

# Minimization Time and Battery for a Straight Movement in MANET

Pattama Longani

**Abstract**—In this work we propose two problems that related to a theoretical mobility model previously proposed by Greenlaw and Kantabutra and add some definitions to make the model more complete. The first problem is the *minimum time maximum coverage* (MTMC) problem to minimize time to move a group of sources which covered a terrain to cover another terrain while all sources can still communicate through a movement duration. The second one is the *minimum battery maximum coverage* (MBMC) problem to minimize battery usage to move sources which covered a terrain to cover another terrain while all sources can still communicate through a movement duration. The best movement patterns in time consuming and battery usage for both problems are shown by techniques of proof. In addition, we focus on maximum starting and ending covered terrain in straight line fashion only.

**Index Terms**—Wireless mobile communications, ad-hoc networks, MANET algorithm, complexity.

## I. INTRODUCTION

MOBILE wireless network become parts of everyone life since it is comfortable to use anywhere. In Ad-hoc network any signal providers can dynamically communicate with each other in a group and many communication patterns are also proposed. In [5] Majeed and Mahmood apply the game theory to analysis general topological position of sources and relay nodes located in the network. In [6] Ye et al. use V2V to prove and simulate communication properties for one-dimensional vehicular data download. In [7] proposes and analyzes protocol and topology of hybrid network, cellular and ad hoc, to improve the network capacity for multicast communication. Today MEMS are interested a lot and even more useful in using with ad hoc network. In [8] Chaumette presents relationships between mobile ad hoc and large MEMS. He also shows some properties and resources related to the network such as time and service.

In some situations which one wants only a temporary network to cover a target area, the moveable wireless providers are needed. For example, the situation that a military wants to chase some terrorists who hide in a big forest and needs to communicate among all members in the team throughout the duration of the chasing time, the situation that a group of scientists want to navigate some parts of country's area while co-operate within the team, the situation that a paramotor trainer wants to communicate with his trainee team while they are in the sky, or even in the situation that a snorkel team takes the precaution of any sea danger to their buddies when they dive to discover the Titanic.

Manuscript received July 23, 2012; revised August 16, 2012. This work has financial support from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program (Grant No. PHD/0003/2551).

P. Longani is with the Department of Computer Science, Faculty of Science, Chiang Mai University, Chiang Mai, Thailand, 50200 e-mail: pattama.longani@gmail.com

Energy or battery usage is another important factor in wireless communication. Because the battery supplements can be run out, these situations can make a network down. The last longer battery power is more expensive and usually bigger in size. For these reasons, we want to save battery power as much as possible especially in the case that we want to go to the place far away from any energy filler. There are many researches relate to mobile battery in ad hoc networks. In [9] proposes algorithms to minimize battery power and life time of a specific network. In [10] proposes a new experimental algorithm to save more power than the IEEE 802.11 standard for ad hoc network.

In this work, we are interested in minimize the time consuming and the battery usage for a group of sources to keep communication throughout a duration time. We focus on maximum starting and ending covered terrain in straight line fashion and define the problems mathematically according to a theoretical mobility model proposed previously by Greenlaw and Kantabutra [1]. We remind the model and add some definitions to make Greenlaw and Kantabutra's model more complete in section II. Next we define two mathematical problems relate to the minimize time and battery power in section III and section IV accordingly. We also propose some theorems relate to minimize time consuming algorithm and battery using algorithm for both problems we proposed. Finally we give the conclusion and offer some other theoretical mobile ad-hoc network problems which can be applied in real applications for any future works.

## II. PRELIMINARIES AND DEFINITION

The theoretical mobility model relate to this work was first proposed in [1] and was defined in two-dimensional grid. This model include 8 tuples which are defined as follow:

A *mobility model*  $M$  is an eight-tuple  $(\mathcal{S}, \mathcal{D}, \mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$ , where

- 1) The set  $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$  is a finite collection of *sources*, where  $m \in \mathbb{N}$ . The value  $m$  is the *number of sources*. Corresponding to each source  $s_i$ , for  $1 \leq i \leq m$ , an *initial location*  $(x_i, y_i)$  is specified, where  $x_i, y_i \in \mathbb{N}$ .
- 2) The set  $\mathcal{D} = \{000, 001, 010, 101, 110\}$  is called the *directions*, and these values correspond to no movement, east, west, south, and north, respectively.
- 3) The set  $\mathcal{U} = \{u_1, u_2, \dots, u_p\}$  is a finite collection of *mobile devices*, where  $p \in \mathbb{N}$ . The set  $\mathcal{U}$  is called the set of *users*. The value  $p$  is called the *number of users*. Corresponding to each user  $u_i$ , for  $1 \leq i \leq p$ , an initial location  $(x_i, y_i)$  is specified, where  $x_i, y_i \in \mathbb{N}$ .
- 4) The vector  $\mathcal{L} = (l_1, l_2, \dots, l_p)$  is a finite collection of "bit strings," where  $\tau \in \mathbb{N}$  and  $l_i \in D^\tau$  for  $1 \leq i \leq p$ .

