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Modeling of Wireless Networks as Queuing System

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***Abstract*—This paper presents the study of special Erlang distribution model in wireless networks and mobile computing. We demonstrate that the Erlang family provides more flexibility in modeling that exponential family, which only has one parameter. In practical situations, the Erlang family provides more flexibility in fitting a distribution to real data that the exponential family provides. The Erlang distribution is also useful in queueing analysis because of its relationship to the exponential distribution. To demonstrate the applicability of the Erlang distribution, we consider queueing model, represented as wireless channel where the interarrival times between failures have the Erlang Distribution.**

***Index Terms*—Erlang distribution, interarrival time between failures, probabilistic approach, queueing model.**

I. Introduction

An accurate estimation of network performance is vital for the success of a network of any kind. Networks, whether voice or data, are designed around many different variables. Two of the most important factors that you need to consider in network design are service and cost. Service is essential for maintaining customer satisfaction. Cost is always a factor in maintaining profitability. One way that you can factor in some of the service and cost elements in network design is to optimize circuit utilization.

Also to a large extent, the success of a network depends on the development of effective congestion control techniques that allow for optimal utilization of a network's capacity. Performance modeling is necessary for deciding the type of congestion control policies to be implemented. Performance models in turn, require very accurate traffic models that have the ability to capture the statistical characteristics of the actual traffic on the network.

The design of robust and reliable networks and network services is becoming increasingly difficult in today's world. The only path to achieve this goal is to develop a detailed understanding of the traffic characteristics of the network.

Managing performance of networks involves optimizing the way networks function in an effort to maximize capacity, minimize latency and offer high reliability regardless of bandwidth available and occurrence of failures. Network performance management consists of tasks like measuring, modeling, planning and optimizing networks to ensure that they carry traffic with the speed, capacity and reliability that is expected by the applications using the network or required in a particular scenario.

Networks are of different types and can be categorized based on several factors. However, the factors that affect the performance of the different networks are more or less the same.

If the underlying traffic models do not efficiently capture the characteristics of the actual traffic, the result may be the under-estimation or over-estimation of the performance of the network. This would totally impair the design of the network. Traffic Models are hence, a core component of any the performance evaluation of networks and they need to be very accurate. Depending upon the type of network and the characteristics of the traffic on the network, a traffic model can be chosen for modeling the traffic.

II. Traffic Models and Erlang distribution

Special Erlang Distribution lends itself well to modeling packet interarrival time for a number of reasons. The first is the fact that The Erlang distribution is a continuous probability distribution with wide applicability primarily due to its relation to the exponential and Gamma distributions. The exponential function is a strictly decreasing function of t. This means that after an arrival has occurred, the amount of waiting time until the next arrival is more likely to be small than large.

To begin modeling we define T as fixed length of packet. Simple traffic consists of single arrivals of discrete entities, packets. This kind of traffic can be expressed mathematically as a Point Process. Point processes can be described as a Counting Process or Inter-Arrival Time (IAT) Process.

We also assume that T is the length of the messages, let n be the number of packets in the message; we define l to be a number of phases in the Erlang distribution of failures, i.e. there is the scheme of failures arrival, according to which the failures must go through l phases (stages), before they actually will arrive;

$F(u)=1(t-\tau_b)$ is Distribution function (DF) of packet length with a cyclic check redundancy (CRC), where 1(t) – unit function, and τ_b=T/n block length; r- the number of allowed repetitions of transmitting a block, G(u)- Distribution function of recovery time; α- the distribution intensity of each phase, i.e. the duration of time intervals between subsequent moments of occurrence of failure follow the Erlang Distribution given by :

$$A(u)=\frac{\alpha(\alpha u)^{l-1}e^{-\alpha u}}{(l-1)!}$$

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Our task is to find Distribution Function of transfer time of fixed length message depending on number of the packets included in it and number of their retransmissions for a given network characteristics.

