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Risk Management for a Risk-averse Firm with Contingent Payment

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***Abstract*—This paper studies the contingent sales price risk mitigation problem of a risk-averse firm which procures some kind of commodity from the spot market as raw material for making certain product. The payment received by this firm depends on the underlying commodity spot price which is unknown until the product is physically delivered. In order to reduce the volatility stemming from the contingent payment, a financial hedging strategy requiring commodity futures contracts is proposed. This approach allows the firm to rebalance the commodity futures position dynamically. This study shows that the optimal strategy can be obtained when the firm adopts the exponential or mean-variance utility**

***Index Terms*—commodity, financial hedging, risk aversion, volatile raw material price**

I. Introduction

NOWADAYS, small-and-medium-sized firms are facing great challenges in maintaining a healthy financial status, due to the pressures from both the selling side and the sourcing side. On the selling side, powerful buyers are demanding a low procurement payment and that put these firms' profit under pressure. On the sourcing side, the firms suffer from the severe price changes of the input commodity as they rely more on the spot market for the acquisition of raw materials (Ansberry, 2002). It is reported by the Efficio Consulting that over 55% of the interviewees, who are procurement professionals at various levels, considered the commodity price instability as their single biggest challenge (Jenkinson, 2011). To help reduce these pressures and benefit the supply chain, various forms of contract have been designed to effectively deal with the volatile commodity risks (to name a few, Martínez-de-Albéniz and D. Simchi-Levi (2006), Fu et al. (2010), Shi et al. (2011), Zhang et al. (2014), etc.). This paper, however, instead of designing contracts, explores ways to mitigate risks due to the volatile commodity price through hedging in the commodity futures market. The particular risk to deal with is incurred from the contingent payment required by the buyer. In our setting, at the time the contract is signed, the future payment received by the firm depends on the future spot price of the commodity, which is required for production. Such cases are found to be common in real practices, e.g. a small-and medium-sized component supplier serving a large contract manufacturer such as Foxconn. The component supplier will be paid based on the raw material price on the delivery date instead of the date signing the contract (Zhang et al. (2014)).

This paper considers a risk-averse small-and-medium-sized firm that produces products for powerful buyers. The firm satisfies the customer demand in a make-to-order fashion. To simplify the analysis, we assume that the firm procures the input commodity from the spot market only. In this study, the contingent payment rule to the firm is decided by the powerful buyer, e.g. such as Foxconn. This kind of contingent payment belongs to the so-called flexible contract, dynamic contract, or index-linked payment contract in Caldentey and Haugh (2009), Swinney and Netessine (2009), and Zhang et al.(2014), respectively. Details on the contingent payment are illustrated in the next section.

Instead of focusing on contract design in the supply chain framework, this paper studies the contingent sales price risk management problem of the firm using commodity futures. In a similar study, Ni et al. (2012) proposed a multi-stage hedging strategy to mitigate the volatility of procurement cost arising from erratic commodity spot price under quadratic utility criterion. While they studied a problem with long planning horizon with unknown demand we consider a situation where the make-to-order firm receives uncertain contingent payment from the buyer. In this study, the exponential and mean-variance utility functions are adopted to reflect the risk-averse attitude of the firm and terminal wealth is used to denote the sum of the given initial monetary wealth and the revenue received from operational and hedging activities over the entire planning horizon. The goal of the firm is to maximize the expected utility with respect to the terminal wealth. Following the approach of Anderson and Danthine (1983), we propose an effective financial hedging strategy for the firm to mitigate the terminal wealth volatility with respect to exponential and mean-variance utility.

