On the Numerical Solution of Three-Dimensional Diffusion Equation with an Integral Condition

A. Cheniguel and M. Reghioua

Abstract— **In this paper, we investigate solution of three-dimensional diffusion equation with non local condition using decomposition method. This method is reliable and gives a solution in a series form with high accuracy. It also guarantees considerable saving of calculation volume and times as compared to traditional methods. The obtained results show that the decomposition method is efficient and yields a solution in a closed form.**

Index Terms—**Adomian decomposition method, non local boundary conditions conditions, exact solution, partial differential equations.**

I. Introduction

OVer the last few years, various processes in science and engineering have led to the non classical parabolic initial/boundary value problems which involve non-local integral terms over the spatial domain [1-10, 12,14]. These include chemical diffusion, heat conduction , population dynamics and control. Up to now partial differential equations with non local boundary conditions have been one of the fastest growing areas in various fields. In this paper we consider a three-dimensional diffusion equation with a non local boundary condition. The two-dimensional case was solved by many authors using traditional numerical techniques such as finite difference method, finite elements method, spectral techniques, etc.. for example Siddiq [7] proposed a fourth-order finite difference padé scheme and Cheniguel [2] has solved the same problem using new techniques the obtained results are all exact.

The aim of this work is to study and to implement the decomposition method for solving a three-dimensional diffusion equation with non local condition[11,13-15]. The decomposition method can also be applied to a large class of system of partial differential equations with approximates that converges rapidly to accurate solutions. The implementation of the method has shown reliable results in that few terms are needed to obtain either exact solution or to find an approximate solution of a reasonable degree of accuracy in real physical models. Numerical example are presented to illustrate the efficiency of the decomposition method, the obtained results are in good agreement with exact ones. We consider the three-dimensional diffusion equation given by:

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$$u_t = u_{xx} + u_{yy} + u_{zz}, 0 < x, y, z < 1, t > 0 \quad (1)$$

Initial condition is given by:

$$u(x, y, z, 0) = f(x, y, z), (x, y, z) \epsilon \Omega \cup \boldsymbol{\partial\Omega}$$

And the dirichelet time-dependent boundary conditions are

$$u(0, y, z, t) = \psi_0(y, z, t), 0 \le y, z \le 1, 0 \le t \le T \quad (2)$$

$$u(1, y, z, t) = \psi_1(y, z, t), 0 \le y, z \le 1, 0 \le t \le T$$

$$u(x, 0, z, t) = \varphi_0(x, z, t) \times \gamma(t), 0 \le x, z \le 1, 0 \le t \le T$$

$$u(x, 1, z, t) = \varphi_1(x, z, t), 0 \le x, z \le 1, 0 \le t \le T$$

$$u(x, y, 0, t) = \emptyset_0(x, y, t), 0 \le x, y \le 1, 0 \le t \le T$$

$$u(x, y, 1, t) = \emptyset_1(x, y, t), 0 \le x, y \le 1, 0 \le t \le T$$

And non local boundary condition

$$\int_0^1 \int_0^1 \int_0^1 u(x, y, z, t) dxdydz = m(t), (x, y, z) \in \Omega \cup \partial\Omega \quad (3)$$

Where $f, \psi_0, \psi_1, \varphi_0, \varphi_1$, and m are known functions and γ(t) is to be determined.

II. Adomian Decomposition Method

A. Operator form

In this section we outline the steps to obtain a solution to the above problem using Adomian decomposition method, which was initiated by G. Adomian [11,13,15]. For this purpose we reformulate the problem in an operator form:

$$L_t(u) = L_{xx}(u) + L_{yy} + L_{zz} \quad (4)$$

Where the differential operators $L_t(.) = \frac{\partial}{\partial t}(.)$ and $L_{xx} = \frac{\partial^2}{\partial x^2}, L_{yy} = \frac{\partial^2}{\partial Y^2}, L_{zz} = \frac{\partial^2}{\partial z^2}$

assuming that the inverse L_t^{-1} exists and is defined as:

$$L_t^{-1} = \int_0^t (.)dt \quad (5)$$

B. Application to the problem

Applying the inverse operator on both the sides of equation (4) and using the initial condition yields:

$$u(x, y, z, t = L_t^{-1}(L_{xx}(u(x, y, z, t) + L_{yy}(u(x, y, z, t) + L_{zz}(u(x, y, z, t))$$