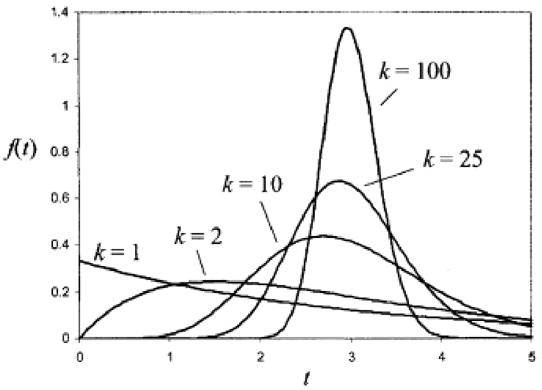

Figure 1. A family of Erlang distributions with mean 3

We denote by $\Phi_j^{(k\nu)}(t,x,T)$ the probability that the message transfer of fixed length of T (consisting of n packets, each of length of τ_b) starting from j-th block will be completed in a time less than t under the condition that: 1) at the t=0 moment data channel (DCH) is in K phase according to failures and x-th part ($x\in[0,\tau_б]$) of j-th block has been transferred without distortion; 2) An attempt was made to transfer the j-the block without distortion ν times.

By definition:

$$\Phi_j^{(k\nu)}(t,x,T)=\begin{cases}\hat{O}_{j+1}^{(k\nu)}(t,0,T), & \text{under } x=\tau_b\\ 0 & \text{under } x>\tau_b\end{cases}$$

$k=\overline{1,l}$, $\nu=\overline{1,r}$, $j=\overline{1,n}$;

We denote by Φ(t,T) the DF of the probability of transfer time of fixed length message ($T=n\tau_б$) and by $\widetilde{F}(u)$ Df of random length message, then we have the following notation:

$$\Phi(t)=\int_0^\infty \Phi(t,u)d\widetilde{F}(u)$$

where

$$\Phi(t,u)=\sum_{\nu=1}^{r}\sum_{k=1}^{l}\Phi_1^{(k\nu)}(t,0)/l$$

is DF od transfer time of fixed length message of u

The model mentioned above can be described by the following system of equations:

$$\bar{F}(x)\Phi_j^{(k\nu)}(t,x)=\int_0^t e^{-\alpha u}\Phi_{j+1}^{k1}(t-u,0,T)F(x+u)du+$$

$$+\int_0^t \alpha e^{-\alpha u}\bar{F}(x+u)\Phi_j^{(k+1,\nu)}(t-u,x+u)du\ , \qquad (2.4.1)$$

$j=\overline{1,n}$, $k=\overline{1,l-1}$, $\nu=\overline{1,r}$;

$$\Psi_j^{(l\nu)}(t,x)=\int_0^t e^{-\alpha u}\Phi_{j+1}^{(l1)}(t-u,0)d_uF(x+u)+$$

$$\sum_{k=1}^{l}\sum_{p=0}^{\infty}\int_0^t \alpha e^{-\alpha u}du\int_0^{t-u}d_\upsilon F(x+u+\upsilon)\left[B_*^{(pl+k-1)}(\upsilon)+B_*^{(pl+k)}(\upsilon)\right]\times$$

$$\times\Phi_j^{(k,\nu+1)}(t-u-\upsilon,0),\ (2.4.2)$$

$\nu=\overline{1,r-1}$; $j=\overline{1,n}$;

$$\Psi_j^{(lr)}(t,x)=\int_0^t e^{-\alpha u}\Phi_{j+1}^{(l1)}(t-u,0)d_uF(x+u)+$$

$$+\int_0^t\left(1-e^{-\alpha u}\right)d_uF(x+u)\int_0^{t-u}\Phi_j^{(1,1)}(t-u-\upsilon,0)dG(\upsilon) \qquad (2.4.3)$$

$j=\overline{1,n}$

where $B_*^{(k)}(\upsilon)$ - k-fold convolution and $B(\upsilon)=1-e^{-\alpha\upsilon}$;

$\psi_j^{(k\nu)}(t,x)=\bar{F}(x)\Phi_j^{(k\nu)}(t,x,T)$; $\bar{F}(x)=1-F(x)$; $F\left(\tau_b^+\right)=1$; $F\left(\tau_b^-\right)=0$

Let us assume

$$\psi_j^{(k\nu)}(t,x)=\begin{cases}\Phi_j^{(k\nu)}(t,0), & x=0\\ \Phi_{j+1}^{(k\nu)}(t,0), & x=\tau_b\\ 0, & x>\tau_b\end{cases}$$