Each group of three bits in  $l_i$  beginning with the first three defines a step in a given direction for the user  $u_i$ 's movement or no movement at all if the string is 000. The value  $\tau$  is called the *duration of the model*.

- 5) Let  $t(i) \in \mathbb{N}$  for  $1 \leq i \leq m$ . The vector  $\mathcal{R} = (r_1, r_2, \dots, r_m)$  is a finite collection of "bit strings," where  $r_i \in D^{t(i)}$  for  $1 \leq i \leq m$ . Each group of three bits in  $r_i$  beginning with the first three defines a step in a given direction for the source  $s_i$ 's movement or no movement at all if the string is 000. The vector  $\mathcal{R}$  is called the *walks* of the mobility model.
- 6) The vector  $\mathcal{V} = (v_1, v_2, \dots, v_m)$  is a finite collection of numbers, where  $v_i \in \mathbb{N}$ . The value  $v_i$  is the corresponding number of steps from  $r_i$  per unit time that  $s_i$  will take. This vector is called the *velocities*.
- 7) The vector  $\mathcal{C} = (c_1, c_2, \dots, c_m)$  is a finite collection of lengths, where  $c_i \in \mathbb{N}$ . The value  $c_i$  is the corresponding diameter of the circular coverage of source  $s_i$ . This vector is called the *coverages*. Each  $c_i \leq c_{\max}$ , where  $c_{\max}$  is a constant in  $\mathbb{N}$ .
- 8) The set  $\mathcal{O} = \{o_1, o_2, \dots, o_d\}$ , where  $o_i = (x_1, y_1, x_2, y_2) \in \mathbb{N}^4$  and  $x_2 > x_1, y_2 > y_1$ , is a finite collection of rectangles in the plane. This set is called the *obstacles*. All coordinates  $x_i, y_i \leq o_{\max}$ , where  $o_{\max}$  is a constant in  $\mathbb{N}$ .

In the theoretical mobility model each source in the set  $\mathcal{S}$  provides wireless signal and can move in a different direction and speed within a time. All possible directions that one source can move in a two-dimensional grid is defined in the set  $\mathcal{D}$ . All directions and speeds for each source to move through the duration time are defined in the set  $\mathcal{R}$  and  $\mathcal{V}$  accordingly. If a source which has velocity of size two wants to move to the left at the time step  $t$ , at that time  $t$  the source will move to the left by two grid. Since each source can provide different signal strength, the coverage set  $\mathcal{C}$  is defined. In a mobile wireless communication, user in the set  $\mathcal{U}$  can move and the movement directions for each user to move through the duration time is defined in the set  $\mathcal{L}$ . This model assume that each user can move by one grid within a time only for easy. Because there are some stuffs that can block a source's signal, the set of obstacles  $\mathcal{O}$  is defined.

### A. Operational Definitions

This section show the definition of the objects, sources and obstacles, in the mobility model and its communication protocol. Because the mobility model is defined on a grid, the circle coverage for a source and the rectangular coverage for an obstacle are defined by discrete lattice points, as follow.

*Definition 2.1 (Coverage Representation):* A coverage of radius  $c$  located at a fixed grid point is represented by the set of lattice points within the coverage and on its boundary.

*Definition 2.2 (Obstacle Representation):* An obstacle in a grid is represented by the set of lattice points within the obstacle and on its boundary.

Because any two wireless signal providers can be connected when there is an intersection of their wireless signal, the the overlap coverage area is defined.

*Definition 2.3 (Overlapping Coverage Area):* Let  $s, s'$  be a coverage or an obstacle in a grid and  $s \cap s' = z$ . We say  $s$  overlaps  $s'$  if and only if  $|z| \geq 2$ . We call  $z$  an overlapping coverage area.

Turn to communication protocol of the model. Let  $k \in \mathbb{N}$  and  $k > 2$ .