Or

$$u(x, y, z, t) = u(x, y, z, 0) + L_t^{-1}(L_{xx}(u(x, y, z, t) + L_{yy}(u(x, y, z, t) + L_{zz}(u(x, y, z, t)) \quad (6)$$

Now, we decompose the unkown function $u(x, y, z, t)$ as a sum of components defined by the series :

$$u(x, y, z, t) = \sum_{k=0}^{\infty} u_k(x, , y, z, t) \quad (7)$$

Where u_0 is identified as $u(x, y, z, 0)$. Substituting equation (7) into equation (6) one obtains:

$\sum_{k=0}^{\infty} u_k(x,y,z,t) =$
$L_t^{-1}\{(L_{xx}+L_{yy}+L_{zz})(\sum_{k=0}^{\infty} u_k(x,y,z,t))\}$ (8)
Or:
$u_0(x,y,z,t) = f(x,y,z)$ (9)
And:
$u_{k+1}(x,y,z,t) =$
$L_t^{-1}\left(L_{xx}\left(u_k(x,y,z,t)\right) + L_{yy}(u_k(x,y,z,t) + L_{zz}(u_k(x,y,z,t))\right),\ k \geq 0$ (10)

The components are obtained by the recursive formula:

$u_0(x,y,z) = f(x,y,z)$ (11)
$u_{k+1}(x,y,z,t) =$
$L_t^{-1}(L_{xx}\left(u_k(x,y,z,t) + L_{yy}(u_k(x,y,z,t) + L_{zz}(u_k(x,y,z,t))\right))$ (12)

From equation (9) and (10) we obtain the first few terms as:

$$u_1(x,y,z,t) = L_t^{-1}(L_{xx}\left(u_0(x,y,z,t)\right) + L_{yy}\left(u_0(x,y,z,t)\right) + L_{zz}(u_0(x,y,z,t)$$

$u_2(x,y,z,t) =$
$L_t^{-1}(L_{xx}\left(u_1(x,y,z,t)\right) + L_{yy}(u_1(x,y,z,t) + L_{zz}(u_1(x,y,z,t)))$
$u_3(x,y,z,t) =$
$L_t^{-1}(L_{xx}\left(u_2(x,y,z,t)\right) + L_{yy}\left(u_2(x,y,z,t)\right) + L_{zz}(u_2(x,y,z,t)))$

and so on. As a result, the components u_0 ,$u_1, u_2, \ldots$ are identified and the series solution is thus entirely determined. However, in many cases the exact solution in a closed form may be obtained as we can see in our examples:

III. EXAMPLES

A. Example 1

We consider the three-dimensional diffusion equation :

$u_t = u_{xx} + u_{yy} + u_{zz}$

In which $u = u(x,y,z,t)$. The Dirichelet time-dependent boundary conditions on the boundary $\partial\Omega$ of the cube Ω defined by the lines
$x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$
Are given by:
$u(0,y,z,t) = e^{y+z+3t}, 0 \leq y,z \leq 1, 0 \leq t \leq T$ (13)
$u(1,y,z,t) = e^{1+y+z+3t}, 0 \leq y,z \leq 1, 0 \leq t \leq T$
$u(x,0,z,t) = e^{x+z+3t}, 0 \leq x,z \leq 1, 0 \leq t \leq T$
$u(x,1,z,t) = e^{1+x+z+3t}, 0 \leq x,z \leq 1{,}0 \leq t \leq T$
$u(x,y,0,t) = e^{x+y+3t}, 0 \leq x,y \leq 1{,}0 \leq t \leq T$
$u(x,y,1,t) = e^{1+x+y+3t}, 0 \leq x,y \leq 1{,}0 \leq t \leq T$
And non local boundary condition
$\int_0^1\int_0^1\int_0^1 u(x,y,z,t)dxdydz = (e-1)^3 e^{3t}$ (14)
With the initial condition:
$u(x,y,z,0) = e^{x+y+z}$ (15)
Analytic solution is given by:
$u(x,y,z,t) = e^{x+y+z+3t}$ (16)
Using the decomposition method, described above, equation (9) gives the first component
$u_0(x,y,z,t) = f(x,y,z) = e^{x+y+z}$ (17)
And equation (10) gives the following components of the series :
$u_1(x,y,z,t) =$
$L_t^{-1}\left(L_{xx}(u_0(x,y,z,t)) + L_{yy}(u_0(x,y,z,t) + L_{zz}(u_0(x,y,z,t))\right) = \int_0^t 3e^{x+y+z}\,dt = 3te^{x+y+z}$ (18)

