput.  
TABLE V

TABLE III  
UNIVARIATE STATISTICS FOR AVERAGE POINT TO POINT QUEUING DELAY

Attribute	Min	Max	Average	Std-dev	Std-dev/avg
PQ	0.000632124	0.00155863	0.0010	0.0004	0.3544
WFQ	0.000632124	0.0015495	0.0010	0.0004	0.3516
PQ/RSP	0.000632124	0.00155866	0.0010	0.0004	0.3541
WFQ/RSP	0.000632124	0.0015495	0.0010	0.0004	0.3516
PQ/RSP/FW	0.000376282	0.00104663	0.0008	0.0003	0.3596
WFQ/RSP/FW	0.000753022	0.00155838	0.0012	0.0003	0.2382

UNIVARIATE STATISTICS FOR AVERAGE POINT TO POINT UTILIZATION

Attribute	Min	Max	Average	Std-dev	Std-dev/avg
PQ	0.00226312	41.0785	10.3311	14.3173	1.3858
WFQ	0.00226312	41.0785	10.3311	14.3173	1.3858
PQ/RSP	0.00226312	41.0785	10.3310	14.3173	1.3859
WFQ/RSP	0.00226312	41.0785	10.3311	14.3173	1.3858
PQ/RSP/FW	0.00401009	6.38712	1.6110	2.2234	1.3802
WFQ/RSP/FW	0.00401009	41.0795	10.3336	14.3163	1.3854

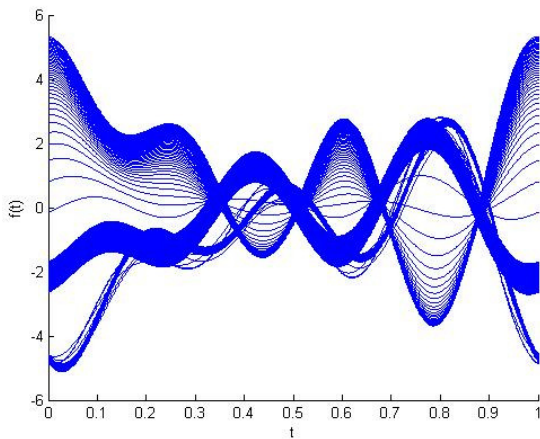


Fig 5. Standardized Plot for Average Point to Point Queuing Delay.

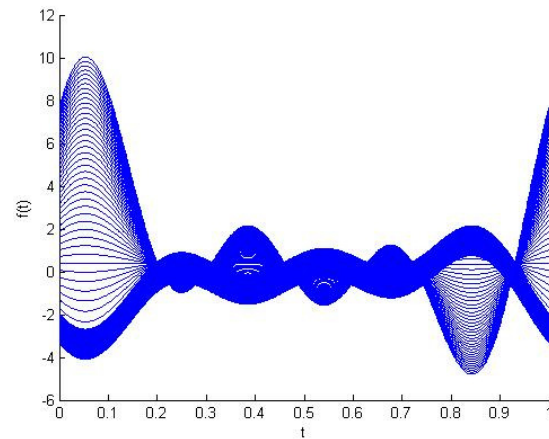


Fig 7. Standardized Plot for Average Point to Point Throughput.  
Subsequently we apply Kohonen-SOM on the trained dataset using a grid size of 2 & 3 with classification of instances into 6 groups (clusters). The learning rate is 0.20 with standard seed generator we observe following record. The data is standardized by division of each variable by their standard deviation.

TABLE IV  
UNIVARIATE STATISTICS FOR AVERAGE POINT TO POINT THROUGHPUT

Attribute	Min	Max	Average	Std-dev	Std-dev/avg
PQ	0.0287356	254.332	63.9997	88.6374	1.3850
WFQ	0.0287356	255.896	64.3972	89.1948	1.3851
PQ/RSP	0.0287356	254.326	63.9391	88.5829	1.3854
WFQ/RSP	0.0287356	255.896	64.3972	89.1948	1.3851
PQ/RSP/FW	0.0383142	205.398	51.6933	71.5641	1.3844
WFQ/RSP/FW	0.0383142	255.43	64.3565	89.0874	1.3843

