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Credit Derivatives and Global Financial Crisis

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***Abstract*—In this conference paper, we solve an optimization problem involving investor payoffs and credit derivatives such as credit default swaps (CDSs) and mortgage-related collateralized debt obligations (CDOs). In a continuous-time framework, this model enables us to solve a stochastic optimal credit default insurance problem that has investor consumption and investment in structured mortgage products as controls. Finally, we provide numerical results involving mezzanine CDO tranches being hedged by CDSs and explain their link with the global financial crisis (GFC).**

***Index Terms*—credit derivatives; collateralized debt obligation; credit default swap; credit risk; systemic risk; global financial crisis.**

I. Introduction

The period prior to the GFC was characterized by financial product development intended to achieve objectives such as offsetting a particular risk exposure (such as mortgage default) or obtain financing. Examples pertinent to this crisis include the pooling of subprime mortgages into mortgage-backed securities or collateralized debt obligations (CDOs) for investment via securitization and a form of credit default insurance known as credit default swaps (CDSs). In particular, CDO issuance grew from an estimated \$ 20 billion in Q104 to its peak of over \$ 180 billion by Q107, then decreased to under \$ 20 billion by Q108. Further, the credit quality of CDOs declined from 2000-2007, as the level of subprime and other non-prime mortgage debt increased from 5 % to 36 % of CDO assets. In addition, CDOs and portfolios of CDSs called synthetic CDOs enabled a theoretically infinite amount to be wagered on the finite value of mortgages. In this regard, buying a CDS to insure a CDO ended up giving the seller the same risk as if they owned the CDO when the CDO market imploded. This boom in credit derivatives was accompanied by more complexity (compare with the IDIOM hypothesis postulated in [12]). This process increased the number of agents – such as mortgage brokers, specialized originators, special purpose vehicles and their due diligence firms, managing agents and trading desks as well as investors, insurances and repo funding providers – related to mortgage originations. The disconnect from the underlying mortgages resulted in these agents relying on indirect information that included FICO scores on creditworthiness, appraisals, organizational due diligence checks as well as computer models of rating agencies and risk management desks. Instead of spreading risk this provided the ground for fraudulent acts, misjudgments and finally market collapse.

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Some relevant literature about the GFC and its relationship with credit derivatives is given below. Subprime mortgage-related problems were exacerbated by CDO distribution methods, off-balance sheet vehicles, derivatives that resulted in negative basis trades moving CDO risk as well as derivatives that created additional long exposure to subprime mortgages (see, for instance, [3] and [7]). Determining the extent of this risk is also difficult because the effects on expected mortgage losses depend on house prices as the first order risk factor. Simulating the effects of this through the chain of interacting securities is very difficult (see, for instance, [2]). On the other hand, [11] (see, also, [2] and [5]) shows that credit risk transfer through the derivatives market resulted in the origination of inferior quality mortgages by originators. We believe that mortgage standards became slack because securitization gave rise to moral hazard, since each link in the mortgage chain made a profit while transferring associated credit risk to the next link (see, for instance, [8] and [11]). The increased distance between originators and the ultimate bearers of risk potentially reduced originators' incentives to screen and monitor mortgagors (see [2]). The increased complexity of residential mortgage-backed securities (RMBSs) and markets also reduces the investor's ability to value them correctly (see, for instance, [8]).

In this paper, we solve an optimal control problem that depends on a stochastic dynamic model for investor payoff that incorporates credit default insurance and CDO dynamics (see Subsection II-C of Section II). In particular, we are able to set-up an optimal credit default insurance problem that seeks to establish the optimal rate of consumption, k_t^*, over a random term, $[t, \tau]$, and terminal payoff for CDO investors at τ (see Theorem 2.2 in Section II). In terms of the GFC, we consider problematic issues such as the reduction of incentives for monitoring SPEs, incentives to destroy value, credit derivative market opacity, industry self-protection and systemic risk as well as the mispricing of credit (see Theorem 2.2 in Section II).