$$u_2 = L_t^{-1}(L_{xx}\left(u_1(x,y,z,t)\right) + L_{yy}(u_1(x,y,z,t) + L_{zz}\left(u_1(x,y,z,t)\right)) = \int_0^t 9te^{x+y+z}dt = \frac{3^2}{2!}t^2e^{x+y+z}$$

$u_3 = L_t^{-1}(L_{xx}\left(u_2(x,y,z,t)\right) + L_{yy}\left(u_2(x,y,z,t)\right) + L_{zz}\left(u_2(x,y,z,t)\right)) = \int_0^t \frac{27}{2}t^2e^{x+y+z}dt = \frac{3^3}{3!}t^3e^{x+y+z}$ (19)
...
And so on.
Then the solution in the series form is given by:

$$u(x,y,z,t) = \sum_{k=0}^{\infty} u_k(x,y,z,t)$$

With the above results:
$u(x,y,z,t) = e^{x+y+z}\left(1 + \frac{3t}{1!} + \frac{3^2}{2!}t^2 + \frac{3^3}{3!}t^3 + \cdots\right)$
Which can be rewritten as:
$u(x,y,z,t) = e^{x+y+z+3t}$ (20)
It can be easily observed that (20) is equivalent to the exact solution. Table I shows the numerical results for $h_x = h_y = h_z = \frac{1}{10}, h_t = \frac{1}{250}$

B. Example 2

Consider the three-dimensional non homogeneous diffusion problem:

$$u_t = u_{xx} + u_{yy} + u_{zz} - e^{-t}(x^2+y^2+z^2+4), 0 < x,y,z < 1, t > 0$$

with the initial condition
$u(x,y,z,0) = 1 + x^2 + y^2 + z^2$ (21)
And the boundary conditions
$u(0,y,z,t) = 3 + (y^2+z^2-2)e^{-t}, 0 \leq y,z \leq 1, 0 \leq t \leq T$
$u(1,y,z,t) = 3 + (-1+y^2+z^2)e^{-t}, 0 \leq y,z \leq 1, 0 \leq t \leq T$
$u(x,0,z,t) = 3 + (x^2+z^2-2)e^{-t}, 0 \leq x,z \leq 1, 0 \leq t \leq T$
$u(x,1,z,t) = 3 + (-1+x^2+z^2)e^{-t}, 0 \leq x,z \leq 1, 0 \leq t \leq T$
$u(x,y,0,t) = 3 + (x^2+y^2-2)e^{-t}, 0 \leq x,y \leq 1, 0 \leq t \leq T$
$u(x,y,1,t) = 3 + (x^2+y^2-1)e^{-t}, 0 \leq x,y \leq 1, 0 \leq t \leq T$
And the non local boundary condition
$\int_0^1\int_0^1\int_0^1 u(x,y,z,t)dxdydz = 3 - e^{-t}, 0 \leq t \leq T$ (22)
Theoretical solution is given by:
$u(x,y,z,t) = 3 + (x^2+y^2+z^2-2)e^{-t}$
Writing the problem in operator form and applying the inverse operator one obtains:
$L_t^{-1}\left(L_t\left(u(x,y,z,t)\right)\right) = L_t^{-1}(L_{xx}(u(x,y,z,t) + L_{yy}\left(u(x,y,z,t) + L_{zz}\left(u(x,y,z,t)\right)\right) + L_t^{-1}(-e^{-t}(x^2 + y^2 + z^2 + 4))$ (23)
$L_t^{-1}\left(L_t\left(u(x,y,z,0)\right)\right) = u(x,y,z,0)$ (24)
From which we obtain :
$u(x,y,z,t) = u(x,y,z,0) + L_t^{-1}(L_{xx}\left(u(x,y,z,t)\right) + L_{yy}\left(u(x,y,z,t)\right) + L_{zz}\left(u(x,y,z,t)\right) + L_t^{-1}(-e^{-t}(x^2 + y^2 + z^2 + 4))$ (25)
Using Adomian decomposition, the zeroth component is given by:
$u_0(x,y,z,t) = u(x,y,z,0) + L_t^{-1}(-e^{-t}(x^2+y^2+z^2 + +4))$ (26)