- At a given instance in time any two sources with overlapping-coverage areas may communicate with each other in full-duplex fashion as long as the intersection of their overlapping-coverage areas is not completely contained inside obstacles. We say that these two sources are *currently in range*. A series  $s_1, s_2, \dots, s_k$  of sources are said to be *currently in range* if  $s_i$  and  $s_{i+1}$  are currently in range for  $1 \leq i \leq k - 1$ .
- We say that two sources  $s_1$  and  $s_2$  can *communicate at time  $t$*  if and only if the two sources  $s_1$  and  $s_2$  can communicate through a sequence of sources that are currently in range.
- Two mobile devices can not communicate directly with each other.
- A mobile device  $D_1$  always communicates with another mobile device  $D_2$  through a source or series of sources as defined next. The mobile devices  $D_1$  at location  $(x_1, y_1)$  and  $D_2$  at location  $(x_2, y_2)$  *communicate through a single source  $s$*  located at  $(x_3, y_3)$  if at a given instance in time the lines between points  $(x_1, y_1)$  and  $(x_3, y_3)$ , and points  $(x_2, y_2)$  and  $(x_3, y_3)$  are within the area of coverage of  $s$ , and do not intersect with any obstacle from  $\mathcal{O}$ . The mobile devices  $D_1$  at location  $(x_1, y_1)$  and  $D_2$  at location  $(x_2, y_2)$  *communicate through a series of sources  $s_1$*  at location  $(a_1, b_1)$ ,  $s_2$  at location  $(a_2, b_2)$ ,  $\dots$ , and  $s_k$  at location  $(a_k, b_k)$  that are currently in range if the line between points  $(x_1, y_1)$  and  $(a_1, b_1)$  is inside  $s_1$ 's coverage area and does not intersect any obstacle from  $\mathcal{O}$  and the line between points  $(x_2, y_2)$  and  $(a_k, b_k)$  is inside  $s_k$ 's coverage area and does not intersect any obstacle from  $\mathcal{O}$ .

### B. Additional Definitions

In this work, we add some definitions to the Greenlaw and Kantabutra's model to make the model more complete. Because sources can move and can have different signal strengths, If a source's signal coverage is completely inside the other source's coverage, the bigger source can manage all the works that the smaller source can do. Moreover, in any mobile wireless networks if signal providers are, for example, robots or humans, and each one carry a source, the signal providers can stay almost the same position in the grid. At that point the smart robots or people can easily share data. In addition, any users who carry the source which has lower battery can turn his source's signal off and uses the data from the other users to keep communication. We define the situation of sharing sources as follow.

*Definition 2.4 (Sharing Signals):* Let  $L_1$  be the set lattice points of source  $s_1$  at location  $x_1, y_1$  and  $L_2$  be the set of lattice points of source  $s_2$  at location  $x_2, y_2$ . The source  $s_1$  can turn its signal off and share data with the source  $s_2$  when  $L_1 \subseteq L_2, x_1 = x_2, \text{ and } y_1 = y_2$ .

Battery is another important factor that drive any mobile wireless networks to work. Battery management for each source may different and sometimes different in the source's life time. A signal provider can turn on, represented by 1, to provide service signal or can turn off, represented by 0, to save its energy. We also interest in the battery consuming, so we define the battery consuming as follow.

**Definition 2.5** (Battery Consuming): The set  $\mathcal{B} = \{b_1, b_2, \dots, b_m\}$  is a finite collection of *batteries* corresponding to each source in  $\mathcal{S}$ . Each  $b_i \in \{1, 0\}^\tau$  for  $1 \leq i \leq m$ ,  $\tau$  is the duration of the model. The sum of 1 in  $b_i \leq \mathcal{T}$ , where  $\mathcal{T}$  is called *battery life time*.

We also define the set of coverage terrain by the set of lattice points as below:

**Definition 2.6** (Connected Coverage Terrain): Coverage of terrain is the set of grid points covered by any connected sources.

Next section, we propose two problem definitions related to the time complexity of the theoretical mobility model and some theorem relate to the time minimization.

### III. MINIMUM TIME MAXIMUM COVERAGE

When a team of scientists has a mission to explore a new area which is far away and lack of any modern technologies or facilities, the small mobile signal provider tools seem to be useful. Each scientist carries a source to keep communicable among the members in the group. At the destination terrain, each member in the team move separately to cover as most terrain as possible. Still all members need to keep communicate all the time. Between moving from an area to another, the scientists can share some sources to save batteries, and they also want to move as fast as possible to save times. We defined this problem as the *general minimum time maximum coverage* (GMTMC) as follow.