II. CDSs hedging credit risk from CDO tranches

In this section, we construct a stochastic model for investor payoff that incorporates CDSs. The economic agents involved are investor banks that purchase subprime CDOs and protection from monoline insurers.

A. Structured mortgage products and their losses

In this subsection, we consider subprime CDOs and their losses. We suppose that a investor bank can invest in riskless Treasuries whose price at time t, given by $\mathtt{T}$ follows the process

$$d\mathtt{T}_t = r^{\mathtt{T}}\mathtt{T}_t dt, \text{ for some } r^{\mathtt{T}} \geq 0. \tag{1}$$

The investor can also invest in a risky CDO whose price at t is given by $\mathtt{P}$ and follows the process

$$d\mathtt{P}_u = \mathtt{P}_u\left[r^P du + \sigma dZ_u\right], \tag{2}$$

where the CDO rate $r^P > r^{\mathrm{T}}$ and σ are constants. In addition, Z_t is a standard Brownian motion with respect to a filtration, $(\mathcal{G}_t)_{t\geq 0}$, of the probability space $(\Omega,\ \mathbf{G},\ (\mathcal{G}_t)_{0\leq t\leq T},\ \mathbf{P})$.

Also, the investor is subject to an insurable credit risk modeled as a compound Poisson process, in which N is a Poisson process with deterministic parameter $\phi(t)$ and the CDO loss process, S. Assume that N is independent of Z, which is the Brownian motion of the CDO process. Also, the random CDO loss amount S is independent of N.

B. *Investor payoff under credit default insurance*

In this subsection, we present a stochastic differential equation describing the dynamics of investor payoff under mortgage securitization as well as the risks that can be associated with components of this equation.

1) Model for investor payoff under credit default insurance: At time t, let Π_t and ψ_t be the investor payoff and the amount that the investor invests in CDOs, respectively. The investor earns an exogeneous income rate of $\iota(t)$ and consumes at a rate of k_t at t. For a CDO portfolio with unhedged credit risk we have that

$$\begin{aligned} d\Pi_u^u = &\left[r^{\mathrm{T}}\Pi_u + (\mu - r^{\mathrm{T}})\psi_u + \iota(u) - k_u\right]du + \sigma\psi_u dZ_u \\ &-S(\Pi_u,u)dN_u, u \geq t, \quad \Pi_t = \pi, \end{aligned} \tag{3}$$

where $\mu = r^P - r^T$ for the transaction costs rate, r^T.

Next, we consider a CDO portfolio in which credit risk is hedged via CDSs. In this case, if the investor suffers a loss, S, from CDO default, then it is paid $\mathtt{C}$ at time t. The result is that

$$\begin{aligned} d\Pi_u = &\left[r^{\mathrm{T}}\Pi_u + (\mu - r^{\mathrm{T}})\psi_u + \iota(u) - k_u\right]du + p(u)du \\ &+\sigma\psi_u dZ_u - \left[S(\Pi_u,u) - \mathtt{C}_u(S(\pi_u,u))\right]dN_u, \\ &u \geq t, \quad \Pi_t = \pi, \end{aligned} \tag{4}$$

where the CDS premium payment leg rate and default payment leg are given by

$$p(u) = -(1+\lambda(u))\phi(u)\mathbf{E}^{\mathbf{P}}[\mathtt{C}_u(S)] \text{ and } \mathtt{C}_u(S(\Pi_u,u)), \tag{5}$$

respectively.

2) Numerical results for investor payoff under credit default insurance: In this subsection, a motivating example is provided by the relationship between the insurer AIG (CDS protection seller) and Merrill Lynch (CDO tranches buyer). The latter's major losses in 2008 were attributed in part to the drop in value of its unhedged CDO portfolio after AIG ceased offering CDSs on CDOs. The loss of confidence of trading partners in Merrill Lynch's solvency and its ability to refinance its short-term debt eventually led to its acquisition by the Bank of America. Subsequently, values for investor payoffs are considered for 2000 to 2008. The simulation is obtained via the Euler-Maruyama numerical method. Firstly, we consider the dynamics of investor payoff where credit risk from CDO tranches is unhedged.