TABLE VI  
MAP TOPOLOGY – AVERAGE VIDEO DELAY

	1	2	3
1	12	4	3
2	6	11	5

TABLE VII  
MAP TOPOLOGY – AVERAGE VOICE DELAY

	1	2	3
1	0	0	0
2	41	0	0

TABLE VIII  
MAP TOPOLOGY – QUEUING DELAY

	1	2	3
1	25	12	1
2	0	3	56

TABLE XI  
MAP TOPOLOGY - THROUGHPUT

	1	2	3
1	8	8	3
2	13	7	58

TABLE X  
MAP TOPOLOGY - UTILIZATION

	1	2	3
1	8	7	57
2	14	8	3

TABLE XIV  
NUMBER OF INSTANCES IN EACH CLUSTER - THROUGHPUT

Attribute	n°1	n°2	n°3	n°4	n°5	n°6
PQ	181.06	133.92	36.58	230.18	79.66	0.4859
WFQ	182.20	134.77	36.75	231.63	80.13	0.4876
PQ/RSP	180.90	133.61	36.55	230.12	79.51	0.4839
WFQ/RSP	182.20	134.77	36.75	231.63	80.13	0.4876
PQ/RSP/FW	146.20	108.14	29.50	185.88	64.30	0.4192
WFQ/RSP/FW	182.07	134.75	36.76	231.31	80.14	0.5090

*n\* clusters*

TABLE XV  
NUMBER OF INSTANCES IN EACH CLUSTER - UTILIZATION

Attribute	n°1	n°2	n°3	n°4	n°5	n°6
PQ	28.36	11.53	0.024	36.81	20.55	4.356
WFQ	28.36	11.53	0.024	36.81	20.55	4.356
PQ/RSP	28.36	11.53	0.024	36.81	20.55	4.356
WFQ/RSP	28.36	11.53	0.024	36.81	20.55	4.356
PQ/RSP/FW	4.412	1.796	0.010	5.724	3.198	0.6842
WFQ/RSP/FW	28.37	11.53	0.028	36.81	20.56	4.357

*n\* clusters*

Applying K-Means clustering approach for 6 clusters & 10 maximum iterations with 5 trials we observe following cluster size with WSS.

TABLE XI  
NUMBER OF INSTANCES IN EACH CLUSTER – VIDEO DELAY

Attribute	n°1	n°2	n°3	n°4	n°5	n°6
PQ:	0.307169	0.332701	0.197014	0.339538	0.346099	0.343106
WFQ:	0.767492	1.391036	0.214426	1.794713	2.895606	2.243338
PQ/RSPVP:	0.512892	0.598909	0.220326	0.616879	0.636727	0.628506
WFQ/RSPVP:	0.767492	1.391036	0.214426	1.794713	2.895606	2.243338
PQ/RSPVP/FW:	3.665622	6.881338	0.723996	8.897137	14.337231	11.114409
WFQ/RSPVP/FW:	0.767517	1.393029	0.214426	1.798952	2.883873	2.243455

*n\* clusters*

TABLE XII  
NUMBER OF INSTANCES IN EACH CLUSTER – VOICE DELAY

Attribute	n°1	n°2	n°3	n°4	n°5	n°6
PQ	-99999	-99999	-99999	0.004419	-99999	-99999
WFQ	-99999	-99999	-99999	0.004393	-99999	-99999
PQ/RSP	-99999	-99999	-99999	0.004426	-99999	-99999
WFQ/RSP	-99999	-99999	-99999	0.004393	-99999	-99999
PQ/RSPVP/FW	-99999	-99999	-99999	0.000696	-99999	-99999
WFQ/RSPVP/FW	-99999	-99999	-99999	0.004397	-99999	-99999

*n\* clusters*

TABLE XIII  
NUMBER OF INSTANCES IN EACH CLUSTER – QUEUING DELAY

Attribute	n°1	n°2	n°3	n°4	n°5	n°6
PQ	.001533	.001417	.000993	-99999	.001192	.000741
WFQ	.001524	.001410	.000992	-99999	.001189	.000741
PQ/RSP	.001534	.001419	.000993	-99999	.001192	.000742
WFQ/RSP	.001524	.001410	.000992	-99999	.001189	.000741
PQ/RSP/FW	.000405	.000527	.000899	-99999	.000737	.000970
WFQ/RSP/FW	.001535	.001441	.001159	-99999	.001282	.000963