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And

$u_{k+1}(x,y,z,t) = L_t^{-t}(L_{xx}(u_k(x,y,z,t)) + L_{yy}(u_k(x,y,z,t)) + L_{zz}(u_k(x,y,z,t)))$ (27)

Applying these formula, we obtain the components of the series solution as:

$u_0(x,y,z,t) = 1 + x^2 + y^2 + z^2 + \int_0^t -e^{-t}(x^2 + y^2 + z^2 + 4)dt = -3 + (x^2 + y^2 + z^2 + 4)e^{-t}$ (28)

$u_1(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_0(x,y,z,t)) + L_{yy}(u_0(x,y,z,t)) + L_{zz}(u_0(x,y,z,t))\right) = \int_0^t 6e^{-t}\,dt = 6 - 6e^{-t}$ (29)

$u_2(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_1(x,y,z,t)) + L_{yy}(u_1(x,y,z,t)) + L_{zz}(u_1(x,y,z,t))\right) = \int_0^t 0dt = 0$ (30)

Then :

$u_k(x,y,z,t) = 0, k \geq 2$ (31)

Finally , we obtain the approximate solution:

$u(x,y,z,t) = u_0(x,y,z,t) + u_1(x,y,z,t)$

$u(x,y,z,t) = -3 + (x^2 + y^2 + z^2 + 4) + 6 - 6e^{-t}$

Or:

$u(x,y,z,t) = 3 + (x^2 + y^2 + z^2 - 2)e^{-t}$ (32)

And we can observe that the obtained result is exact.

Table II shows the numerical results for $h_x = h_y = h_z = \frac{1}{10}, h_t = \frac{1}{250}$

C. Example 3

Consider the problem

$u_t = u_{xx} + u_{yy} + u_{zz}, 0 < x,y,z < 1, t > 0$ (33)

Subject to the initial condition

$u(x,y,z,0) = (1-y-z)e^x, 0 \leq x,y,z \leq 1$ (34)

And the boundary conditions

$u(0,y,z,t) = (1-y-z)e^t, 0 \leq y,z \leq 1, 0 \leq t \leq 1$

$u(1,y,z,t) = (1-y-z)e^{1+t}, 0 \leq y,z \leq 1, 0 \leq t \leq 1$

$u(x,0,z,t) = (1-z)e^{x+t}, 0 \leq x,z \leq 1, 0 \leq t \leq 1$

$u(x,1,z,t) = -ze^{x+t}, 0 \leq x,z \leq 1, 0 \leq t \leq 1$

$u(x,y,0,t) = (1-y)e^{x+t}, 0 \leq x,y \leq 1, 0 \leq t \leq 1$

$u(x,y,1,t) = -ye^{x+t}, 0 \leq x,y \leq 1, 0 \leq t \leq 1$ (35)

And the local boundary condition

$\int_0^1 \int_0^1 \int_0^{x(1-x)} u(x,y,z,t)dxdydz = \frac{15}{2}(1-e)e^t$ (36)

Consider the equation (33) in an operator form

$L_t(u(x,y,z,t)) = L_{xx}(u(x,y,z,t)) + L_{yy}(u(x,y,z,t)) + L_{zz}(u(x,y,z,t))$ (37)

Where, L_t, L_{xx}, L_{yy}, L_{zz}, L_t^{-1} are defined as above. Assume that the inverse operator L_t^{-1} exists operating with L_t^{-1} on both sides of equation (37) we obtain