II. Model Formulation

This section describes a two-period hedging model with three trading time points. t is used to index the trading points, $t \in \{1, 2, 3\}$. At the beginning of the planning horizon ($t = 1$), given the initial wealth W_0, the firm procures the commodity from the spot market at the price of S_1 for production to satisfy the customer order Q. At the same time, the firm initializes the position, θ_1, in the futures

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market. At $t = 2$, the profit or loss of the futures contracts entered at $t = 1$ is realized. This implies that the future contracts are assumed to be "marked to market", i.e., all profit or loss of a futures position are realized at the end of each trading time point. There is no operation decisions needed to be made in this period. At $t = 3$, i.e., at the end of the planning horizon, the firm will not hold any futures contract, i.e., $\theta_3 = 0$. The spot market price of the commodity, S_3, is observed. The production process is completed and the processed product is then delivered to the buyer at the price of $S_3 + p$, where p represents the exogenous unit markup. The aforementioned delivery price is the realization of the predetermined uncertain contingent payment at the beginning of the planning horizon. In other words, the contingent payment received by the firm in this model is the sum of a unit markup and the market value of one unit input commodity at the time when products are transferred. Note that the real material acquisition cost is determined at the beginning of the production horizon, which is S_1. This type of contingent payment is a special case in Caldentey and Haugh (2009) and Zhang et al. (2014). Without loss of generality, one unit of commodity is taken to produce one processed product and the firm's production cost is zero. The result also holds true for a more general case with nonzero production cost as the term of production cost can be eliminated by adjusting the term of markup.

For model tractability, the financial market is assumed to be complete. In our setting, there exists a futures contract written on the same commodity used for production and its maturity date is the same as the end of the horizon, say $t = 3$. The complete financial market assumption and the existence of the futures contract with perfect match maturity date imply that no basis risk exists in the hedge. In this paper, terminal wealth, which is denoted by W_3, is adopted to denote the sum of the firm's initial wealth and the profit or loss from both operation and trading in commodity futures during the planning horizon. The objective of the firm is to maximize the expected utility of the terminal wealth. This assumption implies that the firm is only concerned with the terminal wealth at the end of the planning horizon. This is true in the sense that firms in practice cares more about the profit every quarter (half year or year). In addition, we assume the firm has sufficient working capital to maintain the position of the futures contracts during the entire planning horizon.

The mathematical notions for the model are listed as follows.

Q the demand of the product, which is known at the beginning of the production horizon, i.e., $t = 1$.

p the unit fixed markup which is exogenous and deterministic

r the constant risk-free interest rate

S_t the spot price of the input commodity at t

F_t the time-t price of the futures contracts maturing at the end of the horizon

θ_t the position of the futures contracts at t (a long position is represented as $\theta_t < 0$)

W_t the wealth of the firm at t, W_0 is the initial wealth

Following the sequence of events and using the notations defined, the uncertain terminal wealth of the firm can be obtained as follows. The wealth at $t = 1$ is equal to the firm's initial wealth subtracting the ordering cost from the spot market, i.e.,

$$W_1 = W_0 - S_1 Q$$

The firm's wealth at $t = 2$ (immediately after θ_1 is executed but before θ_2 is initiated) is

$$W_2 = (1+r)W_1 + (F_1 - F_2)\theta_1$$

where θ_1 is the size of the firm's position in this futures contract from $t = 1$ to $t = 2$. Note that $\theta_2 - \theta_1$ is the amount of futures the firm sells at $t = 2$.

The firm's terminal wealth at $t = 3$ is

$$W_3 = (1+r)W_2 + (F_2 - F_3)\theta_2 + (p + S_3)Q$$

The terminal wealth can be rewritten as

$$\begin{aligned} W_3 &= (1+r)^2 W_0 + (1+r)(F_1 - F_2)\theta_1 \\ &\quad + (F_2 - F_3)\theta_2 + (p + S_3)Q - (1+r)^2 S_1 Q \end{aligned}$$

Notice that S_3 is equal to F_3 in our model due to the absence of basis risk.

The optimal hedging strategy can thus be obtained by solving the following two-period hedging problem:

$$\max_{\theta_1, \theta_2} E_1\left[U(W_3)\right]$$

where $E_1[\cdot]$ represents that the expectation is taken conditional on the spot price of the input commodity at $t = 1$, i.e., S_1.