*n\* clusters*

TABLE XVI  
CLUSTER SIZE & WSS – VIDEO DELAY

Cluster	Description	Size	WSS
cluster n°1	c_kmeans_1	5	1.8420
cluster n°2	c_kmeans_2	11	3.0898
cluster n°3	c_kmeans_3	9	1.7600
cluster n°4	c_kmeans_4	6	0.7075
cluster n°5	c_kmeans_5	3	3.6356
cluster n°6	c_kmeans_6	7	0.8780

TABLE XVII  
CLUSTER SIZE & WSS – VOICE DELAY

Cluster	Description	Size	WSS
cluster n°1	c_kmeans_1	5	2.3632
cluster n°2	c_kmeans_2	14	4.6022
cluster n°3	c_kmeans_3	9	1.7048
cluster n°4	c_kmeans_4	1	0.0000
cluster n°5	c_kmeans_5	3	6.2459
cluster n°6	c_kmeans_6	9	5.4025

TABLE XVIII  
CLUSTER SIZE & WSS – QUEUING DELAY

Cluster	Description	Size	WSS
cluster n°1	c_kmeans_1	3	0.9219
cluster n°2	c_kmeans_2	9	1.5949
cluster n°3	c_kmeans_3	39	0.0000
cluster n°4	c_kmeans_4	17	1.4608
cluster n°5	c_kmeans_5	0	0.0000
cluster n°6	c_kmeans_6	29	1.2244

TABLE XIX  
CLUSTER SIZE & WSS – THROUGHPUT

Cluster	Description	Size	WSS
cluster n°1	c_kmeans_1	14	3.1140
cluster n°2	c_kmeans_2	7	1.0406
cluster n°3	c_kmeans_3	5	0.5262
cluster n°4	c_kmeans_4	57	0.0362
cluster n°5	c_kmeans_5	4	0.3488
cluster n°6	c_kmeans_6	10	1.9720

TABLE XX  
CLUSTER SIZE & WSS – UTILIZATION

Cluster	Description	Size	WSS
cluster n°1	c_kmeans_1	14	3.1217
cluster n°2	c_kmeans_2	7	1.0414
cluster n°3	c_kmeans_3	5	0.5260
cluster n°4	c_kmeans_4	57	0.0364
cluster n°5	c_kmeans_5	4	0.3487
cluster n°6	c_kmeans_6	10	1.9680

TABLE XXIV  
BEST CLUSTER SELECTION – THROUGHPUT

Clusters	BSS ratio	Gap
1	0.0000	0.0000
2	0.8306	4.3694
3	0.9329	0.3792
4	0.9720	0.1640
5	0.9838	0.0413
6	0.9887	0.0295

TABLE XXV  
BEST CLUSTER SELECTION – UTILIZATION

Clusters	BSS ratio	Gap
1	0.0000	0.0000
2	0.8739	4.8959
3	0.9318	0.1038
4	0.9724	0.1693
5	0.9848	0.0582
6	0.9876	0.0163

Further using Hierarchical Clustering Approach (HCA) we record the best cluster [11].

TABLE XXI  
BEST CLUSTER SELECTION – VIDEO DELAY

Clusters	BSS ratio	Gap
1	0.0000	0.0000
2	0.6189	2.8412
3	0.7643	0.0923
4	0.8944	0.6411
5	0.9175	0.0931
6	0.9252	0.0460

TABLE XXII  
BEST CLUSTER SELECTION – VOICE DELAY

Clusters	BSS ratio	Gap
1	0.0000	0.0000
2	0.3576	0.0191
3	0.7121	1.3689
4	0.8384	0.4567
5	0.8886	0.1613
6	0.9118	0.0199

TABLE XXIII  
BEST CLUSTER SELECTION – QUEUING DELAY

Clusters	BSS ratio	Gap
1	0.0000	0.0000
2	0.8887	4.9762
3	0.9480	0.1606
4	0.9806	0.1365
5	0.9904	0.0445
6	0.9928	0.0006

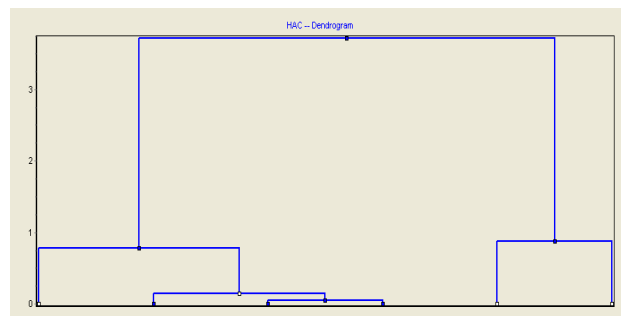


Fig 8. Dendrogram for Average End to End Delay (sec) for Video.

