Modeling and Simulation of Partial Blocks of Flexible Energy System in MATLAB&Simulink for Temperature Control of Steam/Air Mixture

Martin Pies, Stepan Ozana, Radovan Hajovsky, and Petr Vojcinak

Abstract—The presented article describes two basic technological blocks of so called Flexible power unit which is located in the grounds of the Vítkovice company Ostrava in the Czech Republic. The experimental unit was established to research the use of potential energy in a mixture of dry air and superheated steam, which is called air/steam mixture. Creation of air/steam mixture is carried out by mixer whose mathematical model is described in this article. Connection of the mixer to the surrounding FES technology is carried out by steel pipes whose mathematical model is also described in this article. This article aims to outline the temperature control of air/steam mixture at selected technological blocks of flexible energy system. The control is performed through linearized model of the mixer in operating point, which corresponds to the operating point of mixer in a real energy unit. Mathematical models of mixer and steel pipelines are implemented in S-functions with the aim of easy simulation of these blocks in Simulink.

Index Terms—Heat exchangers, Mathematical models, Complex systems, Steam, Temperature control.

I. Introduction

THE paper deals with mathematical model of experimental unit of the power plant with low power, see Fig. 1. The whole mathematical model consists of combination of particular mathematical models of the following technological components:

- superheaters of steam/air mixture
- unheated areas (pipelines)
- mixer
- turbine unit

The mathematical model of experimental power plant is presented. The model serves as a base for simulation of both the power plant dynamics and control principles. The paper describes the scheme of the power plant, the structure of its mathematical model, and software implementation of the model by a computer. Superheaters of steam/air mixture and unheated areas (pipelines) are described by sets of nonlinear partial differential equations and thus represent distributed-parameters systems. Mixers serve for heating/cooling of the mixture or adjustment of the concentration. Their mathematical models are based on heat and enthalpy balance equations, and they use *XSteam* library for Matlab with thermodynamic properties of the media. Turbine unit is composed of compressor, compressor turbine and power turbine.

1…compressor
2…compressor turbine
3…power turbine
4…generator
5…overboard bleed valve
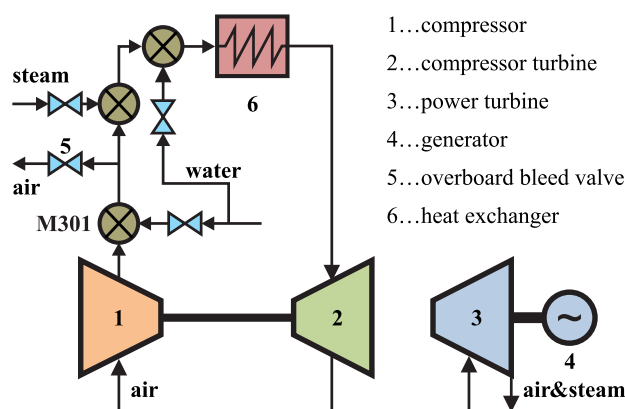6…heat exchanger

Fig. 1. Simplified model of Flexible Energy System

Dynamics of particular components is modeled by use of Simulink S-functions and FDM (finite difference method) to transfer sets of partial differential equations (PDE) into sets of ordinary differential equations (ODE).

II. Mathematical model of heat exchanger

Heat exchanger transfers heat energy from a heating media to a heated media. In a typical power plant heat exchanger a tube bundle is located into a gas channel.

The tube bundle transports the heated fluid, the gas channel transports the heating fluid or vice versa. Heat from the heating media is transmitted to the heated media through the walls of the steel tubes. Fig. 2 shows the principal scheme of the physical state variables of a counter-flow heat exchanger.

In case of simulation in the designed operating point there are not so rapid changes of pressure and power gas temperature. For this purposes, it is possible to divide the heat exchanger described in [1] into temperature and pressure parts. Temperature part of the heat exchanger is described in Fig. 2.

Heat transfer between power gas and the wall of heat exchanger, and between flue gas and the wall of heat exchanger are described by formulas (1) - (3). Unheated area relates to (1) and modified formula (3) that does not include flue gas temperature T_2.