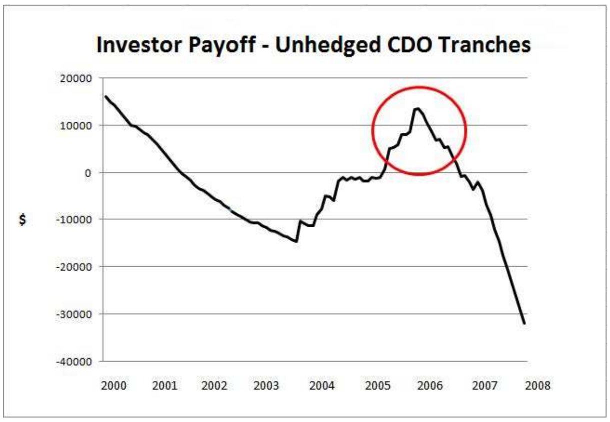

Fig. 1: Investor Payoff–Unhedged Credit Risk from CDO Tranches

Next, we present investor payoff dynamics where credit risk from CDO tranches is hedged by CDSs.

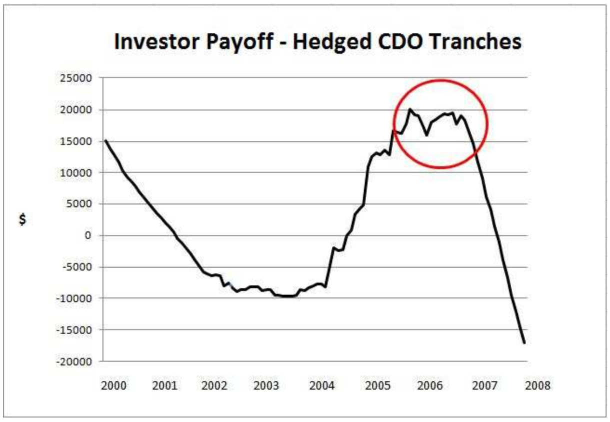

Fig. 2: Investor Payoff–Hedged Credit Risk from CDO Tranches

Figures 1 and 2 reflect investor payoff dynamics when credit risk from CDO tranches was unhedged and hedged by CDSs. If we consider the components of (3) and (4), low interest rates prevailed in 2001-2004 preceded by an increase in interest rates by the Federal Reserve Bank that scuppered the ability of mortgagors to refinance. Defaults increased dramatically in the U.S. in late 2006 and triggered a global financial crisis from 2007 onwards. The downturn of the housing market caused mortgage losses to increase significantly. Notice that losses were significantly less when CDO tranches were hedged than when they were not (compare Figures 1 and 2). The region inside the red circle in Figure 2 bears testimony to an increase in protection seller payments to compensate for counterparty defaults. Simultaneously, the investor's payoff decreased significantly due to subprime bond losses as well as mortgage defaults, foreclosures, etc.

Figures 1 and 2 also support the hypothesis of [3] where it was observed that new issuance of CDOs came to an abrupt halt in early 2007. This took place subsequent to

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the implosion and re-pricing of credit risk in the capital markets. Here it was found that the market inefficiencies were substantial, given the size of the CDO market and the magnitude of CDO fees. During the GFC, CDOs were used to arbitrage a substantial price discrepancy in the mortgage markets and to convert existing mortgages that are priced accurately into new fixed income mortgage-related instruments that are overvalued. Also, the aforementioned figures are related to [7] that uses data on privately-securited subprime mortgages to examine study the increase in defaults after 2007.

C. Optimal credit default insurance

In this section, we solve an optimal credit default insurance problem related to the stochastic model of investor payoff with hedged credit risk given by (4). Because of the unpredictable shutdown of CDO markets, the solution to the insurance problem is determined for a random term $[t, \tau]$.