#### GENERAL MINIMUM TIME MAXIMUM COVERAGE (GMTMC)

**GIVEN:** A mobility model  $M = (\mathcal{S}, \mathcal{D}, \mathcal{U} = \emptyset, \mathcal{L} = \emptyset, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$ , and a variable  $a, t \in \mathbb{N}$ .

**PROBLEM:** Can we move the  $m$  sources which lay to cover terrain  $A$  to cover another terrain  $A'$   $a$  steps away in at most  $t$  time steps while all sources can still communicate through the movement duration?

The GMTMC is quite complex because sources can be laid in any patterns, can be moved in any fashions, and can be placed in any environments. In addition, because any mobile wireless networks use movable tools in communication, it usually relates with some movement patterns and some destination terrains. Thus, we scope the problem into the case that all sources are laid to cover maximum terrain in straight line fashion, we will clarify later. The sources also need to move to cover another terrain  $a$  steps away in the same straight pattern. We assume that each source has equal velocity of size one and has equal coverage of size two. All sources can move to only one direction or no movement. The definition of the new problem, straight line fashion, and  $a$  steps away movement are defined as follow. Let  $m, i, x_i, y_i, x'_i, y'_i \in \mathbb{N}$

#### MINIMUM TIME MAXIMUM COVERAGE (MTMC)

**GIVEN:** A mobility model  $M = (\mathcal{S}, \mathcal{D} = \{000, 001\}, \mathcal{U} = \emptyset, \mathcal{L} = \emptyset, \mathcal{R}, \mathcal{V} = \{v_1 = 1, v_2 = 1, \dots, v_m = 1\}, \mathcal{C} = \{c_1 = 2, c_2 = 2, \dots, c_m = 2\}, \mathcal{O} = \emptyset)$ , and a variable  $a, t \in \mathbb{N}$ .

**PROBLEM:** Can we move the  $m$  sources which the initial location for sources  $s_1, s_2, \dots, s_m \in \mathcal{S}$  are  $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$ ,  $x_i = x_{i+1} - 1$  and  $y_i = y_{i+1}$ ,  $1 \leq i \leq m - 1$  which lay cover a terrain  $A$  to cover another terrain  $A'$  where the layout of

each source in  $\mathcal{S}' = s'_1, s'_2, \dots, s'_m$  is at the location  $(x'_1, y'_1), (x'_2, y'_2), \dots, (x'_m, y'_m)$ ,  $x'_i = x'_{i+1} - 1$  and  $\min(x'_i) = \min(x_i) - a$  and  $y'_i = y'_{i+1} = y_i$  in at most  $t$  time while all sources can communicate through the movement duration?

**Definition 3.1** (Straight Line Connected): Sources  $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$  with coverage of size two in a two-dimensional grid are laid in *straight line* if the sources' positions are  $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$ ,  $x_i = x_{i+1} - 1$  and  $y_i = y_{i+1}$  accordingly,  $1 \leq i \leq m$ .

**Definition 3.2** (Steps away): Let  $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$  and  $\mathcal{S}' = \{s'_1, s'_2, \dots, s'_m\}$  be the set of connected sources in straight line fashion at the position  $(x_i, y_i)$  and  $(x'_i, y'_i)$  accordingly,  $1 \leq i \leq m$ . The group of sources  $\mathcal{S}$  go  $a$  steps away to  $\mathcal{S}'$  in  $x$ -axis if  $\min(x'_i) = |\min(x_i) - a|$  and  $y'_i = y'_{i+1} = y_i$ .

The straight line connected for sources is the way that sources connect among each others like a line, and the  $a$  steps away is the distance that a group of sources which are connected in the straight line manner move. The Theorem 3.3 shows that the maximum coverage terrain of  $m$  sources is equal to  $3m + 2$  lattice points. Therefore, we know that the stright line connected fashion is one of the layout patterns to lay sources to cover maximum terrain.