$k=\overline{1,l}$, $\nu=\overline{1,r}$, $j=\overline{1,n}$;

The boundary conditions have the form:

$\Psi_{n+1}^{(k\nu)}(t,x)=\Phi_{n+1}^{(k\nu)}(t,0)=1$, $k=\overline{1,l}$, $\nu=\overline{1,r}$, $j=\overline{1,n}$;

By Integrating the product of the probabilities of events according to u, υ and summing the probabilities of incompatible events, we obtain (2.4.2).

Using the Laplace Transform in (2.4.2), we obtain:

$$\int_0^\infty e^{-st}dt\int_0^t e^{-\alpha u}d_uF(x+u)\hat{O}_{j+1}^{(k1)}(t-u,0)=$$

$$\int_0^\infty e^{-\alpha u}d_uF(x+u)\int_u^\infty e^{-st}\hat{O}_{j+1}^{(k1)}(t-u,0)dt=$$

$$=\int_0^\infty e^{-\alpha u}d_uF(x+u)\int_0^\infty e^{-s(z+u)}\hat{O}_{j+1}^{(k1)}(z,0)dZ=$$

$$\hat{O}_{j+1}^{(k1)}(s,0)\int_0^\infty e^{-(\alpha+s)u}d_uF(x+u)=$$

$$=e^{-(\alpha+s)(\tau_b-x)}\hat{O}_{j+1}^{(k1)}(s,0)$$

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x+u=τ_6; t-u=z; t=z+u.

$$\alpha\int_0^\infty e^{-st}dt\int_0^\infty e^{-\alpha u}\psi_j^{(k+1,\nu)}(t-u,x+u)du =$$

$$\alpha$$

$$\int_0^\infty e^{-\alpha u}du\int_0^\infty e^{-st}\psi_j^{(k+1,\nu)}(t-u,x+u)dt =$$

$$=\alpha\int_0^\infty e^{-\alpha u}du\int_0^\infty e^{-s(u+z)}\psi_j^{(k+1,\nu)}(z,x+u)dZ =$$

$$=\alpha\int_0^\infty e^{-(s+\alpha)u}du\int_0^\infty e^{-sz}\psi_j^{(k+1,\nu)}(z,x+u)dZ =$$

$$=\alpha\int_0^\infty e^{-(s+\alpha)u}d_u\psi_j^{(k+1,\nu)}(s,x+u) =$$

$$\alpha\int_x^\infty e^{-(s+\alpha)(\tau-x)}\psi_j^{(k+1,\nu)}(s,\tau)d\tau =$$

$$=\alpha e^{-(s+\alpha)x}\int_x^\infty e^{-(s+\alpha)\tau}\psi_j^{(k+1,\nu)}(s,\tau)d\tau,$$

$$t-u=z$$

Denoting by:

$$\overline{\Psi}_j^{(k\nu)}(s,x)=\int_0^\infty e^{-st}\psi_j^{(k\nu)}(t,x)dt\,;$$

$$\overline{\Phi}_j^{(k\nu)}(s,x)=\int_0^\infty e^{-st}\Phi_j^{(k\nu)}(t,x)dx\,;$$

We obtain:

$$\psi_j^{(K\nu)}(s,x)=e^{-(\alpha+s)(\tau_b-x)}\hat{O}_{j+1}^{(k1)}(s,0)+$$

$$+\alpha e^{-(\alpha+s)x}\int_x^\infty e^{-(\alpha+s)\tau}\psi_j^{(k+1,\nu)}(s,\tau)d\tau$$

or

$$e^{(s+\alpha)x}\psi_j^{(k\nu)}(s,x)=e^{-(\alpha+s)\tau_b}\hat{O}_{j+1}^{(k1)}(s,0)+ \qquad (2.4.4)$$

$$+\alpha\int_x^{\tau_b} e^{-(\alpha+s)\tau_b}\psi_j^{(k+1,\nu)}(s,\tau)d\tau$$

Moving on to differential equation (2.1.4), we obtain:

$$\frac{d\overline{\Psi}_j^{(k\nu)}(s,x)}{dx}-(s+\alpha)\overline{\Psi}_j^{(k,\nu)}(s,x)+\alpha\overline{\Psi}_j^{(k+1,\nu)}(s,x)=0 \qquad (2.4.5)$$

$$\overline{\Psi}_j^{(k\nu)}(s,0)=\overline{\Phi}_j^{(k\nu)}(s,0),\quad j=\overline{1,n}\,,\quad k=\overline{1,l-1}\,,\quad \nu=\overline{1,r}$$