A set of partial differential equations (1) - (3) can be derived for concurrent superheater. It assumes that there is no temperature gradient in the pipeline wall in the radial direction, heat doesn't conduct along axial direction and heat transfer between media and wall occurs in radial direction only [2], [3].
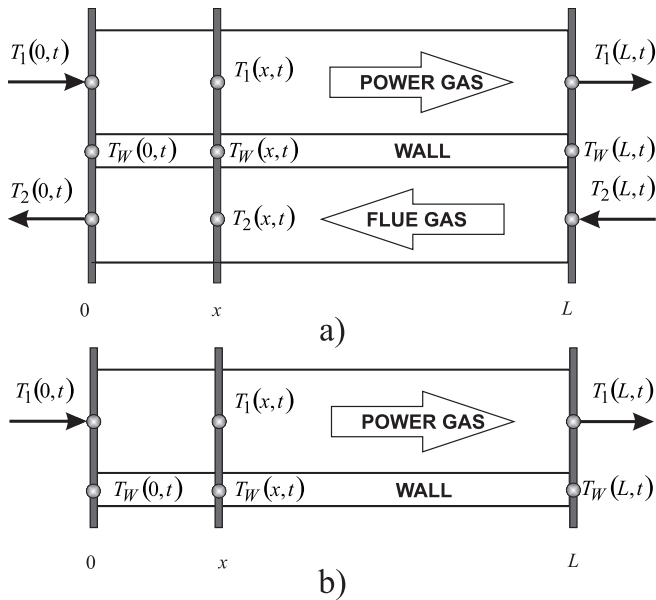
Fig. 2. Principal scheme of the physical state variables at a counter-flow heat exchanger a) and pipeline b)

TABLE I
HEAT EXCHANGER PARAMETERS

symbol	description	unit
c_1	heat capacity of power gas	[J/kg/K]
c_2	heat capacity of flue gas	[J/kg/K]
c_W	heat capacity of superheater's wall material	[J/kg/K]
G	weight of wall per unit of length in x direction	[kg/m]
L	active length of the wall	[m]
M_1	power gas mass flow rate	[kg/s]
M_2	flue gas mass flow rate	[kg/s]
O_1	surface of wall per unit of length in x direction for power gas	[m]
O_2	surface of wall per unit of length in x direction for flue gas	[m]
u_1	velocity of the power gas in x direction	[m/s]
u_2	velocity of the flue gas in x direction	[m/s]
α_{W1}	heat transfer coefficient between the wall and the power gas	$[\text{W/m}^2\text{/K}]$
α_{W2}	Heat transfer coefficient between the wall and the flue gas	$[\text{W/m}^2\text{/K}]$

- For power gas temperature

$$T_W - T_1 = \tau_1 \left[u_1 \frac{\partial T_1}{\partial x} + \frac{\partial T_1}{\partial t} \right] \quad (1)$$

- For flue gas temperature:

$$T_W - T_2 = \tau_2 \left[u_2 \frac{\partial T_2}{\partial x} + \frac{\partial T_2}{\partial t} \right] \quad (2)$$

- For wall temperature:

$$\frac{T_1 - T_W}{\tau_{W1}} + \frac{T_2 - T_W}{\tau_{W2}} = \frac{\partial T_W}{\partial t} \quad (3)$$

where

$$\tau_1 = \frac{c_1 M_1}{\alpha_{W1} O_1 |u_1|} \quad \text{and} \quad \tau_{W1} = \frac{c_W G}{\alpha_{W1} O_1} \quad (4)$$

and further

$$\tau_2 = \frac{c_2 M_2}{\alpha_{W2} O_2 |u_2|} \quad \text{and} \quad \tau_{W2} = \frac{c_W G}{\alpha_{W2} O_2} \quad (5)$$

It is obvious that temperatures T_1, T_2 and T_W depend both on time t and distance x. This distance determines current position of the slice in heat exchanger, as shown in Fig. 2.

Pressure effects in the mathematical model of the power plant yet cannot be neglected. Therefore, heat exchanger model is linked to the pressure part which is described by a static linear model of the lumped system. Then power gas pressure in the heat exchanger $p_1(x,t)$ can be referred to as power gas temperature at the heat exchanger input $p_1(\text{MP}_{\text{in}},t)$ and power gas temperature at the heat exchanger output $p_1(\text{MP}_{\text{out}},t)$, where MP stands for measuring point. Pressure loss Δp over the heat exchanger, respectively over unheated area, is calculated from mass flow rate of power gas M_1 and pipeline load.