**Theorem 3.3:** The maximum number of grid points that are covered by  $m$  sources of size two is equal to  $3m + 2$ .

**Proof: Base Case:** There is only one source. The number of lattice points of a source with coverage of size two are equal to five. The maximum coverage lattice points of one source is equal to  $(3 * 1) + 2 = 5$  points. Thus, the base case is true. **Inductive Hypothesis:** Suppose the number of lattice points to cover maximum area of  $m$  sources are equal to  $3m + 2$ . To add another source, the new source must devote two lattice points for making connection with the previous  $m$  connected sources, so the new source can cover only three more new grid points. There are two possible patterns to connect a new source to one of the sources in the previous  $m$  connected sources as shown in Figure 1. For these patterns, we can always position the new source to cover three more grid points to cover the maximum terrain. Therefore, the number of lattice points to cover the new area are equal to  $(3m + 2) + 3 = 3(m + 1) + 2$  points, which is maximum. Thus, the theorem holds. ■

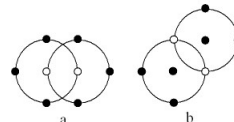


Fig. 1. Two possible patterns to connect a new source to  $m$  connected sources to cover any maximum terrains.

In MTMC we want to find minimum time consuming in moving a group of sources to a destination. the alorithm are proposed according to the Theorem 3.4.

**Theorem 3.4:** To minimize time to move  $m$  sources which connected in a straight line fashion and cover a terrain  $A$   $a$  steps away to cover another terrain  $A'$  in straight line fashion when all sources can communicate throughtout the duration time, will take  $a$  time steps and the order of the sources in moving from a terrain  $A$  to another terrain  $A'$  must be the same.

*Proof:* At first sources are placed and ordered in straight line fashion. We claim that to move the sources from  $A$  to cover another terrain  $A'$   $a$  steps away all sources must go ahead to the destination area in every time step to minimize the time consuming. The source order to cover  $A'$  must be the same order as to cover  $A$ . If the order of connected sources are not the same, some sources must stop moving to let the other sources pass. When there are any sources stop moving to the destination, this situation can not use less time than every sources move to the destination in every time step with its maximum velocity, which equal to one. Thus, the theorem holds. ■

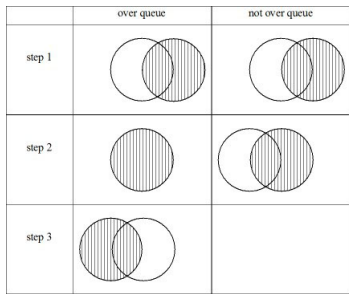


Fig. 2. The pictures on the left column show the situation that one source stop moving to let the other source pass, and the picture on the right column show the situation that all sources move to the destination in every time steps. Both picture show the sources moment from the same starting point to the same destination.

We have proposed some movement pattern definitions and focus on some sources moving time in this section. In the next section, we focus more in battery consuming.

#### IV. MINIMUM BATTERY MAXIMUM COVERAGE

Sources must have some kinds of powers to provide communication signal. In mobile wireless network batteries are usually attached to each source with a limit of the battery life time. Sometimes batteries are allowed to be recharged to recover its signal, but there are some situations which battery rechargers are rare or even do not have any, so an efficient battery usage patterns are very important to keep a communication network can work as long as possible. We define the problem as follow.

##### GENERAL MINIMUM BATTERY MAXIMUM COVERAGE (GMBMC)

GIVEN: A mobility model  $M = (\mathcal{S}, \mathcal{D}, \mathcal{U} = \emptyset, \mathcal{L} = \emptyset, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$ , and a variable  $a, b \in \mathbb{N}$ .

PROBLEM: Can we move  $m$  sources which lay to cover a terrain  $A$  to cover another terrain  $A'$   $a$  steps away which use at most  $b$  battery units while all sources can still communicate through the movement duration?

The GMBMC is quite complex, so we scope the battery usage problem into the case that all sources are connected over a straight line fashion, have velocity of size one, and have coverage of size two. All sources can move to only one direction or no movement in each step of time. Sources also suppose to cover maximum destination area  $a$  steps away. We define the battery usage patterns for this situation as follow:

##### MINIMUM BATTERY MAXIMUM COVERAGE (MBMC)

GIVEN: A mobility model  $M = (\mathcal{S}, \mathcal{D} = \{000, 001\}, \mathcal{U} = \emptyset, \mathcal{L} = \emptyset, \mathcal{R}, \mathcal{V} = 1, \mathcal{C} = 2, \mathcal{O} = \emptyset)$ , and a variable  $a, b \in \mathbb{N}$ .

PROBLEM: Can we move  $m$  sources  $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$  with the initial location  $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$ ,  $x_i = x_{i+1} - 1$  and  $y_i = y_{i+1}$ ,  $1 \leq i \leq m$  which cover a terrain  $A$  to cover another terrain  $A'$  where the layout of each sources in  $\mathcal{S}' = \{s'_1, s'_2, \dots, s'_m\}$  are at the location  $(x'_1, y'_1), (x'_2, y'_2), \dots, (x'_m, y'_m)$ ,  $x'_i = x'_{i+1} - 1$  and  $y'_i = y'_{i+1} = y_i$  and  $x'_i - a \leq x_j$  which use at most  $b$  battery units while all sources can still communicate through the movement duration?