$\psi_j^{(k\nu)}(s,x)$ - monotonic and continuous function according to x in the range

$$0^+\le x\le \tau_b^-,$$

$$\psi_j^{(k\nu)}(s,x)=0, \qquad \text{under}\quad x\ge\tau_b^+$$

We apply the Laplace transform to (2.4.2). For this purpose, first of all, let u+υ=y; dυ=dy. We rewrite (2.4.2) as follows:

$$\psi_j^{(l\nu)}(t,x)=\int_0^\infty d_uF(x+u)e^{-\alpha u}\hat{O}_{j+1}^{(l1)}(t-u,0)+$$

$$+\sum_{k=1}^{l}\sum_{p=0}^{\infty}\alpha e^{-\alpha u}du\int_u^t \begin{matrix} d_yF(x+y)[B_*^{(pl+k-1)}(y-u)- \\ -B_*^{(pl+k)}(y-u)]\hat{O}_j^{(k,\nu+1)}(t-y,0)\end{matrix}\;;$$

and changing the order of integration in the second member, we get:

$$\psi_j^{(l\nu)}(t,x)=\int_0^t e^{-\alpha u}\hat{O}_{j+1}^{(l1)}(t-u,0)du\mathrm{F}(x+u)+$$

$$+\sum_{k=1}^{l}\sum_{p=0}^{\infty}\alpha\hat{O}_j^{(k,\nu+1)}(s,0)\int_0^\infty e^{-sy}d_yF(x+y)\int_0^y e^{-\alpha u}x$$

$$x\left[B_*^{(pl+k-1)}(y-u)-B_*^{(pl+k)}(y-u)\right]$$

Applying the Laplace-Stieltjes, we get:

$$\overline{\psi}_j^{(l\nu)}(s,x)=e^{-(\alpha+s)(\tau_b-x)}\overline{\hat{O}}_{j+1}^{(l1)}(s,0)+$$

$$+\sum_{k=1}^{l}\sum_{p=0}^{\infty}\alpha\overline{\hat{O}}_j^{(k,\nu+1)}(s,0)\int_0^\infty e^{-sy}d_yF(x+y)\times$$

$$\times\int_0^y e^{-\alpha u}\left[B_*^{(pl+k-1)}(y-u)-B_*^{(pl+k)}(y-u)\right]du$$

If we denote:

$$\varphi_1(s)=\int_0^\infty e^{-sy}d_yF(x+u)=e^{-s\left(\tau_b-x\right)}$$

$$\varphi_2(s)=\int_0^\infty e^{-sy}dy\int_0^y\left[B_*^{(pl+k-1)}(y-u)-\right.$$

$$\left.-B_*^{(pl+k)}(y-u)\right]e^{-\alpha u}du==\frac{1}{(s+\alpha)^2}\left(\frac{\alpha}{s+\alpha}\right)^{pl+k-1}$$

Thus, we have:

$$\overline{\Psi}_j^{(l\nu)}(s,x)=e^{-(\alpha+s)\left(\tau_b-x\right)}\Phi_{j+1}^{(l1)}(s,0)+$$

$$+\sum_{k=1}^{l}\sum_{p=0}^{\infty}\alpha\overline{\Phi}_j^{(k,\nu+1)}(s,0)\times\left[\frac{1}{2\pi i}\int_{C^*-i\infty}^{C^*+i\infty}\varphi_1(s-w)\varphi_2(w)dw\right] \qquad (2.4.7)$$

where C * is the abscissa of convergence of the improper integral, which lies in the domain of analyticity of under the integral sign function; $i=\sqrt{-1}$