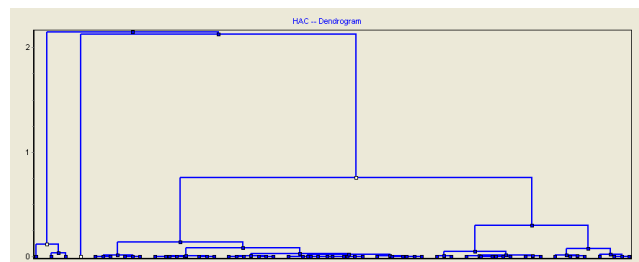


Fig 9. Dendrogram for Average End to End Delay (sec) for Voice.

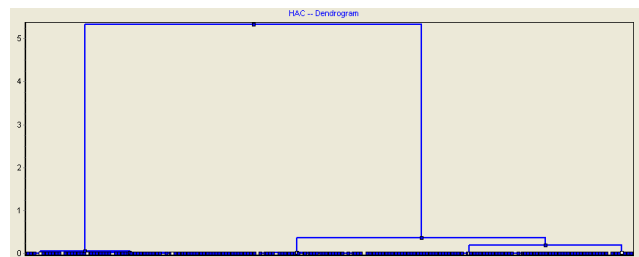


Fig 10. Dendrogram for Average Point to Point Queuing Delay.

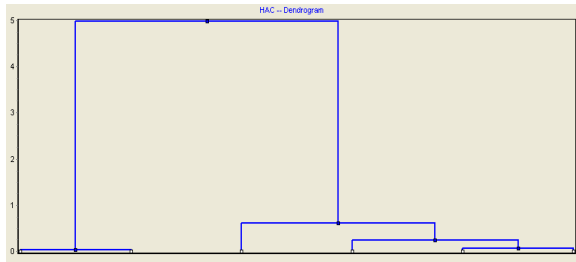


Fig 10. Dendrogram for Average Throughput.

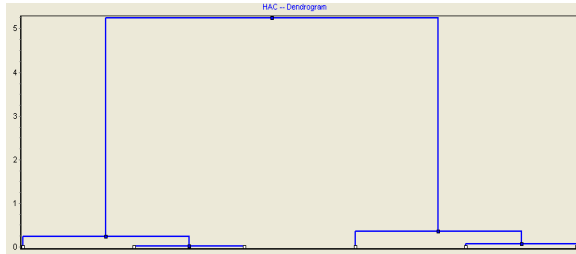


Fig 11. Dendrogram for Utilization.

## V. DISCUSSION

After reviewing univariate statistics for video conferencing, voice, queuing delay, throughput & utilization we observe that combination of PQ with RSVP gives the best result. Mean delay in case of PQ is least for video & if we combine it with RSVP we are further able to add to its performance. Further standard deviation is lowest in case of PQ. While analyzing Kohonen-SOM statistics for end to end video delay we observe number of instances for PQ/RSP cluster is the least as compared with WQ/RSP. Firewall based PQ/RSP shows better performance than Firewall based WQ/RSP strategy. In case of queuing delay observations, we come across that PQ/RSP/FW combination has marked better results than WFQ/RSP/FW combination. In case of end to end delay for video conferencing, we observe that 88.11% of total sum of squares (TSS) is encompassed over 6 classes. When we compare it using Principle Component Analysis (PCA) we encounter that PQ & WFQ accounts for 99.78% of variability [12].

From KMeans statistics for video conferencing we observe that the qualified part of the TSS comprehended by the partitioning is 95.157%. There is some discreet variation with SOM which is at 88.11%. The delay in case of PQ is the least which is closely followed by WFQ. Comparison of Kohonen-SOM & K-Means demonstrates that PQ with RSVP will be better able to capture the video delay within acceptable limits. For voice communication TSS for Kohonen-SOM is -167.13% with PCA value of 92.28% while K-Means statistics for voice stand at 91.74%. Hence for voice there is large relative prediction error & hence we are unable to state with confidence about the voice communication outcomes with PQ/RSP/FW method.