Concentration of the steam w_1 in the power gas in the mathematical model of the heat exchanger does not influence the calculation of heat distribution in the heat exchanger. The value of the concentration is copied to the output of the heat exchanger with a delay calculated from the ratio of gas power

velocity u_1 and length L. This concentration of the steam in the power gas is then used for further calculations in the injectors and turbine unit.
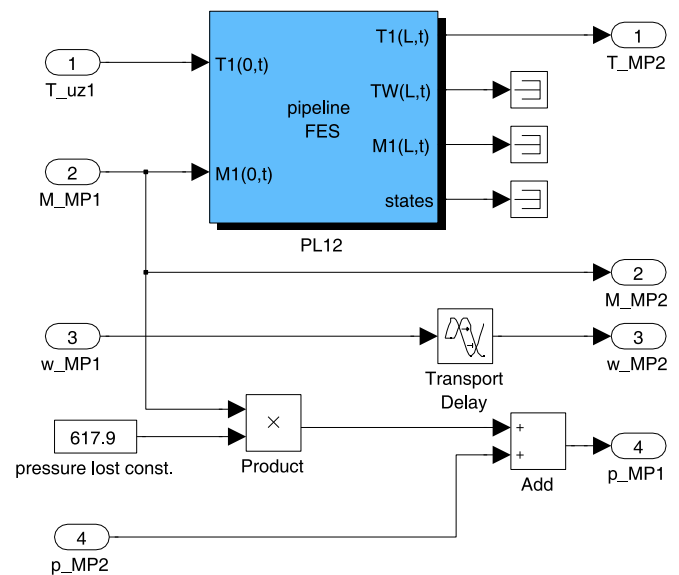

Fig. 3. Mathematical model of unheated area in Simulink with embedded pressure loss over the pipeline.

Fig. 3 shows the mathematical model of unheated area implemented in Simulink environment. Notation **PL12** stands for mathematical model of pipeline (unheated area) placed between compressor **1** and safety valve **5**, see Fig. 1. Location of particular unheated areas in mathematical model of the power plant is shown in Fig. 5 in chapter IV. The inputs of mathematical model of unheated area, see Fig. 3, and of the heat exchanger, are agitated by mass flow rate of power gas M_1 and mass flow rate of flue gas M_2. These signals affect time constants τ_1 and τ_2, see formulas (4), (5).

III. MATHEMATICAL MODEL OF THE MIXER FOR HUMID AIR/WATER

Mathematical model of the mixer for humid air/water, referred to as mixer **M301** in Fig. 1 includes algebraic equations describing blending of the water and humid air compressed by a compressor turbine. Mathematical model supposes that the temperature of compressed air coming into mixer **M301** is higher than boiling point of the water at a given pressure. Only fulfillment of this condition allows entire evaporation of the water being injected into hot air.

The output of mixer **M301** is power gas whose composition is determined by concentration of dry air w_{da} and by concentration of the steam w_s.

The following table II describes the physical quantities involved in mathematical model of mixer **M301**.

TABLE II
PHYSICAL QUANTITIES INVOLVED IN MATHEMATICAL MODEL OF MIXER

symbol	description	unit
h	enthalpy	[kJ/kg]
M	mass flow rate	[kg/s]
p	pressure	[Pa]
Q	heat added / drained per second	[kJ/s]
T	temperature	[°C]
w	concentration of mixture components	[kg/kg]

Indexes

da	corresponds to dry air
pg	corresponds to power gas
ma	corresponds to moist air
s	corresponds to steam
sd	corresponds to steam in moist air
wr	corresponds to water

The assumption to be fulfilled is that both incoming media have the same pressure p_{pg}, corresponding the power gas pressure at the mixer output. The first step includes determination of water steam concentration w_s and dry air concentration w_{da} in power gas. Quantity of the steam in power gas is determined by a relative air humidity d in humid air and quantity M_{wr} of the water being evaporated. Ratios of particular concentrations are illustrated in Fig. 4.