$u(x,y,z,t) = L_t^{-1}(L_{xx}(u(x,y,z,t)) + L_{yy}(u(x,y,z,t)) + L_{zz}(u(x,y,z,t)))$ (38)

Using the decomposition method, the zeroth component is given by

$u_0(x,y,z,t) = u(x,y,z,0)$ (39)

And

$u_{k+1}(x,y,z,t) = L_t^{-1}(L_{xx}(u_k(x,y,z,t)) + L_{yy}(u_k(x,y,z,t)) + L_{zz}(u_k(x,y,z,t)))$ (40)

Applying these formula, we have

$u_0(x,y,z,t) = (1-y-z)e^x$

$$u_1(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_0(x,y,z,t)) + L_{yy}(u_0(x,y,z,t)) + L_{zz}(u_0(x,y,z,t))\right) = \int_0^t (1-y-z)e^x\,dt = (1-y-z)e^x t$$

$u_2(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_1(x,y,z,t)) + L_{yy}(u_1(x,y,z,t)) + L_{zz}(u_1(x,y,z,t))\right) = \int_0^t (1-y-z)e^x\,tdt = (1-y-z)e^x\frac{t^2}{2!}$

$u_3(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_2(x,y,z,t)) + L_{yy}(u_2(x,y,z,t)) + L_{zz}(u_2(x,y,z,t))\right) = \int_0^t (1-y-z)e^x\frac{t^2}{2!}dt = (1-y-z)e^x\frac{t^3}{3!}$

..

$$u_k(x,y,z,t) = L_t^{-1}\left(L_{xx}(u_{k-1}(x,y,z,t)) + L_{yy}(u_{k-1}(x,y,z,t)) + L_{zz}(u_{k-1}(x,y,z,t))\right) = \int_0^t (1-y-z)e^x\frac{t^{k-1}}{(k-1)!}dt = (1-y-z)e^x\frac{t^k}{k!}$$

And so on , once the components are determined then, the series solution is given by:

$u(x,y,z,t) = \sum_{k=0}^{\infty} u_k\,(x,y,z,t) = (1-y-z)e^x(\sum_{k=0}^{\infty}\frac{t^k}{k!})$

Or equivalently:

$u(x,y,z,t) = (1-y-z)e^{x+t}$

This result is in good agreement with the exact one.

Table III shows the numerical results for $h_x = h_y = h_z = \frac{1}{10}, h_t = \frac{1}{250}$.

IV. Conclusion

In this work, we have detailed the study of the Adomian decomposition method ADM and using it for finding the solution of the three-dimensional heat equation with energy specification . This method is employed without using linearization, discretization, transformation, or restrictive assumptions. It is very much compatible with the diversified and versatile nature of physical problems, the results obtained are all in good agreement with the exact solutions under study. Moreover this method is efficient, reliable, accurate, easier to implement as compared to the traditional techniques.

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Table I : Example 1

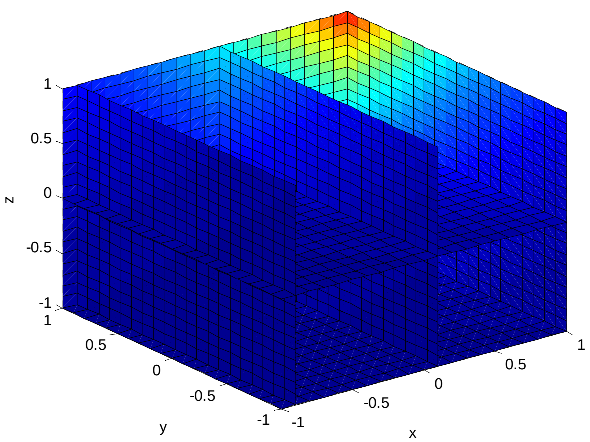
x_i	y_j	z_k	u_{ex}	u_{Ad}	$\lvert u_{ex}-u_{Ad}\rvert$
0.0	0.0	0.0	1.021	1.021	0.0
0.1	0.1	0.1	1.3662	1.3662	0.0
0.2	0.2	0.2	1.8441	1.8441	0.0
0.3	0.3	0.3	2.4893	2.4893	0.0
0.4	0.4	0.4	3.3602	3.3602	0.0
0.5	0.5	0.5	4.5358	4.5358	0.0
0.6	0.6	0.6	6.1227	6.1227	0.0
0.7	0.7	0.7	8.2648	8.2648	0.0
0.8	0.8	0.8	11.156	11.156	0.0
0.9;	0.9	0.9	15.059	15.059	0.0
1.0	1.0	1.0	20.328	20.328	0.0

