A New Stochastic Damage Model for Fatigue Crack Growth

M. H. Hafezi, A.A Shariff

Abstract—The fatigue process of mechanical components under service loading is stochastic in nature. Fatigue life prediction and reliability evaluation is still a problematic issue. A new damage model is presented and some difficulty in fatigue crack growth under variable amplitude loading involved in load interaction effects have been kept in a memory function $M(a)$. A technique is introduced where both deterministic and probabilistic viewpoints in fatigue modelling are combined in a simple manner.

Keywords—Stochastic, Damage, fatigue, crack

I. INTRODUCTION

According to some observations, it is generally accepted that a two-parameter model using driving force combining the applied maximum SIF, and the applied stress intensity factor should be the main focus in order to explain shortcomings in crack closure tip concept [1]. The crack growth expressions proposed by Donald and Paris [2] and Kujawski [3] are most useful for investigating stochastic expressions. It should be noted that in recent years the UniGrow model [4] (similar to the above-mentioned models except for use of total maximum stress intensity factor and total stress intensity range) have been tried to account effectively for the residual stress induced at the crack tip by cyclic plastic deformation. It is very clear that deterministic prediction models such as UniGrow model considering mixed driving force do not give us any information about the controlling of driving force during the propagation time of crack. In other words, best correlation of fatigue crack growth (FCG) data and driving force has clarified transition effects (retardation and acceleration) under a variety of deterministic loading conditions as well as known material properties.

This is the contribution of those models in helping to understand crack physics mechanism and fracture behavior. Finally, on the predicting of number of cycles to failure those models are going to help in finding reliable components much more effectively than before. Hence, the question arises of how we can use all the benefits of deterministic models to explain complex conditions in a stochastic process, where the represented model has an ability to provide similar clarification, but can better accommodate the parameters which are not under our control. On the other hand, uncertainty in load variation (random loading) and in material properties can be better understood [5].

II. DEFINITIONS

A. Two-dimensional plane stress

This section is an overview on the key assumptions of fatigue crack growth model, which is used to establish the damage rate expression. In the linear theory of elasticity a plate loaded in its midplane is said to be in a state of plane stress [6]. The following assumption governs plane stress in two-dimensional problems:

- All loads applied to the plate act in the midplane direction, and are symmetric with respect to the midplane.
- All support conditions are symmetric about the midplane.
- In-plane displacements, strains and stresses can be taken to be uniform through the thickness.
- The normal and shear stress components in the $z$ direction are zero or negligible.
- The plate is fabricated of the same material through the thickness. Such plates are called transversely homogeneous.

All analysis has been carried out assuming plane stress at the crack tip plane.

B. Stress intensity factor

Considering that the reversed plastic deformation around the crack tip induced at relatively high stress ratio (greater than 0.5) and relatively small stress intensity ranges (near the threshold FCG), are usually not sufficient to produce compressive residual stresses, therefore, the residual stress intensity factor is close to zero and the total SIFs are the same as applied values [4]. It therefore deduces that:

M. H. Hafezi received his M. Sc from National University of Malaysia in 2012. He currently is in research collaboration with Dr. Shariff in Mathematics Division, Centre for Foundation Studies in Science, University of Malaya, 50603 Kuala Lumpur, Malaysia. His research interest is fatigue crack growth modeling. (Corresponding e-mail: hafezi@eng.ukm.my).

A.A. Shariff is an Associate Professor at the Mathematics Division, Centre for Foundation Studies in Science, University of Malaya, 50603 Kuala Lumpur, Malaysia. She received her B. Sc from University of Malaya (1984), her M. Sc. is from Sussex University, UK (1987), and her Ph.D. degree is from University of Newcastle Upon Tyne, UK (1998). Her interests are engineering mathematics and statistical methods. (e-mail: gsna@um.edu.my)
Material block:

For the elementary material block size, the fatigue crack growth rate can be determined as

\[
d a = \frac{\rho}{N_f} \]

Fatigue life analysis

The number of cycles to failure for the material over the elementary material block has been obtained from the Coffin-Manson curve combined with the Smith-Watson-Topper (SWT), fatigue damage parameter.

\[
D = \frac{\sigma_{max}^a \Delta N}{2} = \frac{\left(\frac{\sigma_{max}^a}{2}\right)^{2n} + \sigma_f \epsilon_f (2N_f)^{b+C}}{2n} \quad (8)
\]

Two-parameter FCG model expression

\[
\Delta \sigma = \frac{\sigma_f - \sigma_m}{E} + \epsilon_f \left(\frac{\sigma_f - \sigma_m}{\sigma_f}\right)^{c/b} (2N_f)^{b} \quad (11)
\]

\[
d a = C \left(\frac{\Delta \sigma}{E}\right)^{2n} \left(\frac{\epsilon_f}{\sqrt{2\pi\rho}}\right)^{1.5} \quad (12)
\]

\[
C = 2\rho \left[ \frac{k'}{E^\sigma} \left(\frac{\psi_{y,1}}{\sqrt{2\pi\rho}}\right)^{\frac{3n+1}{2}} \left(\frac{1}{\sigma_f - \sigma_m}\right)^{1.5} \right]^{-\frac{1}{b}} \quad (13)
\]

\[
p = \frac{n'}{n'+1} \quad (14)
\]

Final analytical derivation of the UniGrow model depends predominantly on considering plastic or elastic material behavior at the crack tip, as well as elastic-plastic behavior at the crack tip. In fact, the model proposes that it is the elastic and plastic driving force and mixed driving force which take into account the combination of the elastic and plastic stress-strain material behavior. It is shown that using mixed driving force in the final expression we can correlate FCG data at various R-ratios for the FCG rates spanning from the near threshold to the high growth rate regime. Regarding these assumptions, only the plastic terms of the Ramberg-Osgood and Neuber equations for the loading reversal, and only the elastic terms for unloading reversal have been applied. The maximum stress and the strain range at the crack tip are determined by the following expressions:

For the sake of brevity, we did not repeat the fatigue crack growth final expression here. Details and some discussions can be found in [4].