For Queuing delay the TSS for Kohonen-SOM is 93.77% with PCA value of 99.95% while K-Means statistics for same stands out at 99.10%. The variation is less when compared with video conferencing case. In case of throughput TSS value for Kohonen-SOM is 98.45% with PCA for PQ is 100%. KMeans statistics for same is 98.79%, which clearly demonstrates the agreement between clustering by Kohonen-SOM & K-Means. For utilization, Kohonen-SOM generates value of 89.60% with PCA for first factor is 100%. The KMeans statistics stand at 98.79%,

hence our analysis shows better agreement for utilization factor. K-Means cluster size & WSS statistics demonstrates that for PQ based RSVP scheme the queuing delay is 0 which clearly reflects that this scheme is efficient as compared to WFQ based RSVP.

## VI. CONCLUSION

Our analysis reflects that on the large for most of QoS parameters, PQ combination with RSVP is better able to address the requirements. We used Kohonen-SOM & K-Means clustering techniques to quantify the agreement between results for voice & video delay along with queuing delay, throughput and utilization. We used principle component analysis & hierarchical clustering approach to better visualize the expectations from different queuing disciplines along with RSVP. The experimental setup and strategy presented is competent enough to ratify the conclusion even under firewall implementations.

## REFERENCES

- [1] R. Balmer, F. Baumgartner, and T. Braun, "A concept for RSVP over diffserv," in Proc. Nineth ICCCN'2000, (Las Vegas, NV), October 2000.
- [2] T. Chahed, G. Hebuterne, and C. Fayet, "On mapping of QoS between Integrated services and Differentiated services," in International Workshop on Quality of Service, pp. 173-175, (Pittsburgh), June 5-7, 2000.
- [3] Cox, T.F., Cox, M.A.A.: *Multidimensional Scaling. Monographs on Statistics and Applied Probability*. Chapman & Hall (1995).
- [4] Jolliffe, I.T., *Principal Component Analysis*. Springer Verlag (1986).
- [5] Zanero, S., *Improving self organizing map performance for network intrusion detection*, In: SDM 2005 Workshop on "Clustering High Dimensional Data and its Applications", submitted for publication. (2004).
- [6] Yamanishi, K., Jichi Takeuchi, J., Williams, G.J., Milne, P., *On-line unsupervised outlier detection using finite mixtures with discounting learning algorithms*. In: Proc. of the 6th ACM SIGKDD Int'l Conf. on Knowledge Discovery and Data Mining, (2000) 320-324.
- [7] Bandyopadhyay S., Giannella C., Maulik U., Kargupta H., Liu K., and Datta S. *Clustering Distributed Data Streams in Peer-to-Peer Environments*. Information Sciences (in press), 2005.
- [8] Han J. and Kamber M. *Data Mining, Concepts and Techniques*. Morgan Kaufman Publishers, San Francisco, CA, 2001.
- [9] S. Haykin, "Neural Networks: A comprehensive Foundation", Prentice-Hall, USA, 2nd edit.
- [10] A. Ultsch, "Data Mining and Knowledge Discovery with Emergent SOMs for Multivariate Time Series", In Kohonen Maps, (1999), pp. 33-46.
- [11] A. Ultsch, "Maps for Visualization of High-Dimensional Data Spaces", Proc. WSOM, Kyushu, Japan, (2003), pp. 225-230.
- [12] Wolff R. and Schuster A, *Association Rule Mining in Peer-to-Peer Systems*. IEEE Transactions on Systems, Man and Cybernetics - Part B, 34(6):2426-2438, 2004.
- [13] P. Tamayo, D. Slonim, J. Mesirov, Q. Zhu, S. Kitzarewan, E. Dmitrovsky, E. Lander, and T. Golub. "Interpreting patterns of gene expression with self-organizing maps: Methods and application to hematopoietic differentiation", Proceedings of the National Academy of Science, 1999.
- [14] J. MacQueen, "Some methods for classification and analysis of multivariate observations", In L. M. Le Cam and J. Neyman, editors, Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, CA, 1967. University of California Press.
- [15] P. Demartines and J. Herault, "Curvilinear component analysis: A self-organizing neural network for nonlinear mapping of data sets", IEEE Transactions on Neural Networks, 1997.
- [16] O. Du Merle, P. Hansen, B. Jaumard, and N. Mladenovic. "An interior point algorithm for minimum sum-of-squares clustering", SIAM Journal on Scientific Computing, 2000.
- [17] A. Ultsch, "Knowledge extraction from self-organizing neural networks", In O. Opitz, B. Lausen, and R. Klar, editors, Information and Classification, pages 301-306. Springer, Berlin, 1993.