Total quantity of power gas is given by formula (6).

$$M_{pg} = M_{da} + M_s \qquad (6)$$

Concentration of humid air coming to the mixer **M301**, is given by formula (7).

$$w_{ma} = \frac{M_{ma}}{M_{ma} + M_{wr}} \qquad (7)$$

Concentration of the water coming to the mixer **M301**, is given by formula (8).

$$w_{wr} = \frac{M_{wr}}{M_{ma} + M_{wr}} \qquad (8)$$

Humid air includes relative humidity d which must be converted into the form of concentration of the steam in humid air (9).
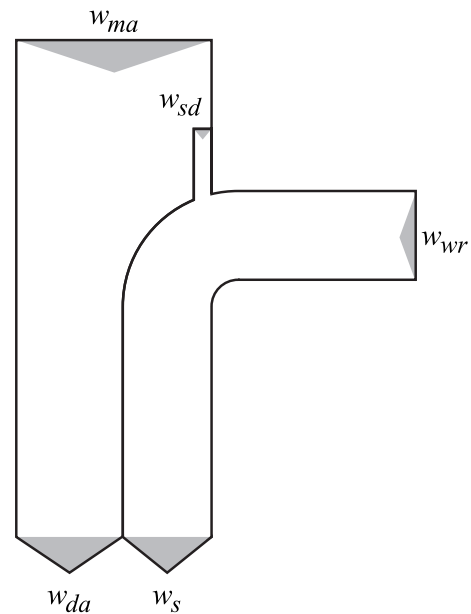
$$w_{sd} = \frac{d}{1 + d} \qquad (9)$$

Fig. 4. Definition of power gas in the block representing mixer **M301**

The concentration of steam in humid air w_{sd} is computed in previous block compressor. Entire concentration of water steam in power gas w_s is computed according (10).

$$w_s = (1 - w_{wr}) \cdot w_{sd} + w_{wr} \qquad (10)$$

Concentration of dry air in power gas w_{da} is then a supplement to one.

$$w_{da} = 1 - w_s \qquad (11)$$

Partial pressure of the water steam and a dry air in power gas are determined by (12).

$$p_s = p_{pg} \cdot \left[1 - \frac{w_{da} \cdot r_{da}}{w_{da} \cdot r_{da} + w_s \cdot r_s} \right]$$
$$p_{da} = p_{pg} - p_s \qquad (12)$$

where r_{da} and r_s are specific gas constants of dry air and water vapor.

Enthalpy of power gas h_{pg}, created as a mixture of humid air and the water is composed of three enthalpy elements. The first one is the enthalpy of dry air h_{da}. This enthalpy can be determined by set of the tables stated in [4] using this command:

```
hda = humde(d,Tma)
```

d	relative humidity level [kg/kg] ($d = 0$)
T_{ma}	temperature of moist air [°C]

The second element is the enthalpy of water steam h_s contained in a humid air. This enthalpy can be determined by set of the tables stated in [5] using this command:

```
hs = xsteam(h_pT,psin,Tma)
```

$p_{s,\text{in}}$	partial pressure of the steam in a humid air [bar] (conversion Pa → bar necessary)
T_{ma}	temperature of moist air [°C]

Partial pressure of the water steam in humid air $p_{s,\text{in}}$ expresses partial pressure of the water steam in power gas before mixing the water and humid air. This partial pressure is computed according formulas (13) and (14).

For $w_{wr} = 0$ according (10) we get

$$w_s = (1 - 0) \cdot w_{sd} + 0 \Rightarrow w_s = w_{sd} \qquad (13)$$

Then

$$p_{s,\text{in}} = p_{pg} \cdot \left[1 - \frac{(1 - w_{sd}) \cdot r_{da}}{(1 - w_{sd}) \cdot r_{da} + w_{sd} \cdot r_s} \right] =$$
$$= p_{pg} \cdot \frac{w_{sd} \cdot r_s}{(1 - w_{sd}) \cdot r_{da} + w_{sd} \cdot r_s}$$
$$p_{da,\text{in}} = p_{pg} - p_{s,\text{in}} \qquad (14)$$

The third component of the mixture enthalpy is the water enthalpy h_{wr} being injected to the humid air. This enthalpy can be determined by set of the tables stated in [5] using this command:

```
hwr = xsteam('h_pT',pwr,Twr)
```

p_{wr} partial pressure of injected water [bar]
T_{wr} temperature of injected water [°C]

Partial pressure of the water p_{wr} means the difference of the partial pressures of the water steam before and after mixing. It can be expressed according (15).

$$p_{wr} = p_s - p_{s,\text{in}} \qquad (15)$$

Thus it is possible to say that particular enthalpies are functions of the following quantity:

$$h_{da} = f\left(T_{ma}\right), \text{ where } d = 0$$
$$h_s = f\left(T_{ma}, p_{s,\text{in}}\right) \qquad (16)$$
$$h_{wr} = f\left(T_{wr}, p_{wr}\right)$$