For the straight line movement, a group of sources can move from one place to another in any patterns. Because any sources at the same location can share battery usage, we can classify the movement patterns into two ways. One is the way that sources do not share batteries with any others. For this case each source will consume one battery unit within a time step. Another case is the situation that a source can share battery with the others. Gathering sources to share batteries will waste a time step to let a source to go overlapping with another one, and also waste the time to separate the sources according to the aim to cover maximum area at the destination. In this part we prove that if the destination area distance is equal to  $a$  steps away, when  $a \leq m$ ,  $m$  is the number of sources, to gather any sources to share the batteries is useless. We also show the case that  $a > m$ , to gather all the sources to share only one batteries, move and extended all the sources at the destination can save the maximum battery. However, one may wonder whether to gather only some sources to share battery, move and separate the gathered sources at the destination can use less time, unluckily we show the prove that to gather some of any number of sources is not save battery better than to gather all sources when  $a > m$ .

*Theorem 4.1:* Minimum battery usage for MBMC is equal to  $a(a+1)$  if  $a \leq m$  and  $m^2 + a$  if  $a > m$ .

*Proof:* For the shortest time movement as the Theorem 3.4 state, if  $m$  sources want to move to cover another terrain  $a$  steps away, each source will use  $a+1$  battery units and all sources battery usage is equal to  $m(a+1)$  units. However, the most battery saving situation for  $m$  sources is to gather all sources to be shared and use only one battery unit, move the gathered sources, and separate all the sources to cover maximum straight terrain at the destination. At the starting time every sources are turned on, so all of the batteries usage are equal to  $m$ . To collect sources, let  $i, j, c \in \mathbb{N}$  and the source  $s_c$  is the source that is going to be combined with the source nearby. Between gathering the source  $s_c$  to the previous source  $s_{c-1}$ , the sources  $s_i$ ,  $i < c$ , have to stop moving and the sources  $s_j$ ,  $c \leq j$  have to move one step to the destination. After this step, only one battery unit can be saved because there are only two new sources overlapped. Notice that we can overlap only one new source within a time, or all the sources will uncommunicable. We collect one new source in each time step until all sources are overlapped, that will use  $m$  steps of time. For all battery usage, at the starting time all the sources are turn on, so the battery usage is equal to  $m$  units, and each gathering time the battery will be saved one unit. Therefore, all batteries usage form starting time until all sources are overlapped are equal to  $m + (m - 1) + (m - 2) + \dots + 1 = \frac{m(m+1)}{2}$  units. After gathering all the sources, we move all sources to the destination and separat

all the overlapping sources to cover the maximum coverage area at the destination. To move the all overlapping sources to the destination will use only one battery unit within a time and the distance before separating sources will equal to  $a - (m - 1) = a - m + 1$  steps, so the battery usage when all overlapped source are moving are equal to  $a - m + 1$  battery units. To separate all the sources to cover the maximum destination terrain, we can separate only one sources with in a time, that mean to increase one battery usage within a time. Therefore, to separate all the sources in  $m - 1$  time steps, the battery usage are equal  $2 + 3 + \dots + m = \left(\frac{m(m+1)}{2}\right) - 1$  units. In this method all battery usage are equal to  $\frac{m(m+1)}{2} + a - m + 1 + \left(\frac{m(m+1)}{2}\right) - 1 = m^2 + a$  battery units.

Since to gather any two sources for sharing battery will waste two steps of time, to gather the sources and to separate the sources to cover maximum destination terrain, but can save only one battery usage. Therefore, we should not gather any sources unless the gathering can save more battery units. That means we will gather and separate any sources when  $m^2 + a > m(1 + a)$ , that is when  $a > m$ . Thus, the Theorem holds. ■

Because gathering any sources can save battery units, but also waste the times. We may wonder whether to gather only some sources can save the battery units more than combine all sources to share only one battery units before move all the sources to the destination or not, because it should be better in the movement time. However, the theorem4.2 shows that it is not useful if the destination target  $a$  steps away is equal to the number of source  $m$ .

**Theorem 4.2:** When the destination distance,  $a$ , is equal to the number of sources,  $m$ , to gather any sources for battery saving is use the same battery units as non gathering one and use  $m(m + 1)$  battery units.