or

$$\overline{\Psi}_j^{(l\nu)}(s,x)=e^{-(\alpha+s)\left(\tau_b-x\right)}\Phi_{j+1}^{(l1)}(s,0)+$$

$$+\sum_{k}^{l-1}\Phi_j^{(k,\nu+1)}(s,0)\alpha^k\left[\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}e^{-(s-w)(\tau_b-x)}\frac{(w+\alpha)^{l-1-k}}{(w+\alpha)^l-\alpha^l}dW\right]+$$

$$+\Phi_j^{(l,\nu+1)}(s,0)\alpha^l\left\{\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}e^{-(s-w)(\tau_b-x)}\frac{dW}{(W+\alpha)\left[(w+\alpha)^l-\alpha^l\right]}\right\} \qquad (2.4.8)$$

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Applying the Laplace-Stieltjes to (2.1.3), we obtain:

$$\overline{\Psi}_j^{(l,r)}(s,x) = e^{-(\alpha+s)(\tau_b - x)}\overline{\Phi}_{j+1}^{(l1)}(s,0) + g(s)\overline{\Phi}_j^{(1,1)}(s,0)\left(e^{-s(\tau_b - x)} - e^{-(\alpha+s)(\tau_b - x)}\right); \quad (2.4.9)$$

$$j = \overline{1,n}$$

(2.4.8) and (2.4.9) contin [(l-1)r+r][j]=lrj - equations, a number of unknowns is equal to 2jer,

$$\tilde{\psi}_j^{(k\nu)}(s,\tau_b) = \Phi_j^{(k\nu)}(s,0);$$
$[\, j = \overline{1,n};\ k = \overline{1,l-1}\,]$.

By the method of deductions, we calculate (2.4.8):

$$\overline{\Psi}_j^{(l\nu)}(s,x) = e^{-(\alpha+s)(\tau_b - x)}\overline{\Phi}_{j+1}^{(l1)}(s,0) + \sum_{k=1}^{l-1}\left[\overline{\Phi}_j^{(k,\nu+1)}(s,0)\alpha^k + \sum_{p=0}^{l} e^{-(s-w_p)(\tau_b - x)} \left(w_p+\alpha\right)^{l-1-k} \Big/ \left(\prod_{\eta=1,\eta\neq p}^{l}\left(w_p - w_\eta\right)\right)\right] + \left[\overline{\Phi}_j^{(l,\nu+1)}(s,0)\alpha^l \sum_{p=0}^{l} e^{-(s-w_p)(\tau_b - x)} \Big/ \left(\prod_{\substack{\eta=1\\ \eta\neq p}}^{l}\left(w_p - w_\eta\right)\right)\right] \quad (2.4.10)$$

where

$w_d = \alpha\left(e^{2\pi di/l} - 1\right)$, $d = \overline{1,l}$ -the zeros of the equation.

$(w+\alpha)^l - \alpha^l = 0$, a $W_0 = -\alpha$;

To simplify further calculations, we introduce a new variable $y = \tau_b - x$ (y-time remaining till the end of the block transfer). By substituting new variables mentioned above in the equations (2.4.5), (2.4.9) and (2.4.10) then they have the following form

$$\frac{d\psi_j^{(k\nu)}(s,\tau_b - y)}{dy}\frac{dy}{dx} - (s+\alpha)\psi_j^{(k\nu)}(s,\tau_b - y) + \alpha\psi_j^{(k+1,\nu)}(s,\tau_b - y) = 0$$

but taking into account that:

$$\psi_j^{(k\nu)}(t,x) = \psi_j^{(k\nu)}(t,\tau_b - y) = \tilde{\psi}_j^{(k\nu)}(t,y) = \overline{\psi}_j^{(k\nu)}(s,y)$$