1) Statement of the optimal credit default insurance problem: We let a set of control processes (laws), $\mathcal{A}$, which is adapted to investor payoff, Π, have the form

$$\mathcal{A} = \{(k_t,\ \psi_t,\ \mathtt{C}_t):\ \text{measurable w.r.t. } \mathcal{F}_t; \quad (4) \text{ has unique solution}\}. \tag{6}$$

The objective function of the stochastic optimal credit default insurance problem is given by

$$J(\pi, t) = \sup_{\mathcal{A}} \mathbf{E}^{\mathbf{P}}\left[\int_t^\tau \exp\{-\delta^r(u-t)\}U^{(1)}(k_u)du + \exp\{-\delta^r(\tau - t)\}U^{(2)}(\Pi_\tau)\right], \tag{7}$$

where, for the first- and second-order differential operators, D and D^2, respectively, we have

$$DU^{(1)}(.) > 0,\ D^2U^{(1)}(.) < 0,\ DU^{(2)}(.) > 0 \text{ and } D^2U^{(2)}(.) < 0.$$

Here, $U^{(1)}$ and $U^{(2)}$ are increasing, concave utility functions and $\delta^r > 0$ is the rate at which the utility functions for consumption, k, and terminal payoff, Π_τ, are discounted. Of course, in principle, one can formulate any utility function. The question then is whether the resulting Hamilton-Jacobi-Bellman equation (HJBE) can be solved (smoothly) analytically ? In the sequel, we obtain an analytic solution for the choice of power utility functions.

We are now in a position to state the stochastic optimal credit default insurance problem for the investor's consumption rate, k, and terminal payoff, Π_τ, for an adjustment term, $[t, \tau]$.

Problem 2.1: **(Optimal Credit Default Insurance):** Suppose that the admissible class of control laws, $\mathcal{A} \neq \emptyset$, is given by (6). Moreover, let the controlled stochastic differential equation for the Π-dynamics be given by (4) and the objective function, $J : \mathcal{A} \to \Re^+$, by (7). In this case, we solve

$$\sup_{\mathcal{A}} J(k_t,\ \psi_t,\ \mathtt{C}_t),$$

and the optimal control law $(k_t^*,\ \psi_t^*,\ \mathtt{C}_t^*)$, if it exists,

$$(k_t^*,\ \psi_t^*,\ \mathtt{C}_t^*) = \arg\sup_{\mathcal{A}} J(k_t,\ \psi_t,\ \mathtt{C}_t) \in \mathcal{A}.$$

The optimal credit default insurance problem determines the optimal consumption rate, k^*, and investor's optimal investment in mortgages, ψ^*, over a random interval. In this regard, Theorem 2.2 provides the general solution to this problem (see Problem 2.1). We note that the objective function in (7) is additively separable in $U^{(1)}$ and $U^{(2)}$ which is not necessarily true for all investors. In our problem, we have a discount rate, δ^r, which is used to discount these utility functions. This discount rate is chosen by the investor and it is not the market discount rate. In the sequel, connections between specific solutions of the optimal credit default insurance problem and the GFC are forged.

2) Solutions to the optimal credit default insurance problem: In this section, we determine a solution to Problem 2.1 in the case where the term $[t, \tau]$ is random. In order to find the optimal control processes, we use the dynamic programming method where we consider an appropriate HJBE. In the sequel, we assume that the optimal control laws exist, with the objective function, J, given by (7) being continuous twice-differentiable. Then a combination of integral calculus and Itô's formula (see, for instance, [14]) shows that J satisfies the HJBE

