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Cash Distribution Using Skip Concept for Heuristics Vehicle Routing Methodology

N. Suthikarnnarunai, *Member, IAENG,* P. Boonsam, *Member, IAENG,* W. Wattanalertrak, K. Engchuan, and C. Taprab

***Abstract*—This article presents new heuristics concept for solving Vehicle Routing Problem with Time Window. New concept, which is a skip concept, derives from a solution of the traditional sweep and nearest neighbor technique, which gives poor vehicle's utilization. Hence, skip – sweep algorithm and skip – nearest neighbor search focus on increasing the vehicle's utilization. The case study of cash distribution has been applied to proof efficiency of the skip concept. New concepts have been resulting in using the same amount of vehicle as the traditional one. The favorable result is the increasing of the vehicle's utilization. The decreasing of total travel time, which is an unexpected result, has found from the case study. It turns to be a favorable result since it associates to the variable cost of fuel.**

***Index Terms*—Vehicle Routing Problem with Time Window, Skip Sweep Algorithm, Skip Nearest Neighbor Search Algorithm, Cash Distribution**

I. Problem Background

Vehicle Routing Problem (VRP) is one of the famous planning techniques used in Logistics as it can effectively reduce cost of transportation. Nowadays, business practices are increasing in complexity, resulting in a complicated transportation problem. Transportation planning can be divided into three levels – strategic planning, tactical planning, and operational planning.

--Strategic planning takes part of organization's processes or decisions. The transportation decisions make in this level may include distribution center or hub location selection and service scope.

--Tactical planning breaks down bigger-picture goals and strategies into narrower. The decision regarding the procurement of vehicles to be used in the transporting goods is in this level.

--Operational planning involves routing of freight to minimize daily distribution cost.

Banking business is just like any other businesses. It needs to transport cash to the bank branches or ATM to meet customer needs promptly at the minimum operation cost. Currently, a competition in the banking business in Thailand is becoming more severe. New comers from other countries come to merge or take over many of Thai banks. The latest acquisition is Bank of Ayudhya which was taken over by Mitsubishi UFJ Financial Group Inc. [1].

The case study bank has an awareness of the entry of a new competitor into the market and uses a transportation planning as a strategic to overcome the competitor. Cash distribution to bank branches and ATM has to be in time at the lowest cost. A VRP software using grouped sweep technique was developed in 2011 [2], [3]. Results from that such program give a daily cash distribution route. However, some vehicles might experience low utilization, due to the algorithm. A branch is selected based on the degree angle which comes first. If the demand of the branch exceeds the vehicle's capacity limit, next vehicle starts, no matter how much capacity of the previous vehicle left.

The researchers have an awareness of such poor results. Therefore, new technique, called a skip technique, is applied to solve this problem. Considering the road network in Bangkok and urban area, it is not a square lattice. The scatter of branches and ATM does not spread very far apart. The neighborhood search technique is in this research. Moreover, the skip technique in conjunction with sweep and neighborhood search techniques are also presents. All techniques, presented in this research, have the objectives in increasing the utilization of the vehicle, reducing in the amount of vehicle used, and minimizing the overall distribution cost.

II. Review of Literature

VRP is a classical transportation problem that has received an attention from many researches for a long time. It has been recognized as a problem that hardly solved. In conjunction with the increasing in complexity of the businesses' operations, several variants of VRP with several solving techniques have developed. From the review of literature, the techniques to solve VRP can classify into three groups – optimization technique [4], [5], [6], meta-heuristics technique [7], [8], [9], and classical heuristics technique [10].

However, not every technique is suitable for any problems. Some techniques may use very long computational time which is not appropriate for daily operation used. The applications of VRP in the real world

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N. Suthikarnnarunai, PhD., is now Director Ph.D. program in Logistics, School of Engineering, University of the Thai Chamber of Commerce, Bangkok,10400, Thailand, (phone:+66859071889, fax:+6626923014, email: nanthi_sut@utcc.ac.th)

P. Boonsam, is now a lecturer in Industrial Management Program, Nakhon Ratchasima Rajabhat University, Nakhon Ratchasima, 30000, Thailand, (phone:+6681046970,email: pratboonsam@yahoo.com)

W. Wattanalertrak, K. Engchuan, and C. Taprab., are now a Bachelor student in Logistics Engineering, School of Engineering, University of the Thai Chamber of Commerce, Bangkok,10400, Thailand

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business include newspaper distribution [11], waste collection [12], post distribution [13], milk distribution [14], etc.

The major variants of VRP include Capacitated VRP (CVRP), VRP with Time Windows (VRPTW), VRP with Backhauls (VPRB), VRP with Pick-up and Delivery (VRPPD), Split Delivery VRP (SDVRP), Open VRP (OVRP), Stochastic VRP (SVRP), and Multi-depot VRP (MDVRP). This section presents a brief detail of each VRP variant stated above.