We have:

$$\frac{d\tilde{\psi}_j^{(k\nu)}(s,y)}{dy} + (s+\alpha)\tilde{\psi}_j^{(k\nu)}(s,y) - \alpha\tilde{\psi}_j^{(k+1,\nu)}(s,y) = 0 \quad (2.4.11)$$

$$\psi_j^{(k\nu)}(t,0) = \tilde{\psi}_j^{(k\nu)}(t,\tau_b) = \Phi_{j+1}^{(k\nu)}(t,0) = 1; \quad t \geq 0.$$

$$\tilde{\psi}_j^{(k\nu)}(s,0) = \psi_{j+1}^{(k\nu)}(s,\tau_b) = \Phi_{j+1}^{(k\nu)}(s,0).$$

$$\overline{\Psi}_j^{(l\nu)}(s,y) = e^{-(\alpha+s)y}\overline{\Phi}_{j+1}^{(l1)}(s,0) + \sum_{k=1}^{l-1}\left[\overline{\Phi}_j^{(k,\nu+1)}(s,0)\alpha^k \sum_{p=0}^{l} e^{-(s-w_p)y}\left(w_p+\alpha\right)^{l-1-k} \Big/ \left(\prod_{\eta=1,\eta\neq p}^{l}\left(w_p - w_\eta\right)\right)\right] + \left[\overline{\Phi}_j^{(l,\nu+1)}(s,0)\alpha^l \sum_{p=0}^{l} e^{-(s-w_p)y} \Big/ \left(\prod_{\substack{\eta=1\\ \eta\neq p}}^{l}\left(w_p - w_\eta\right)\right)\right] \quad (2.4.12)$$

$$\tilde{\overline{\Psi}}_j^{(l,r)}(s,y) = e^{-(\alpha+s)y}\overline{\Phi}_{j+1}^{(l1)}(s,0) + g(s)e^{-sy}\left(1 - e^{-\alpha y}\right)\overline{\Phi}_j^{(1,1)}(s,0); \quad (2.4.13)$$

Applying the Laplace transform of the argument y (respectively, operator ω) к (2.4.11),(2.4.12) и (2.4.13), we obtain:

$$(\omega + s + \alpha)\tilde{\tilde{\psi}}_j^{(k\nu)}(s,\omega) = \overline{\Phi}_{j+1}^{(k\nu)}(s,0) + \alpha\tilde{\tilde{\Psi}}_j^{(k+1,\nu)}(s,\omega),$$

(2.4.14)

$$j = \overline{1,n},\ k = \overline{1,l-1},\ \nu = \overline{1,r};$$

$$\tilde{\Psi}_j^{(l\nu)}(s,\omega) = \Phi_{j+1}^{(l1)}(s,0)/(\omega+s+\alpha) + \sum_{k=1}^{l-1}\left\{\alpha^k\Phi_j^{(k,\nu+1)}(s,0)\sum_{p=1}^{l}\left[\left(w_p+\alpha\right)^{l-1+k} \Big/ \prod_{\substack{\eta=1\\ \eta\neq p}}^{l}\left(w_p - w_\eta\right)\right] \Big/ \left(\omega + s - w_p\right)\right\} + \alpha^l\Phi_j^{(l,\nu+1)}(s,0)\sum_{p=0}^{l}\left\{\left[1 \Big/ \prod_{\substack{\eta=0\\ \eta\neq p}}^{l}\left(w_p - w_\eta\right)\right] \Big/ \left(\omega+s-w_p\right)\right\},$$

(2.4.14)

$$\nu = \overline{1,r-1},\ w_d = \alpha\left(e^{i2\pi d/l} - 1\right),\ d = \overline{1,l},\ w_0 = -\alpha;$$

$$\tilde{\Psi}_j^{(lr)}(s,\omega) = \Phi_{j+1}^{(l1)}(s,0)/(\omega+s+\alpha) + \{\alpha\overline{g}(s)/[(s+\omega)(s+\omega+\alpha)]\}\Phi_j^{(1,1)}(s,0); \quad (2.4.15)$$

$$\Phi_{n+1}^{(k\nu)}(s,0) = \frac{1}{s} \quad (2.4.17)$$

Here

$$\Psi_j^{(k\nu)}(t,x) = \Psi_j^{(k\nu)}(t,\tau_b - y) = \tilde{\Psi}_j^{(k\nu)}(t,y);$$

$$\tilde{\overline{\Psi}}_j^{(k\nu)}(s,y) = \int_0^\infty e^{-st}\tilde{\Psi}_j^{(k\nu)}(t,y)dt;$$

$$\tilde{\Psi}_j^{(k\nu)}(t,0) = \Psi_j^{(k\nu)}(t,\tau_b) = \Phi_{j+1}^{(k\nu)}(t,0);$$

$$\tilde{\overline{\Psi}}_j^{(k\nu)}(s,0) = \Phi_{j+1}^{(k\nu)}(s,0);$$

$$\tilde{\overline{\Psi}}_j^{(k\nu)}(s,\omega) = \int_0^\infty e^{-\omega y}\tilde{\overline{\Psi}}_j^{(k\nu)}(s,y)dy$$