$$\begin{aligned}\delta^r J = J_t &+ \max_k \left[U^{(1)}(k) - kJ_\pi\right] + (r^{\mathrm{T}}\pi + \iota(t))J_\pi \\ &+ \max_\psi \left[(\mu - r^{\mathrm{T}})\psi J_\pi + \frac{1}{2}\sigma^2\psi^2 J_{\pi\pi}\right] \\ &+ \max_{\mathtt{C}} \Big[\phi(t)\Big\{\mathbf{E}^{\mathbf{P}}[J(\pi - (S - \mathtt{C}(S)), t)] - J(\pi, t)\Big\} \\ &\quad -(1 + \lambda(t))\phi(t)\mathbf{E}^{\mathbf{P}}[\mathtt{C}(S)]J_\pi\Big] \\ &+\omega_b(t)\left[U^{(2)}(\pi) - J(\pi, t)\right],\end{aligned} \tag{8}$$

$$\mathbf{E}^{\mathbf{P}}\left[\exp\left\{-\int_t^s (\rho + \omega_b(u))du\right\}J(\Pi_s^*, s)|\Pi_t^* = \pi\right] = 0, \quad \text{as } s \to \infty.$$

In the sequel, J_t, J_π and $J_{\pi\pi}$ denote first and second order partial derivatives of J with respect to the variables t and π. The objective function, J, is increasing and concave with respect to payoff, π, because the utility functions $U^{(1)}$ and $U^{(2)}$ are increasing and concave. In this case, $\omega_b(t)$ is the hazard rate for investor at time t (compare with the hazard rate analysis in [7]). During the GFC, the hazard rate was very high due to dysfunction in the CDO market. It is important to note that the HJBE (8) can be deduced by using the methods contained in [14]. As a consequence, the integrability and regularity conditions that arise in our paper are covered by these contributions. For instance, in our case, we can use the verification theorems in [14] to show that if our objective function, J, has a smooth solution as well as the related HJBE, $\widehat{J}$, then under our regularity conditions, $J = \widehat{J}$.

Theorem 2.2: **(Optimal Credit Default Insurance):** Suppose that the objective function, $J(\pi, t)$, solves the HJBE *(8)*. In this case, a solution to the stochastic optimal credit default insurance problem is

$$\psi_t^* = -\frac{\mu - r^{\mathrm{T}}}{\sigma^2}\frac{J_\pi(\Pi_t^*, t)}{J_{\pi\pi}(\Pi_t^*, t)}, \tag{9}$$

in which Π_t^* is the optimally controlled payoff under credit default insurance. Also, the optimal consumption rate, $\{k_t^*\}_{t\geq 0}$, solves the equation

$$D_k U^{(1)}(k_t^*) = J_\pi(\Pi^*, t), \tag{10}$$

where D_k represents the ordinary derivative with respect to k.

Proof. The proof is completed via standard arguments about static optimization (see, for instance, [14]). □

3) Optimal accrued premiums: We recall that the CDS accrued premium is the amount owing to the protection seller for investor's credit default protection for the period between the last premium payment and default at τ. This premium has a direct influence on optimal CDS represented by $\mathtt{C}^*$. For instance, from insurance theory (see, for instance, [1] and the extension to continuous-time in [9]), we have that the optimal CDS process is related to classical insurance theory. Analogous to [9] where deductibles were discussed, we can show that in the continuous-time setting, optimal CDS is accrued premium CDS. In this regard, we assume that $0 \leq \mathtt{C} \leq S$. Taking our lead from insurance theory and the assumption that $p(u)$ is proportional to the nett CDS premium for a portfolio with mass of type-A CDOs, λ, the optimal CDS contract takes the form

$$\mathtt{C}(S) = \begin{cases} 0, & \text{if } S \leq \eta; \\ S - \eta, & \text{if } S > \eta. \end{cases} \tag{11}$$

Some features of the aforementioned CDS contract are as follows. If $S \leq \eta$, then it would be optimal for the investor not to buy CDS protection. If $S > \eta$, then it would be optimal to buy CDS protection. In the sequel, the maximization of the CDS contract purchased by the investor is now reduced to the problem of determining the optimal accrued premium, η.

