An Improved Cross-Correlation Method Based on Fractional Delay Estimation for Velocity Measurement of High Speed Targets

Xinggan Zhang, Yechao Bai*, Xiaoli Chen

***Abstract*—Velocity estimation of high-speed targets is very important for target detection, identification and imaging. It is difficult to apply the traditional Doppler velocity method into estimation of high speed targets, since the target echoes may not locate at the same range bin, and the Doppler velocity is seriously ambiguous. Currently a wideband cross-correlation method is widely utilized to estimate the velocity. However, the measurement accuracy is limited by the sampling interval. An improved cross-correlation velocity measurement method based on the fractional delay estimation is proposed in this paper, and simulations are carried out. The results show that the estimation accuracy of the proposed method is within 1m/s in the situation of wideband. The proposed method breaks through the limitation of integer sampling interval, and results in higher estimation accuracy.**

***Index Terms*—fractional delay estimation, cross-correlation, velocity measurement, high-speed targets, wideband**

I. Introduction

WITH the rapid development of ballistic missile technology and astrospace detection technology, the detection of high-speed targets and the parameter estimation have become a research hotspot. The velocity estimation of high-speed targets is very important for target detection, identification and imaging [1]-[5].

The velocity of moving targets can be estimated by the phase and envelope of echoes [6]-[8]. Traditionally, we use the Doppler frequency measurement method to estimate the velocity [8]-[10], which applies Fourier transform to the same range bin in different repetition pulse echoes to estimate the Doppler frequency. The estimation range and accuracy of the Doppler frequency are mainly determined by pulse repetition frequency and the coherent processing time. As the echo coherence decrease quickly with the changing range, high-speed targets cannot be chronically accumulated. Wideband radars are usually used for high range resolution radar [11]. However, this is hard for high speed targets, since the target position may migrate through resolution cell at adjacent pulses.

In this paper, we describe the velocity measurement method with cross-correlation first. Then considering the limitation of estimation accuracy due to the sampling interval, an improved method based on fractional delay estimation is proposed. Simulations are also carried out and analyzed in this paper. At last, the conclusion is given out.

II. The Velocity Measurement Method With Cross-Correlation

The cross-correlation method utilizes the change rate of relative distance between the target and the radar to measure the velocity. Since the change of target characteristics in any two adjacent echoes is not significant, the signals in any two adjacent frames have strong correlation. The delay between the signals in any two adjacent echoes can be estimated by the maximum value position of the cross-correlation result of the two echoes. Thereby, the moving distance of the target in the interval of the two echoes can be calculated. And then we can get the instantaneous radial velocity of the target with the pulse repetition period. Multiple scattering points may be contained in one target, when wideband signal is employed. Scattering points in different directions have amplitude fluctuation, which is also disturbed by the clutter and the noise. Correlation utilizes the information from every scattering point comprehensively. In any two adjacent periods, the correlation coefficient of the target echoes is much larger than that of the noise and clutter. So this method has strong anti-interference ability.

The wideband radar has high range resolution, which leads to more accurate cross-correlation peak position. Therefore, the velocity estimation method with cross-correlation is particularly suitable for wideband radar.

Assuming the adjacent echoes after dechirp are $s_1(n)$ and $s_2(n)$, $n = 0,1,2,\ \ldots,\ N-1$, the cross-correlation can be achieved in the frequency domain as follows:

$$\begin{aligned} R_{12}(k)_N &= \sum_{n=0}^{N-1} s_1^*(n) s_2(n+k) \\ &= IFFT[(FFT(s_1))^* \bullet FFT(s_2)] \end{aligned} \tag{1}$$

where the superscript * denotes the complex conjugate.

If the peak position of cross-correlation is k_0, the relationship between pulse repetition period and range difference of adjacent pluses is

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$$\frac{2vT_r}{c} = \frac{k_0}{f_s} \tag{2}$$

where T_r denotes the pulse repetition period, and f_s denotes the sampling frequency.