(2.4.16)

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As a result of solutions of algebraic equations (2.4.14), (2.4.15), (2.4.16) and taking into account (2.4.17) we find $\tilde{\bar{\psi}}_j^{(k\nu)}(s,\omega)$, $(j=\overline{1,n}; k=\overline{1,l}; \nu=\overline{1,r})$. Defining the reverse conversion $\tilde{\bar{\psi}}_j^{(k\nu)}(s,\omega)$ and after substituting in it y=0 (x=τ_b), we determine the value of $\bar{\Phi}_{j+1}^{(k\nu)}(s,0)$. Let us solve the equation (2.4.14) by successive substitutions, starting from k=l-1, k=l-2 ..., let l-m=k; m=l-k, then we have:

$$\tilde{\psi}_j^{(k\nu)} = \frac{\sum_{c=1}^{l-k-1}\left[(\omega+s+\alpha)^{(l-k-c)}\alpha^{(c-1)}\right] + \alpha^{l-k-1} + S\alpha^{l-k}\tilde{\psi}_j^{(l\nu)}(s,\omega)}{S(\omega+S+\alpha)^{l-k}} \tag{2.4.18}$$

Let us calculate the following equations (2.4.15), (2.4.16) and (2.4.18), we give an example for this, where

l=2, r=2, n=1, j=1, $\Phi_2^{(2i)}(s,0) = 1/s$, i=1,2.

$$\tilde{\psi}_1^{(1i)}(s,\omega) = \frac{1+S\alpha\tilde{\psi}_1^{(2i)}(s,\omega)}{S(\omega+S+\alpha)}, \qquad i=1,2.$$

in accordance with (2.4.18)

$$\tilde{\tilde{\Psi}}_1^{(2,2)}(s,\omega) = \frac{1}{s(s+\omega+\alpha)} + \left[\frac{\alpha g(s)}{(s+\omega)(s+\omega+\alpha)}\right]\bar{\Phi}_1^{(1,1)}(s,0)$$

$$\tilde{\tilde{\Psi}}_1^{(2,1)}(s,\omega) = \frac{1}{s(s+\omega+\alpha)} + \alpha\bar{\Phi}_1^{(1,2)}(s,0) \text{ x}$$

$$\times \left[\frac{1}{(w_1-w_2)(s+\omega-w_1)} + \frac{1}{(w_2-w_1)(s+\omega-w_2)}\right] + \alpha^2\bar{\Phi}_1^{(2,2)}(s,0) \times$$

$$\times\left[\frac{1}{(w_0-w_1)(w_0-w_2)(s+\omega-w_0)} + \frac{1}{(w_1-w_0)(w_1-w_2)(s+\omega-w_1)} + \frac{1}{(w_2-w_0)(w_2-w_1)(s+\omega-w_2)}\right];$$

$$\bar{\Phi}_1^{(21)}(s,0) = \frac{1}{s}e^{-(s+\alpha)\tau_b} + \frac{1}{2}\left(e^{-s\tau_b} - e^{-(s+2\alpha)\tau_b}\right)\bar{\Phi}_1^{(12)}(s,0) + \left[\frac{1}{2}e^{-s\tau_b} + \frac{1}{2}e^{-(s+2\alpha)\tau_b} - e^{-(s+\alpha)\tau_b}\right]\bar{\Phi}_1^{(2,2)}(s,0)$$