Proposition 2.3: **(Optimal Accrued Premium):** The optimal CDS contract is either no protection or per-loss accrued premium CDSs, in which the accrued premium, η, varies with time. In particular, at a specified time, the optimal accrued premium, η_t^*, solves

$$J_\pi(\Pi_t^* - \eta_t, t) = [1 + \lambda(t)]J_\pi(\Pi_t^*, t). \tag{12}$$

No CDSs contract is optimal at time t if and only if

$$J_\pi(\Pi_t^* - \text{ess sup} S(\Pi_t^*, t), t) \leq [1 + \lambda(t)]J_\pi(\Pi_t^*, t). \tag{13}$$

Proof. Again the proof is completed via standard arguments about static optimization (see, for instance, [14]). □

In order to determine an exact (closed form) solution for the stochastic optimization problem in Theorem 2.2, we are required to make a specific choice for the utility functions $U^{(1)}$ and $U^{(2)}$. Essentially these functions can be almost any function involving k and π, respectively. However, in order to obtain smooth analytic solutions to the stochastic optimal credit default insurance problem, in the ensuing discussion, we choose power utility function and analyze the result.

From Proposition 2.3, we deduce that the optimal CDS contract coincides with the optimal accrued premium, η^*. In this regard, η^* is attained when the marginal cost of decreasing or increasing η is equals to the marginal benefit of the CDS contract. Moreover, if $\lambda = 0$ then the optimal accrued premium should be zero, i.e., $\eta^* = 0$. In this case, if the investor holds no type-A CDOs, λ, – which is indicative of a high PD for reference mortgage portfolios – then it may be optimal for the investor to purchase a CDS contract which protects against all such losses. However, full protection may also introduce high costs in the event that the protection seller fails to honor its obligations. In particular, during the GFC, many investors that purchased CDS contracts promising to cover all losses, regretted making this decision when the protection sellers were unable to make payments after a credit event. Notwithstanding this, certain investors that bought CDS contracts that only pay when the losses exceed a certain level set by the protection seller found protection beneficial. In particular, they did not experience the same volume of losses as those who purchased full protection (see, for instance, [2]).

D. Optimal credit default insurance with power utility

For a choice of power utility, we have that

$$\overline{U}^{(1)}(k) = \frac{k^\alpha}{\alpha} \text{ and } \overline{U}^{(2)}(\pi) = \gamma\frac{\pi^\alpha}{\alpha}, \tag{14}$$

for some $\alpha < 1$, $\alpha \neq 0$, and $\gamma \geq 0$. The parameter γ represents the weight that the investor gives to terminal payoff versus the consumption rate and can be viewed as a measure of its propensity to retain earnings. This leads to the following result.

Proposition 2.4: **(Optimal Credit Default Insurance with Power Utility):** Let the power utility functions be given by *(14)* and assume that the investor's CDO losses, S, are proportional to the investor's payoff under mortgage securitization so that

$$S(\pi, t) = \varphi(t)\pi,$$

for some deterministic S and severity function, $\varphi(t)$, where $0 \leq \varphi(t) \leq 1$. Under power utility, the objective function may be represented by

$$\bar{J}(\pi, t) = \frac{\pi^\alpha}{\alpha}\vartheta(t), \quad \vartheta(t) > 0, \tag{15}$$

where $\vartheta(t)$ solves the differential equation

$$\vartheta' + G(t)\vartheta + (1-\alpha)\vartheta^{\frac{\alpha}{\alpha-1}} = -\gamma\omega_b(t), \tag{16}$$

with $G(t)$ having the form

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$$
\begin{aligned}
G(t) &= -\delta^r + \frac{(r^{\mathrm{T}}\pi + \iota(t))\alpha}{\pi} + \frac{1}{2}\frac{(\mu - r^{\mathrm{T}})^2}{\sigma^2(1-\alpha)}\alpha \\
&+ \frac{\phi(t)}{\pi^\alpha}\left(\mathbf{E}^{\mathbf{P}}[(\pi - \eta^*)^\alpha] - \pi^\alpha\right) \\
&- (1+\lambda(t))\phi(t)\frac{\alpha}{\pi}\mathbf{E}^{\mathbf{P}}[S - \eta^*] - \omega_b(t)
\end{aligned}
$$