A. Capacitated VRP (CVRP)

CVRP is the basis of the VRP which has a basic assumption as follows [15], [16], [17]:

--The total travel time of each vehicle is limit and is equal in every vehicle.

--Every vehicle starts and ends at depot.

--Each customer's demand must not exceed the vehicle's capacity.

--Each customer can be served by only one vehicle.

If the limitation of total travel time is substituted by total travel distance while the other characteristics still hold, this variant might be called Distance Constraint VRP (DVRP).

B. VRP with Time Windows (VRPTW)

VRPTW has the same condition as CVRP. The difference is the time window or time limitation constraint which indicates the delivery time to customer [18], [19].

C. VRP with Backhauls (VPRB)

VRPB shares all conditions as CVRP, except before returning to depot, vehicle will pick up goods from other location back to depot [20], [21], [22]. VRPB can integrate time window as one of the constraints to be met. This will give another variant as Vehicle Routing Problem with Backhauls and Time Window (VRPBTW).

D. VRP with Pick-up and Delivery (VRPPD)

VRPPD has two ways of implementation: 1). Picking up goods from one customer and send to another customer, 2). Mix service [23], [24]. Both ways can combine with time window constraint. Vehicle Routing Problem with Pick-up and Delivery and Time Window (VRPPDTW) exists. It increases the complexity to the problem.

E. Split Delivery VRP (SDVRP)

SDVRP drops two assumptions of CVRP. The amount of goods to pick up or delivery at each customer's location can exceed the vehicle's capacity. Therefore, it is allowed to split load into more than one vehicle. Other assumptions are still the same as the CVRP [25], [26], [27]. A mathematical model for SDVRP is presented in [28]. When includes time window into this variant, this variation changes to Split Delivery VRP with Time Window (SDVRPTW).

F. Open VRP (OVRP)

In this variant, the vehicle will not return to the depot after finish the operation. A transportation outsource business is the application of OVRP [29], [30], [31], [32].

G. Stochastic VRP (SVRP)

In this case, the value of travel time or distance between each location is not a deterministic. It is from statistical data under various forms of the probability distribution. Therefore, the solution to this problem is likely to be most chance to happen [33], [34], [35], [36]. Ref [28] also presents a mathematical model for OVRP.

H. Multi-depot VRP (MDVRP)

MDVRP is a transportation problem with multiple depots [37], [38]. An allocation technique is firstly employed to split customers into group. Therefore, each group is similar to CVRP or CVRPTW or other variant of VRP described above.

From the review of literature above, each VRP variant has different constraint. Table 1 is a summary of characteristic with it choices which is a guideline to categorize a problem into each VRP variant.

TABLE I
GUIDE OF CHARACTERISTICS TO CLASSIFY THE VARIANT OF VRP

Characteristics	Choices
Number of fleet	- One - Many
Vehicle type	- Homogeneous - Heterogeneous
Number of depot or number of distribution center	- One - Many
Customer's demand	- Deterministic - Stochastic
Source of demand (Demand location)	- At the customer location (on node) - On the road network (on arc)
Vehicle capacity	- Same in every vehicle - Not the same in each vehicle
Maximum route time	- Equal in each route - Not equal in each route
Time windows	- Single side - Double side

III. A VRP CASH DISTRIBUTION CASE STUDY

A case study bank has three cash distribution centers (DC). Each center has a responsibility in distributing cash to its branches. There are a total of 376 branches – 151 members of the first DC, 90 members of the second DC, and 135 members of the last DC. Previously, the operations' assumptions or characteristics in distributing cash to branches of each DC are as follows:

--Every vehicle starts at the depot at 8.00 am and returns to the depot at 5.00 pm. Each branch must receive cash before 2.00 pm. of each day. Therefore, the total operation time is only 360 minutes.

--Each vehicle has one driver and two security guards. In case that the vehicle can not return to the depot on time, the overtime must be paid.

--Each branch might not need cash every day. Cut off time in notifying need of cash is at 5.00 pm. of the previous day.

--A capacity of the vehicle is 200 million baht.

Before 2011, the case study bank has routed the cash distribution without any decision tools. Experienced person made all decision in making such routes. Distribution routes

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are the same in every day. The vehicle just dropped the location without replacement, if there is no need to service. For example, vehicle number one responsible in delivery cash to 13 branches, if six branches do not need cash, this vehicle will service only 7 branches. Operating with such methods gave a low vehicle's utilization with excessive use of vehicles.

Since 2011, the VRP software has been developed to solve the case study's cash distribution problem. At the beginning, the researchers chose grouped sweep algorithm to route a daily cash distribution. All characteristics of the operations are as described at the beginning of this section. The solutions from this method apparently reduce in number of vehicle used in each DC. Table 2 gives the number of vehicles used before and after the improvement.