The velocity can be obtained as

$$v = \frac{k_0 c}{2 f_s T_r} \tag{3}$$

According to the former analyses, the error of the estimated velocity with the cross-correlation method is mainly affected by sampling frequency, pulse repetition period. In the case of high SNR, the highest estimation accuracy of cross-correlation peak position is half of the range bin. The maximum error of estimated velocity is

$$\Delta v = \frac{1}{2} \bullet \frac{1}{2} c T_s / (2T_r) = \frac{c}{8 f_s \bullet T_r} \tag{4}$$

For example, when the bandwidth B=1GHz, the sampling frequency f_s=2GHz, and the pulse repetition frequency is 300Hz, then the maximum velocity error is 5.625m/s.

The sampling interval affects the accuracy of the peak position estimation of cross-correlation, which limits the velocity estimation accuracy. An improved cross-correlation velocity measurement method that based on the fractional delay estimation is proposed to break through the sampling interval limitation.

III. Improved Velocity Measurement Method Based on Fractional Delay Estimation

Generally, interpolation can be used to realize fractional delay estimation to reduce the measuring error caused by range quantization. However, this method suffers heavy computational loads, and is also too hard for engineering realization.

The fractional delay estimation can obtain continuous delay estimation. And the estimation error is much smaller than the sampling interval. A method of weighted linear fitting to the phase of cross power spectrum is utilized to realize the fractional delay estimation, which can improve the estimation accuracy of the cross-correlation peak position.

Suppose the two adjacent echo signals are respectively as follows

$$\begin{aligned} r_1(i) &= s(i) + n_1(i) \\ r_2(i) &= s(i-D) + n_2(i), i = 0,1,\dots N-1 \end{aligned} \tag{5}$$

The parameter D represents the fractional delay to be estimated between the two echoes. $s(i)$ denotes the discrete sample of target signal. $n_1(i)$ and $n_2(i)$ are zero-mean stationary Gaussian processes with variances of σ^2, being uncorrelated with each other and with $s(i)$. N is the number of sampling. And without loss of generality, let N be even.

Let $R_1(k)$ and $R_2(k)$ be the discrete Fourier transform of $r_1(t)$ and $r_2(t)$ respectively, and their cross-spectrum can be expressed as

$$\begin{aligned} R_1(k)R_2^*(k) &= |S(k)|^2 \exp\left(j\frac{2\pi}{N}kD\right) + S(k)N_2^*(k) + \\ & S^*(k)N_1(k)\exp\left(j\frac{2\pi}{N}kD\right) + N_1(k)N_2^*(k) \\ &= |S(k)|^2 \exp\left(j\frac{2\pi}{N}kD\right) + W(k), \\ k &= -N/2+1, -N/2+2, \dots N/2 \end{aligned} \tag{6}$$

where the superscript * denotes complex conjugate, $S(k)$, $N_1(k)$, $N_2(k)$ are the Fourier transforms of $s(i)$, $n_1(i)$, and $n_2(i)$ respectively, and the noise component is

$$\begin{aligned} W(k) = S(k)N_2^*(k) + S^*(k)N_1(k)\exp\left(j\frac{2\pi}{L}kD\right) \\ +N_1(k)N_2^*(k) \end{aligned} \tag{7}$$

From (6) we can see that the time delay appears in the cross-spectrum as a phase function

$$\varphi(k) = \frac{2\pi}{N}kD \tag{8}$$

The phase is a linear function of the time delay D. If there is noise, the cross-spectrum phase fluctuates along the line. Then the time delay can be estimated by fitting a straight line to the cross-spectrum phase. Considering the symmetric cross-spectrum of real functions, the positive frequency part is taken into account only. According to the least squares approach, D is estimated as

$$\hat{D} = \frac{N}{2\pi}\frac{\sum_{k=1}^{N/2-1}\Phi(k)k}{\sum_{k=1}^{N/2-1}k^2} \tag{9}$$

where $\Phi(k)$ is the phase of cross-spectrum.

Each phase contributes the same in (9). Nevertheless, the SNR of cross-spectrum is different at different frequencies, thus it is more reasonable to give different weights when fitting.