*Proof:* Let  $c$  be the number of sources to be gathered for saving batteries through out the movement duration. To gather  $c$  sources, the sources must stop for  $c$  time steps and also need times to extend the sources to cover the destination terrain which is also equal to  $c$  time steps. Each time of gathering a source, one unit of battery can be saved. Therefore, to gather  $c$  sources, the batteries will be save  $\sum_{i=1}^c i = \frac{c(c+1)}{2}$  units. In separating sources also save the same amount of battery units to be saved. When the sources are moving to the destination  $a$  steps away, if there are any  $c$  collecting sources, we must waste  $c$  steps of time to gather the sources and we will use  $m + 1 + c$  steps to move to the destination. Because the time to gather and separate the sources are equal to  $c + c = 2c$ , the time duration which the sources move are equal to  $(m + 1 + c) - (2c) = m - c + 1$ . Battery units that are saved when the  $c$  sources are collect for sharing batteries are equal to  $c$  units, so when the  $c$  collected sources move to the destination will save  $c(m - c + 1)$  battery units. Thus, all battery usages are equal to  $m(m + 1 + c) - 2 \sum_{i=1}^c i - c(m - c + 1) = m(m + 1)$  units which is equal to the battery usage when there are no any collecting sources. Thus, when the moving distance to the destination is equal to the number of sources,  $a = m$ , to collect or not to collect any number of sources are use the same battery units which equal to  $m(m + 1)$ . ■

From the Theorem 4.1 and 4.2, we can conclude that if any sources want to move to the destination  $a < m$  steps away, for efficiency movement each source should move to the same direction one step at each time with out collecting any sources for sharing batteries. In case that  $a \geq m$ , the collecting is need. All sources must be gathered to share only one battery unit before start moving to the destination. To gather sources to share battery unit other than one is useless when we want to save battery or time, according to the Theorem 4.2. We show the graph of battery consuming when there are collecting and no collecting the sources in Figure 3 and Figure 4 accordingly. We also show comparison of battery consuming for each time step for five sources when there are collecting sources and no collecting sources in Figure 5.

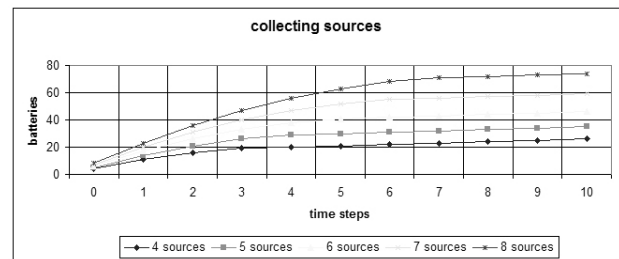


Fig. 3. Battery consuming for each time step for four to eight sources when the sources are collected to share one battery before move to the destination.

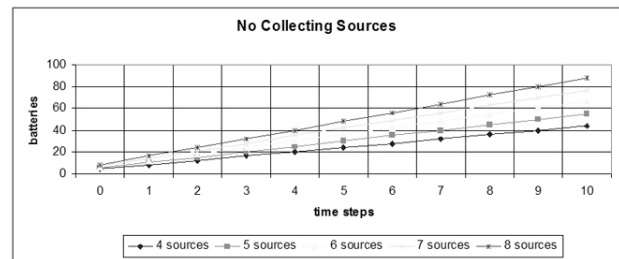


Fig. 4. Battery consuming for each time step for four to eight sources when there are no collecting of any sources before all the sources move to the destination.

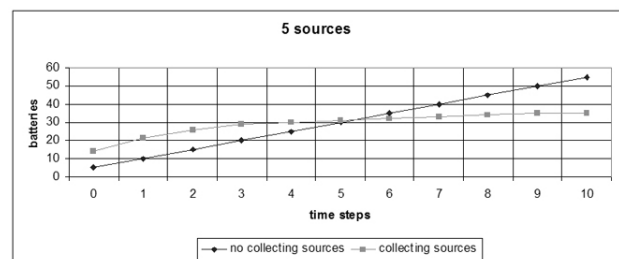


Fig. 5. Battery consuming compare for each time step of five sources when there are collecting sources and no collecting sources.