TABLE II
NUMBER OF VEHICLE USED IN EACH DC

DC	Before Improvement (# of vehicle)	After Improvement in 2011 (# of vehicle)
A	11	10
B	7	7
C	11	9

After the improvement, each DC needs a less of vehicles than before. However, due to limitations of the previous algorithm, as already mentioned in section 1, utilization of the vehicle is still low. In practice, the utilization of the vehicle can be raised by relaxing the time window of DC and branches. However, the overtime, which is a variable cost, must be paid. The impact that might cause by extending time window of branches is a dissatisfaction of customers due to lack of cash on time. In the long run, the impact may be more severe. The customer might switch to another bank. The opportunity loss cost exists.

The variant of this case study is VRPTW because there is a time window on returning to DC and on delivery cash to branches [39]. The VRPTW may be defined mathematically as follows [40], [41]. Let $G = (N, A)$ be a road network where $N = \{0, 1, \dots, n\}$ is the vertex set or customer locations. Vertex 0 is the starting depot and vertex $n+1$ is the returning depot while $N \backslash \{0, n+1\}$ denote set of customers, C, on the road network. Associated with each customer is a demand d_i. The arc set, $A \subseteq N \times N$, is the connection between the depot and the customers and among the customers. There are no arcs ending at vertex 0 and starting from vertex $n+1$. The parameter c_{ij} represents cost between vertices i and j, while t_{ij} represents service time at customer location i. Each vehicle in a fleet of vehicles, V, has a capacity q.

Customer at each location has time window $[a_i, b_i]$, represent the earliest possible departure from the depot and the latest possible arrival at the depot, respectively. The time window at each depot denotes as $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ and are assumed to be identical. All parameters present in the model are non – negative integers and t_{ij} is a positive integer. A decision variables x_{ijk} is a binary decision variable has a value of 1 if vehicle k drives directly from customer location i to location j, and 0 otherwise. A decision variable w_{ik} is a real number denotes the time vehicle k starts to service customer i. The formulation of VRPTW non – linear model is as follows:

$$\min \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk} \tag{1}$$

Subject to:

$$\sum_{k \in V} \sum_{j \in N} x_{ijk} = 1 \quad \forall i \in C \tag{2}$$

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijk} \leq q \quad \forall k \in V \tag{3}$$

$$\sum_{j \in N} x_{0jk} = 1 \quad \forall k \in V \tag{4}$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0 \quad \forall h \in C, \forall k \in V \tag{5}$$

$$\sum_{i \in N} x_{i,n+1,k} = 1 \quad \forall k \in V \tag{6}$$

$$x_{ijk} \left(w_{ik} + t_{ij} - s_{jk} \right) \leq 0 \quad \forall i, j \in N, \forall k \in V \tag{7}$$

$$a_i \leq w_{ik} \leq b_i \quad \forall i \in N, \forall k \in V \tag{8}$$

$$x_{ijk} \in \{0,0\} \quad \forall i, j \in N, \forall k \in V \tag{9}$$

The overall goal of VRPTW is to minimizing total cost as shown in (1). The constraint (2) ensures that each customer is serviced exactly once by vehicle k. Each vehicle has a limit capacity as shown in (3). Equations (4) and (6) ensure that every route must start at depot 0 and return at deport $n+1$, while equation (5) indicates that same vehicle enters and leaves a given customer vertex. The inequalities constrain (7) shows the relationship between the vehicle departure time from a customer j and customer j. Time window constraint is equation (8). Lastly, (9) is an integrality constraint.

Since VRPTW model shown above is a non – linear mathematical model. It creates a critical computational technique which will be very complicated. Therefore, linearization technique can be applied [39], [40] by substitute (7) by (10), where M_{ij} can be decreased to $max\{b_i + t_{ij} - a_j\}, (i, j) \in A$

$$w_{ik} + t_{ij} - M_{ij} \left(1 - x_{ijk} \right) \leq w_{jk} \quad \forall i, j \in N, \forall k \in V \tag{10}$$

IV. A DECOMPOSITION ALGORITHM FOR VRP

As the mathematical formulation shown in the previous section, exact method which gives the optimization solution is one of the choices in solving the problem. However, VRP falls into the category of NP-Hard problems [42]. For such problems, the size of the instances grows the running time for exhaustive search [43], which is not suitable for daily operation planning. Therefore, it is often desirable to adopt alternative methods which give an approximated solution. But, its computational time is significantly faster than the original method. In the recent year, there has been growing interest in designing various algorithms to solve VRP. This research presents new techniques called skip – sweep algorithm and skip – nearest neighbor search method. Both techniques integrate an idea of skipping location that violated the constraints to the next location. The programming language used to develop our software is PHP and MySQL. The maximum computational time for our software is 5 minutes (mostly 2 minutes).