The cross-spectrum is normalized firstly to find the weights for straight line fitting as

$$\begin{aligned} \frac{R_1(k)R_2^*(k)}{|S(k)|^2} &= \exp\left(j\frac{2\pi}{N}kD\right) + \frac{W(k)}{|S(k)|^2}, \\ k &= 1,2,\dots\frac{N}{2}-1 \end{aligned} \tag{10}$$

$N_1(k)$ and $N_2(k)$, k=1,2…N/2-1 follow independent and identical zero-mean complex Gaussian distribution with variance $N\sigma^2$. With high SNR, the last item of $W(k)$ can be omitted. Then $W(k)$ becomes zero-mean complex Gaussian distribution with variance $2N|S(k)|^2\sigma^2$.

Now, we build a model from (10) to find the weights. Since $\exp(j2\pi kD/N)$ has been used to fit a straight line and the task is to find the weights now, $\exp(j2\pi kD/N)$ can be replace with a constant A which we want to estimate by the best linear unbiased estimator (BLUE). The model is shown as

$$x(k) = A + W_n(k), k = 1,2,\dots N/2-1 \tag{11}$$

where $x(k)$ and $W_n(k)$ denote the item at the left side of equal in (10) and the last item of (10) respectively. $W_n(k)$ follows zero-mean complex Gaussian distribution with variance $\sigma_w(k)^2 = 2N\sigma^2/|S(k)|^2$. The best linear unbiased estimator (BLUE) for A is

$$\hat{A} = \frac{\sum_{k=1}^{N/2-1} \frac{x(k)}{\sigma_w(k)^2}}{\sum_{k=1}^{N/2-1} \frac{1}{\sigma_w(k)^2}} \tag{12}$$

The weight of the point with the smallest variance is the largest. If the variances for $x(k)$ at each point are the same, it is clear that the estimator of A is the averaging of $x(k)$, and the weights for $x(k)$ are all $1/(N/2\text{-}1)$. Therefore, the weights caused by different variances are

$$p(k) = \frac{\frac{1}{\sigma_w(k)^2}(\frac{N}{2}-1)}{\sum_{k=1}^{N/2-1} \frac{1}{\sigma_w(k)^2}} = \frac{|S(k)|^2}{\sum_{k=1}^{N/2-1} |S(k)|^2}(\frac{N}{2}-1), \tag{13}$$

$$k = 1, 2, \ldots \frac{N}{2} - 1$$

These weights act to pre-whiten $x(k)$ before averaging.

The estimated time delay D is all contained in A in the model, and A, which equals $\exp(j2\pi kD/N)$, is exactly used to fit the straight line. So the weights shown in (13) can be used in the straight line fitting to the cross-spectrum phase. The weighted least squares approach is used to estimate D. The least squares error is

$$\hat{D} = \frac{N}{2\pi} \frac{\sum_{k=1}^{N/2-1} p(k)\Phi(k)k}{\sum_{k=1}^{N/2-1} p(k)k^2} = \frac{N}{2\pi} \frac{\sum_{k=1}^{N/2-1} \Phi(k)|S(k)|^2 k}{\sum_{k=1}^{N/2-1} |S(k)|^2 k^2} \tag{14}$$

From (8), it is clear that the phase will wrap for k=1,2,…N if D is larger than 1. Even the positive frequency part is employed only, D cannot be larger than 2 to avoid wrapping. The integer part of time delay can be estimated by locating the maximum position of cross-correlation first. Consequently, the fractional part can be estimated by weighted straight line fitting without wrapping. Also, it is hard to obtain $|S(k)|^2$ directly, so the absolute value of the cross-spectrum is used instead. Referring to (6), it is seen that this similarity is advisable when SNR is not very low.

IV. Simulation Results and Analyses

Simulation parameters are set as follows: the center frequency of radar signal f_0 =35GHz, the pulse width T=10 μs, the pulse repetition frequency $PRF=300Hz$. The bandwidth B varies between 20MHz-1GHz, the sampling frequency $f_s=2B$, and the SNR=0dB. Besides, the target distance r_0=100km, the velocity v=14km/s.