Table II Example 2

x_i	y_j	z_k	u_{ex}	u_{Ad}	$\lvert u_{ex}-u_{Ad}\rvert$
0.0	0.0	0.0	1.0080	0.98403	0.02397
0.1	0.1	0.1	1.0976	1.0737	0.0239
0.2	0.2	0.2	1.3665	1.3426	0.0239
0.3	0.3	0.3	1.8148	1.7908	0.024
0.4	0.4	0.4	2.4422	2.4183	0.0239
0.5	0.5	0.5	3.249	3.225	0.024
0.6	0.6	0.6	4.235	4.2111	0.0239
0.7	0.7	0.7	5.4004	5.3764	0.024
0.8	0.8	0.8	6.7450	6.721	0.024
0.9	0.9	0.9	8.2689	8.2429	0.024
1.0	1.0	1.0	9.9721	9.9481	0.024

Table III Example 3

x_i	y_j	z_k	u_{ex}	u_{Ad}	$\lvert u_{ex}-u_{Ad}\rvert$
0.0	0.0	0.0	1.004	1.004	0.0
0.1	0.1	0.1	0.88768	0.88767	0.00001
0.2	0.2	0.2	0.73578	0.73577	0.00001
0.3	0.3	0.3	0.54211	0.54210	0.00001
0.4	0.4	0.4	0.29956	0.29956	0.0
0.5	0.5	0.5	0.0	0.0	0.0
0.6	0.6	0.6	−0.36588	−0.36588	0.0
0.7	0.7	0.7	−0.80873	−0.80872	0.00001
0.8	0.8	0.8	−1.3407	−1.3407	0.0
0.9	0.9	0.9	−1.9756	−1.9756	0.0
1.0	1.0	1.0	−2.7292	−2.7292	0.0



Fig; 1Variation of the approximate solution for different values of x, y and z when t=1/250

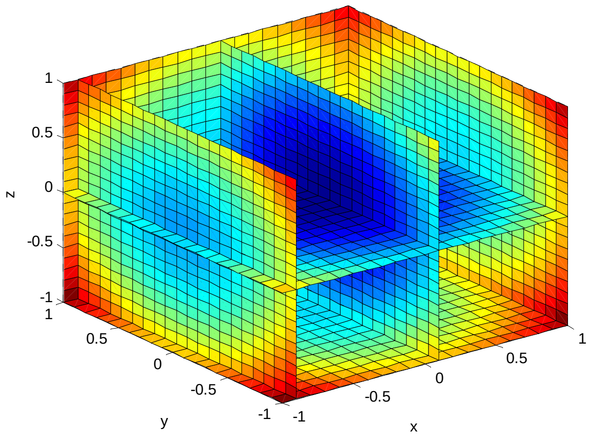


Fig 2Variation of the approximate solution for different values of x,y and z when t=1/2500

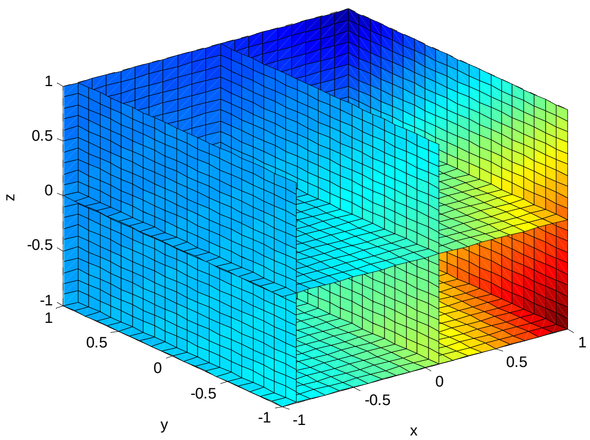


Fig. 3 Variation of the approximate solution for different values of x, y and z when t=1/250

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