Entire enthalpy of power gas after mixing humid air with the water will be:

$$h_{pg} = h_{da} \cdot w_{da} + h_s \cdot (1 - w_{wr}) \cdot w_{sd} + h_{wr} \cdot w_{wr} \qquad (17)$$

In case w_{wr} is zero, then p_{wr} is also zero and relation (17) turns into:

$$w_{s,\text{out}} = (1 - w_{wr}) \cdot w_{sd} + w_{wr}; \ w_{wr} = 0$$
$$w_{s,\text{out}} = (1 - 0) \cdot w_{sd} + 0; \ w_{da,\text{out}} = 1 - w_{sd}$$
$$h_{pg} = h_{da} \cdot w_{da,\text{out}} + h_s \cdot w_{s,\text{out}} \qquad (18)$$

In case air at the mixer output **M301** does not contain relative humidity d ($w_{sd} = 0$), then $p_{s,\text{in}}$ is zero and relation (17) turns into:

$$w_{s,\text{out}} = (1 - w_{wr}) \cdot w_{sd} + w_{wr}; \ w_{sd} = 0$$
$$w_{s,\text{out}} = (1 - w_{wr}) \cdot 0 + w_{wr}; \ w_{da,\text{out}} = 1 - w_{wr}$$
$$h_{pg} = h_{da} \cdot w_{da,\text{out}} + h_s \cdot w_{s,\text{out}} \qquad (19)$$

Resulting heat of power gas is described by (20).

$$Q_{pg} = h_{pg} \cdot M_{pg} \qquad (20)$$

This resulting heat Q_{pg} is one of input parameters for iterative algorithm described in [6].

IV. TEMPERATURE CONTROL IN FES

FES unit so far contains 2 control loops. The first one (small, fast) serves for keeping constant temperature T_{MP6} in measuring point **MP6**, see Fig. 5. Regulation of the temperature in **MP6** is carried out by injection **M301**. The second control loop (big, slow) keeps the constant temperature T_{MP13} at the input of turbine unit **MP13**. Regulation in **MP13** is carried out by increasing and decreasing amount
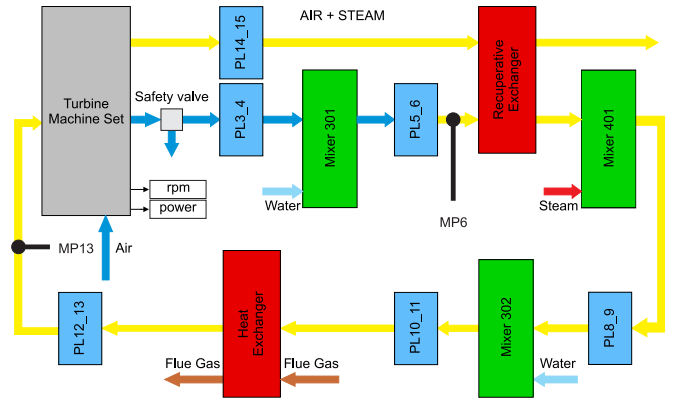

Fig. 5.   Connection of particular blocks with mathematical models of the components of power plant
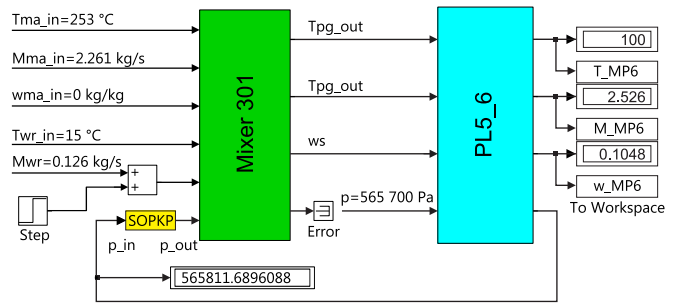

Fig. 6.   Model for identification of controlled plant in a fast loop

of steam/air mixture behind the compressor, see Fig. 5. The paper deals with the fast loop