In this case, the investor's optimal rate of consumption is given by

$$k_t^* = \vartheta^{\frac{1}{\alpha-1}}\pi, \tag{17}$$

and the investor's optimal investment in CDOs is

$$\psi_t^* = \frac{(\mu - r^{\mathrm{T}})}{\sigma^2(1-\alpha)}\pi. \tag{18}$$

Furthermore, under power utility, the optimal accrued premium is given by

$$\eta_t^* = \min\left\{\left[1 - (1+\lambda(t))^{\frac{1}{\alpha-1}}\right], \varphi(t)\right\}\pi. \tag{19}$$

Proof. The proof follows from Theorem 2.2 and Proposition 2.3 as well as (8). Furthermore, a consideration of [13, Chapter V, Section 3] yields a unique solution to (4) under power utility. □

In Proposition 2.4, the optimal controls in (17), (18) and (19) are expressed as linear functions of the investor's optimal profit under mortgage securitization, Π^*. In this case, we see that the optimal consumption rate, k^*, is independent of the frequency and severity parameters ϕ and φ, of the aggregate CDO losses, S, respectively. These results are true because the power utility function exhibits constant relative risk aversion which means that

$$-\frac{\pi D^2\overline{U}^{(2)}(\pi)}{D\overline{U}^{(2)}(\pi)} = 1 - \alpha.$$

Here, we see that if the relative risk aversion increases, the amount invested in CDOs decreases which may be indicative of the fact that the mass of type-A CDOs, λ, is low at that time. The expression for ϑ in (16) reveals that not only the objective function, $\bar{J}$, is affected by the horizon τ, but also the optimal consumption, k^*. Moreover, the investor's optimal investment in CDOs, ψ^*, is affected by the time horizon τ via the optimal consumption rate, k^*, which impacts on the investor's profit. In addition, the expression for ϑ in (16) shows that k^* depends on the frequency and severity parameters, ϕ and φ, of the CDO losses, S, respectively. Furthermore, the investor's optimal investment, ψ^*, is affected by mortgage losses that indirectly involves k^*. From Proposition 2.4, it is clear that the amount invested in CDOs, ψ, depends on the profit, Π. Reference mortgage portfolio defaults will cause a decrease in the investor's profit under mortgage securitization, which will later affect the consumption rate, k. In particular, this may cause a liquidity problem in the secondary mortgage market since μ may decrease as a result of this effect on k.

If profits, Π, decrease, it is natural to expect that some investors will fail as in the GFC. For instance, both the failure of Lehman Brothers investment bank and the acquisition in September 2008 of Merrill Lynch and Bear Stearns by Bank of America and JP Morgan, respectively, was preceded by a decrease in profits from securitization. A similar trend was discerned for the U.S. mortgage companies, Fanie Mae and Freddie Mac, who had to be bailed out by the U.S. government at the beginning of September 2008.

III. Conclusions and Future Work

In this paper, we constructed a stochastic dynamic model for investor payoff that incorporates credit derivatives. This model related to problems experienced with credit derivatives in the GFC such as the reduction of incentives for monitoring SPEs, incentives to destroy value, credit derivative market opacity, industry self-protection and systemic risk as well as the mispricing of credit. In continuous-time, we obtained optimal investor payoff in the presence of CDOs and credit default insurance with consumption, CDO value and credit default insurance as controls. Finally, we were able to explain elements of the GFC in a quantitative way via numerical results involving credit derivatives.

In future, it is important that we increase the sophistication of our model by incorporating interest rate and credit risk more effectively. Also, our model has to accommodate dealing with real financial market interest rates. structuring the securitization and pricing its outcome or for explaining the economic mechanism behind the recent crisis. In this regard, we have to account for important issues such as moral hazard in expanding mortgage portfolios, incomplete information among market players about their counterparties, myopia in decision making in subprime mortgage market and monetary policy incentives boosting the growth of the subprime market.

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