The sweep algorithm is a method for clustering customers (bank branches in this case) into groups so that branches in the same group are geographically close together and can be served by the same vehicle [44]. In our case, the skip concept is integrated. The skip – sweep algorithm uses the following steps.

1. Locate the depot as the center of the two-dimensional plane.
2. Compute the polar coordinates of each branch with respect to the depot.
3. Start sweeping all branches by increasing polar angle.
4. Assign each branch encompassed by the sweep to the current cluster.
5. Skip branch which violate the maximum capacity of the vehicle or time constraint.
6. Stop the sweep when adding the next branch would violate the maximum vehicle capacity.
7. Create a new cluster by resuming the sweep where the last one left off.
8. Repeat steps 4 – 6, until all branches have been included in a cluster.

Nearest neighbor search (NNS) is another famous method for clustering customer into groups. The closest point search technique is another suitable name of this method [45]. Skip technique is also applied to this method. Hence, the algorithm for skip-nearest neighbor search is as the following steps.

1. Start assigning branch which is closest to the depot into cluster.
2. Assigning another branch which is the closest to previous branch into the cluster.
3. Repeat step 2 until the capacity and time window constraints are violated.
4. Skip a branch that violated the constraints stated in step 3, at most 3 locations.
5. Repeat step 2 until the capacity constraint or time constraint can not be hold.
6. Create a new cluster by resuming the process in steps 1 – 4, until all branches have been included in a cluster.

The difference between traditional method and skip concept method is a skipping allowance for the customer locations which violate the capacity and time constraint. However, this research allows a maximum of three locations to be skipped. The researchers have tested these methods with other test sets. It found that the travel distance might be very large if 1). The number of skipped location is not limited, and 2). The maximum skipped distance is not limited. The transportation cost and delay time might be increased.

V. Finding and Contribution

The results of all methodologies have shown in Table 3 – 5. The cost presents in the table includes fixed cost and variable cost. The vehicle leasing cost, driver's salary, and guards' salary are fixed cost. The fuel cost per kilometer is a variable cost.

TABLE III
A COMPARISON OF THE RESULTS USING DIFFERENT METHOD OF THE FIRST DC

Method	No. of Vehicle	Total Distance (Km)	Cost (Bath)	Increasing of Utilization (%)
Sweep Algorithm	11	4023.4	628,070.20	40%
Skip Sweep Algorithm	11	3989.6	627,968.80	
Nearest Algorithm	9	3162.6	513,487.80	48%
Skip Nearest Algorithm	9	3090.9	513,272.70	

TABLE IV
A COMPARISON OF THE RESULTS USING DIFFERENT METHOD OF THE SECOND DC

Method	No. of Vehicle	Total Distance (Km)	Cost (Bath)	Increasing of Utilization (%)
Sweep Algorithm	7	2439	399,317.00	34%
Skip Sweep Algorithm	7	2224	398,672.00	
Nearest Algorithm	6	2114.8	342,344.40	40%
Skip Nearest Algorithm	6	2113.6	342,340.80	

TABLE V
A COMPARISON OF THE RESULTS USING DIFFERENT METHOD OF THE THIRD DC

Method	No. of Vehicle	Total Distance (Km)	Cost (Bath)	Increasing of Utilization (%)
Sweep Algorithm	11	2802.6	624,407.80	29%
Skip Sweep Algorithm	11	2802.6	624,407.80	
Nearest Algorithm	8	2719.9	456,159.70	40%
Skip Nearest Algorithm	8	2724.9	456,174.70	

From the results in Table 3 – 5, the nearest neighbor search is likely to be a better method in solving VRP in all aspects. The skip concept has been resulting in using the same amount of vehicle in every DC. The noticeable result is the increasing of the vehicle's utilization. The travel distances received from skip concept decreased in the first two DCs but not in the last DC. The latter result is favorable because it affects the variable cost which is the fuel price.

By closely considering the results described above and the scatter of the members in each DC, it found that the average distance between branches in the first two DCs is less than in the third DC. This might be the cause of the decline of the overall travel distance received by the skip method.

However, it still needs the further research to confirm this assumption.

VI. Summary and Recommendation

This research aims to present the efficient method for daily of cash distribution. All methodologies present in this paper focuses on cost reduction and vehicle's utilization. The skip concept has integrated with two classical heuristics – sweep algorithm and nearest neighbor search. The new concept is likely to give higher vehicle's utilization and less traveling distance. The result in the latter case is still in doubt. The relationship between the scatter of the members and the total traveling distance needs to be proofed. The optimal skipped location allowance also needs to be investigated.

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