## V. CONCLUSION

In this work, we propose two problems relate to a mobility model which was previously propose theoretically by Greenlaw and Kantabutra. We also add some definitions to make the model more complete. The first problem is the MTMC to minimize time to move sources which cover a

terrain to cover another terrain while all sources can still communicate through the movement duration. Another one is the MBMC to minimize battery usage to move sources which cover a terrain to cover another terrain while all sources can still communicate through the movement duration. For both problems, we prove some properties and show some algorithms to minimize the time consuming and the battery using which can be applied in real applications.

There are many problems related to the mobile wireless network proposed in [1], [2], [3], [4]. For example, to find some moving patterns of sources to cover maximum area, to remove as least obstacles which block the wireless signal as possible but all users can still communicate, to find an approximation algorithm to make all pairs of interested user can communicate without sharing any sources, and to find minimum number of sources to make any two users can communicate throughout a duration time. These problems are still left open and want to be solved and to be known its complexity.

#### ACKNOWLEDGMENT

Special thanks to Professor Vites Longani, Professor Raymond Greenlaw and Professor Sanpawat Kantabutra for many useful comments and suggestions.

#### REFERENCES

- [1] R. Greenlaw and S. Kantabutra, *A Mobility Model for Studying Wireless Communication*, Proceedings of the 15th International Conference of Forum for Interdisciplinary Mathematics on Interdisciplinary Mathematical and Statistical Techniques (IMST), Shanghai, P. R. China, May 20-23, 2007.
- [2] P. Longani and S. Kantabutra, *Time-Optimal User Communication and Source Reachability Algorithms in a Two-Dimensional Grid Wireless Mobility Model*, Proceedings of The IEEE International Conference in Electrical Engineering/Electronics, Computer, Telecommunications, and Information Technology (ECTI-CON 2008), May 14-17, 2008, Krabi, Thailand.
- [3] S. Kantabutra and P. Longani, *The Complexity of the Grid Wireless Mobility Model*, Proceedings of The IEEE/ACIS International Conference on Software Engineering, Artificial Intelligence, Networking, and Parallel/Distributed Computing (SNPD2008), August 6-8, 2008, Phuket, Thailand.
- [4] R. Greenlaw, S. Kantabutra, and P. Longani, *A Mobility Model for Studying Wireless Communication and the Complexity of Problems in the Model*, Networks, Wiley, New Jersey, USA. In Press.
- [5] S. J. Majeed and H. Mahmood, *Topological Importance as an Incentive to Cooperate in Mobile Ad Hoc Networks: A Game Theoretic Analysis*, Proceedings of The 14<sup>th</sup> International Conference on the Advanced Communications Technology (ICAC2012), February 19-22, 2012, PyeongChang, South Korea.
- [6] F. Ye, S. Roy, and H. Wang, *Efficient Data Dissemination in Vehicular Ad Hoc Networks*, Selected Areas in Communications, IEEE Journal, Volume:30, Issue: 4, Page(s): 769-779, May 2012.
- [7] X. Chen, W. Huang, X. Wang, and X. Lin, *Multicast Capacity in Mobile Wireless Ad Hoc Network with Infrastructure Support*, Proceedings of The 31st Annual IEEE International Conference on Computer Communications (IEEE INFOCOM 2012), Florida, USA, March 25-30, 2012.
- [8] S. Chaumette, *Can highly dynamic mobile ad hoc networks and distributed MEMS share algorithmic foundations*, Proceedings of The Second Workshop on Design, Control and Software Implementation for Distributed MEMS (dMEMS), Besanon, France, April 2-3, 2012.
- [9] P. Gupta, P. Saxena, A.K. Ramani, R. Mittal, *Optimized Use of Battery Power in Wireless Ad hoc Networks*, Proceedings of The 12th International Conference on Advanced Communication Technology (ICACT 2010), Gangwon-Do, South Korea, February 7-10, 2010.
- [10] A. K. Sharma, A. Gupta, A. K. Misra, *Optimized Power Saving Mechanism for Wireless Ad Hoc Networks*, Proceedings of The 1st International Conference on Recent Advances in Information Technology (RAIT2012), India, March 15-17, 2012.
- [11] C. Ma and Y. Yang, *A Battery-Aware Scheme for Routing in Wireless Ad Hoc Networks*, Vehicular Technology, IEEE Transactions, Volume: 60, Issue: 8, Page(s): 3919-3932, Oct, 2011.
- [12] C. Priyadharshini, K. Thamarai Rubini, *Predicting Route Lifetime for Maximizing Network Lifetime in MANET*, Proceedings of The International Conference on Computing, Electronics and Electrical Technologies [ICCEET], Tamilnadu, India, March 21-22, 2012.