Out of the FES unit, for identification of the plant set up in a fast loop out injection **M301** and unheated areas **PL5_6** have been chosen, see Fig. 6. Step signal of the water flow was brought to the input of the injection. The step height was 10 %, so the water flow M_{wr} increased from initial value 0.12 kg/s to 0.132 kg/s. Temperature response to that change in **MP6** is shown in Fig. 7.
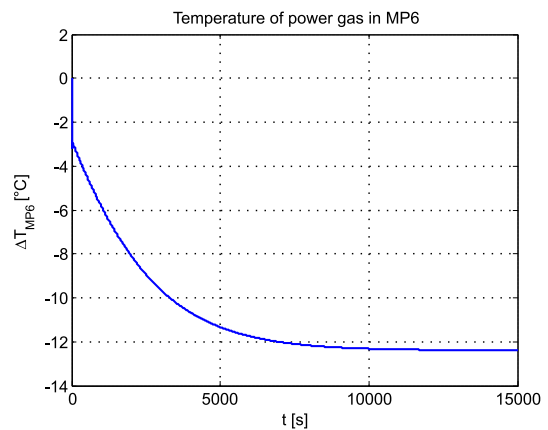

Fig. 7.   Temperature response in **MP6** to 10 % water flow increase at **M301**

The resulting transfer function of the fast loop after identification works out as follows:

$$G\left(s\right) = -1039.1 \frac{488.33s + 1}{(2382.1s + 1)(2.93s + 1)^2} \qquad (21)$$

Comparison between measured and identified responses works out 97.07 %. For a given plant decribed by transfer

function (21) a controller has been designed by use of *pidtool* function, resulting in its transfer function (22).

$$G_R\left(s\right) = -5 \cdot 10^{-5} - \frac{2 \cdot 10^{-4}}{s} \qquad (22)$$

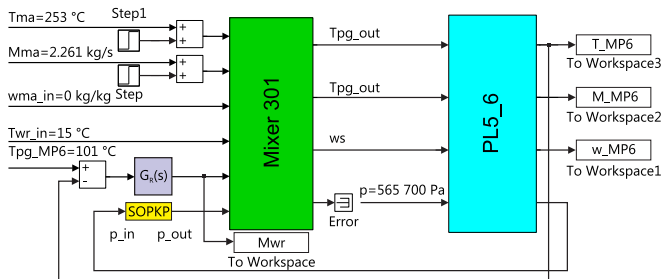Resulting fast loop can be seen in Fig. 8.

Fig. 8.   Fast loop in FES

The control loop keeps the temperature of the media in **MP6** at $T_{\mathrm{MP6}} = 101$ °C. The loop has been tested with two successive disturbances. The first one represents 10 % increase of air temperature at $t = 0$ s. The second one, representing 10 % increase of air mass flow, arose at $t = 200$ s. The resulting responses are shown in Fig. 9.
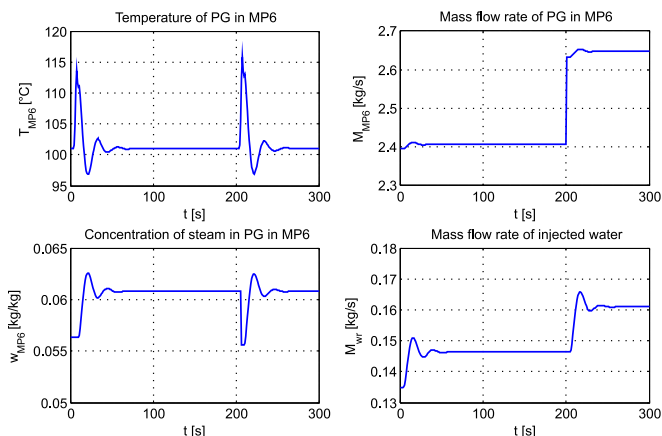

Fig. 9.   Resulting responses in fast loop in FES

V. Conclusion

Presented paper was to show possibilities of identification of particular parts of Flexible Energy System unit in order to design a controller. As an example, injection **M301** with unheated areas **PL5_6** have been chosen. Mathematical models of these parts have been introduced at the beginning of the paper.

Implementation of these blocks is carried out by level-2 Simulink S-functions, introduced in [1], [7], [8], [6]. Similar approach use for the fast loop might be used to design a controller for the slow loop. As it is shown in Fig. 5, FES unit contains 4 actuators determined for regulation, three of them are injections and one is a safety valve determined for fast intervention when regulating temperature at turbine input (measuring point **MP13**).

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