For the first elementary material size the value of \(\psi_{y,1} = 1.633\) [4].

Fatigue crack growth model with mean stress

A mean stress equation can be incorporated into the fatigue crack growth rate expression. The special case of two parameter FCG model (section E) is established. The conclusions of the literature study are as follows:

- Coffin-Manson does not consider the mean stress that is why SWT has been used for showing mean stress effect in fatigue life.
- The SWT method provides good results in most cases, and for aluminum alloys it is somewhat more accurate than the Morrow equation [7].
- The SWT method has the advantage of simplicity and is a good choice for general use.
- The Morrow equation is also reasonably accurate for steels, but often gives grossly non-conservative life estimations for aluminum alloys [7].

If the alternative version of Morrow mean stress [8] is substituted in UniGrow model and also only to be considered for applied value of stress intensity factor (section B) the final expression can be obtained by:

This model is the same as expressions of Donald and Paris [2], Kujawski [9] and UniGrow model [4] models, except that using Modified Morrow means stress equation [7] is incorporated. It is worth noting that both plastic and elastic
terms in modified Morrow mean stress are affected by mean stress terms. The changes appear in fatigue crack growth constant and fatigue crack exponent. Equation (12) is applied for stochastic formation of FCG model.

IV. STOCHASTIC DAMAGE MODEL

Based on the equations (4), (5) and (12), the following expression can be obtained. It is simple but might be tedious.

\[
\left(1 - \frac{S_{\text{min}}}{S_{\text{max}}} \right)^{0.5} \gamma \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right)^{0.5} \gamma \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right)^{0.5} \gamma \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right)^{0.5} \gamma \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right)^{0.5} \gamma
\]

(16)

In this study the stress intensity factor for a compact tension specimen [10] can be calculated by:

\[
k = \frac{P \times F(\alpha)}{B \times W^{0.5}}
\]

(17)

where

\[
\alpha = \frac{a}{W}
\]

(18)

and

\[
F(\alpha) = (2 + \alpha) \left[ 0.868 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4 \right]
\]

(19)

The stress intensity factor (SIF) is a function of crack length. So, it is reasonable if we define \( M(\alpha) \) as a SIF memory function. Until now, the SIF effect can be kept in a memory function. However, in a real engineering material as well as continuum aspect the effect of SIF should be considered with propagation of crack in a length domain. So, the operator \( \psi(\alpha) \) as counter effects can be introduced by:

\[
D = \frac{\psi(\alpha)}{\psi(\alpha_i)}
\]

(24)

Hence, regarding (16), (21), (23) and (24) the damage rate in each cycle can be obtained by:

\[
dD \over dN = \frac{C}{\psi(\alpha_i)} \left( \frac{1 - S_{\text{min}}}{S_{\text{max}}} \right)^{0.5} \gamma
\]

(25)

Metallic components are most often under random loading, meaning that uncertainty in peaks and amplitude are always probable. In order to show the effect of random loading in our stochastic model, the amplitude of loading can be defined as

\[
S = \frac{S_{\text{max}} - S_{\text{min}}}{2}
\]

(26)

Therefore, let’s rewrite (25) in different form containing maximum value of stress and stress amplitude.

\[
dD \over dN = \frac{2S(t)}{\psi(\alpha_i)} \left( \frac{S(t)_{\text{max}}}{S(t)} \right)^{0.5} \gamma
\]

(27)

If the damage in each cycle is assumed to be sufficiently small, it is reasonable to define an equivalent frequency by:

\[
f = \frac{dN}{dt}
\]

(28)

For narrow band random loading the value of equivalent frequency can be considered fixed. Therefore, the final stochastic expression of FCG model can be written as follows.

\[
dD \over dt = f \times \left[ \frac{2S(t)}{\psi(\alpha_i)} \left( \frac{S(t)_{\text{max}}}{S(t)} \right)^{0.5} \gamma \right]
\]

(29)

If load amplitude and maximum stress are considered as random variables, then by integrating equation (29) the damage expression in time domain can be obtained as.

\[
D(t) = \frac{2fC}{\psi(\alpha_i)} \int_0^t S(t) dt
\]

(30)
V. CONCLUSION

The key contributions of this study are:

- From the sequence of equations given, we can see that a mean stress equation can be incorporated into the fatigue crack growth model.
- The presented technique could establish a stochastic form for given deterministic model equation (29) and (30) where the damage rate during the time of propagation of the crack can be determined.
- If the damage per cycle rate is greater than zero, then it can be applicable to finding crack lengths after applying certain number of cycles.

Some suggestions for the future work are:

- Accommodation of uncertainty quantification in the stochastic model is one of the highlighted advantages of this technique, which need further studies.
- This kind of model has a good potential for helping to develop a universal model in fatigue crack growth modeling in the near future.

REFERENCES