The traditional cross-correlation method and the improved method are simulated and compared. The results are showed in detail in Fig.1 and Fig.2 as follows.

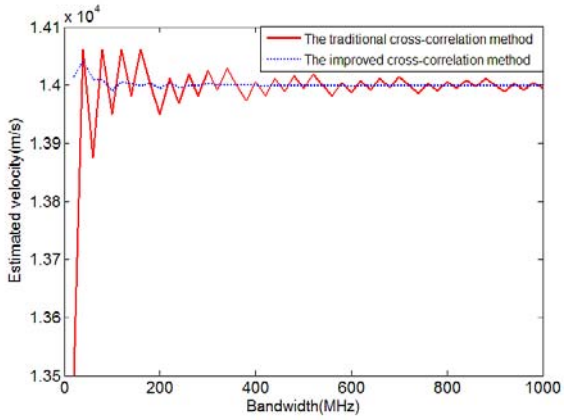

Fig.1 The estimated velocity of traditional cross-correlation method and the improved cross-correlation method

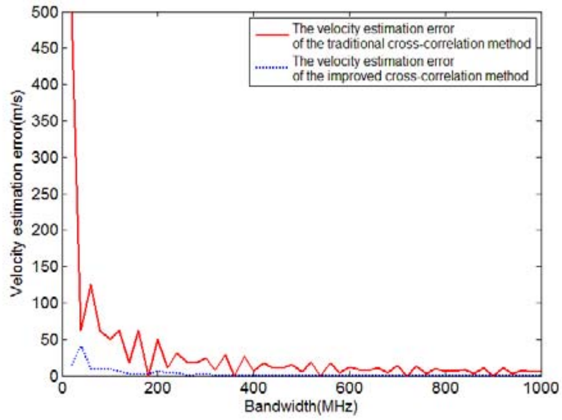

Fig.2 The velocity estimation error of traditional cross-correlation method and the improved cross-correlation method

Fig.1 and Fig.2 show that the velocity estimation method with cross-correlation is more suitable for wideband signals. The estimating accuracy is much higher in the condition of wideband radar. Comparing the two methods, the proposed method can improve the estimation accuracy of cross-correlation peak position by breaking through the limitation of the integer range bin. When the bandwidth varies from 500MHz to 1GHz, the details about velocity measurement error of the two methods are shown in Fig.3.

According to the above theoretical analyses, the cross-correlation velocity estimation error is $\Delta v = \frac{c}{8 f_s \bullet T_r}$.

As can be seen in Fig.3, the trend of the velocity estimation error is consistent with theoretic analyses. However, the velocity estimation error of the improved method is smaller by breaking through the limitation of integer range bin. The velocity estimation error is within 1m/s in the condition of wideband.

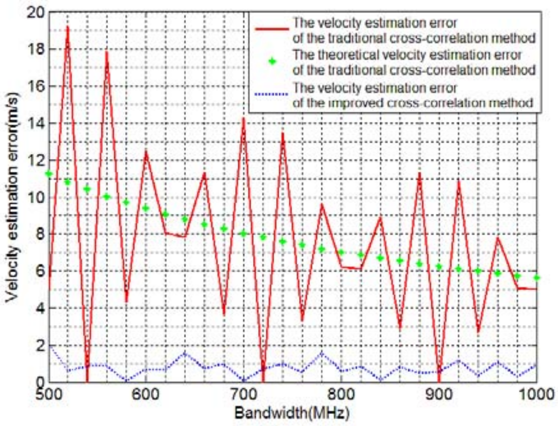

Fig.3 The velocity estimation error of the two methods when bandwidth varies from 500MHz to 1GHz

V. Conclusion

An improved method, based on the fractional delay estimation with weighted linear fitting of the phase of cross power spectrum, is proposed in this paper. The proposed method can improve the estimation accuracy of the velocity measurement method with cross-correlation by breaking through the limitation of integer range bin. Simulations and analyses of both the traditional and proposed methods are accomplished in this paper. The results show that, in the situation of wideband, the velocity estimation accuracy of the proposed method is within 1m/s, and the velocity estimation accuracy can be improved with wider bandwidth.

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