Similar to:

$$\bar{\Phi}_1^{(2,2)}(s,0) = A(s,T) + D(s,T)\bar{\Phi}_1^{(1,1)}(s,0);$$

$$\bar{\Phi}_1^{(1,1)}(s,0) = A(s,T) + E(s,T)\bar{\Phi}_1^{(2,1)}(S,0);$$

$$\bar{\Phi}_1^{(1,2)}(s,0) = A(s,T) + F(s,T)\bar{\Phi}_1^{(2,2)}(S,0);$$

Conditional mean is equal to:

$$-\left|S\hat{O}_1^{(2,1)}(S,0)\right|'_{S=0} =$$

$$= \tau^{(21)}(0) = \tau_b + \frac{1}{2}(1-e^{-2\alpha\tau_b})\tau_1^{(1,2)}(0) + (\frac{1}{2}+\frac{1}{2}e^{-2\alpha\tau_b} - e^{-\alpha\tau_b})\tau_1^{(2,2)}(0);$$

Solving the rest of the equations, we finally obtain:

$$\Phi_1^{(1,1)}(S,0) = b(\alpha)K(\alpha)\frac{M(s)}{S}$$

$$\Phi_1^{(1,2)}(S,0) = \frac{b(\alpha)K(\alpha)e^{-s\tau_b}}{S}[1+d(\alpha)g(s)M(s)]$$

$$\Phi_1^{(2,1)}(S,0) = \frac{K(\alpha)e^{-s\tau_b}}{S}\{1+b(\alpha)e(\alpha)e^{-s\tau_b}[1+d(\alpha)g(s)M(s)]+b(\alpha)f(\alpha)M(s)\}$$

$$\Phi_1^{(2,2)}(S,0) = \frac{K(\alpha)e^{-s\tau_b}}{S}[1+b(\alpha)f(\alpha)M(s)]$$

$$\tau^{(1,1)}(0) = -\left|S\Phi_1^{(1,1)}(S,0)\right|'_{S=0} = b(\alpha)k(\alpha)M'(0)$$

$$\tau^{(1,2)}(0) = -\left|S\Phi_1^{(1,2)}(S,0)\right|'_{S=0} = -\tau_b b(\alpha)k(\alpha)[1-f(\alpha)M(0)]+b(\alpha)f(\alpha)k(\alpha)M'(0)$$

$$\tau^{(2,1)}(0) = -\left|S\Phi_1^{(2,1)}(S,0)\right|'_{S=0} = -k(\alpha)\tau_b[1-2b(\alpha)e(\alpha)]-k(\alpha)b(\alpha)\{e(\alpha)d(\alpha)[\tau_b+\tau_в]-\tau_b f(\alpha)\}M(0)+b(\alpha)k(\alpha)[e(\alpha)d(\alpha)+f(\alpha)]M'(0)$$

$$\tau^{(2,2)}(0) = -\left|S\Phi_1^{(2,2)}(S,0)\right|'_{S=0} = -\tau_b k(\alpha)-\tau_b k(\alpha)b(\alpha)f(\alpha)M(0)+b(\alpha)k(\alpha)f(\alpha)M'(0)$$

где

$a(\alpha)=1/2+e^{-\alpha\tau_б}[(1/2)\, e^{-\alpha\tau_б}-1)$

$b(\alpha)=1+\alpha\tau_б$

$c(\alpha)=1/2-e^{-\alpha\tau_б}[(1/2)\, e^{-\alpha\tau_б}-\alpha\tau_б)$

$d(\alpha)=1+ e^{-\alpha\tau_б}(1-\alpha\tau_б)$

$e(\alpha)=(1/2)[1- e^{-2\alpha\tau_б}]$

$f(\alpha)=1- e^{-\alpha\tau_б}$

$k(\alpha)=e^{-\alpha\tau_б}$

$M(s)=[1+ e^{-s\tau_б}(a(\alpha)b(\alpha)+c(\alpha))]/[1- e^{-2s\tau_б}(d(\alpha)g(s)+ f(\alpha))]$

III. Conclusion

In this paper, we investigated queueing models, represented as wireless system, where time intervals between failures are Erlang distributed. We present some advantages of the Erlang model we proposed for mobility modeling. We show the generality of such model, which can be used to model not only interarriaval time between neighboring failures but also other time variables in wireless networks and mobile computing systems.

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