In this paper, exponential utility and mean-variance utility are employed to capture the firm's attitude toward risk. The specific objectives of the firm under both utility functions and the corresponding optimal hedging strategies are presented in Section 3 and 4, respectively.

III. MEAN-VARIANCE UTILITY CRITERION

This section describes the case in which the firm employs a mean-variance utility function of the following form

$$E_1\left[U(W_3)\right] = E_1\left[W_3\right] - \frac{1}{2}\lambda Var_1(W_3) \qquad (1)$$

where λ is a strictly positive constant representing risk aversion.

The following theorem determines the optimal position of the futures contract for the firm on each trading date and the corresponding optimal utility of the terminal wealth in the presence of financial hedging.

Theorem 1: The optimal hedging strategy with respect to the mean-variance criterion is:

$$\theta_1^* = \frac{Q}{1+r},\ \theta_2^* = Q.$$

The maximal utility of the terminal wealth is given as

$$\max_{\theta_1,\theta_2} E_1\left[U\left(W_3\right)\right] = \left(1+r\right)^2 W_0 + \left(p + F_1 - \left(1+r\right)^2 S_1\right)Q$$

This expression shows that the volatility of the firm's utility with respect to the terminal wealth is perfectly mitigated by the proposed hedging strategy because all the terms are deterministic now.

Proof: The whole problem can be solved backwardly by the following two steps. At $t=2$, the firm should choose the best θ_2 given θ_1, i.e.,

$$\max_{\theta_2} E_2\left[U\left(W_3\right)\right] = \max_{\theta_2}\left\{E_2\left[W_3\right] - \frac{1}{2}\lambda Var_2\left(W_3\right)\right\}$$

The above conditional expectation of the utility of the terminal wealth at $t=2$ is

$$E_2\left[W_3\right] = V\left(Q\right) + \left(F_2 - E_2\left[F_3\right]\right)\theta_2 + E_2\left[S_3\right]Q$$

where

$$V\left(Q\right) = \left(1+r\right)^2 W_0 + \left(1+r\right)\left(F_1 - F_2\right)\theta_1 + \left(p - \left(1+r\right)^2 S_1\right)Q$$

The corresponding conditional variance at $t=2$ is

$$\begin{aligned} Var_2\left(W_3\right) &= E_2\left[\left(W_3 - E_2\left[W_3\right]\right)^2\right] \\ &= \theta_2^2 Var_2\left(F_3\right) - 2\theta_2 Q Cov_2\left(F_3, S_3\right) \\ &\quad + Q^2 Var_2\left(S_3\right) \end{aligned}$$

As there is no basis risk in this model, the above equation can be rewritten as

$$\begin{aligned} Var_2\left(W_3\right) &= \theta_2^2 Var_2\left(F_3\right) - 2\theta_2 Q Var_2\left(F_3\right) \\ &\quad + Q^2 Var_2\left(F_3\right) \end{aligned}$$

Taking the first derivation with respect to θ_2, the necessary and sufficient conditions for the optimal θ_2 are given by the following equation

$$\left(F_2 - E_2\left[F_3\right]\right) - \lambda\left(\theta_2 Var_2\left(F_3\right) - Q Var_2\left(F_3\right)\right) = 0$$

Since there is no arbitrage opportunity, i.e., $F_2 = E_2\left[F_3\right]$, we have

$$\theta_2^* = Q$$

At $t=1$, the firm chooses the best θ_1, i.e.,

$$\max_{Q\ge 0,\theta_1} E_1\left[U\left(W_3\right)\right] = \max_{Q\ge 0,\theta_1}\left\{E_1\left[W_3\right] - \frac{1}{2}\lambda Var_1\left(W_3\right)\right\}$$

Similarly, the expectation and variance of utility of the terminal wealth at $t=1$ are:

$$\begin{aligned} E_1\left[W_3\right] &= \left(1+r\right)^2 W_0 + \left(p - \left(1+r\right)^2 S_1\right)Q \\ &\quad + \left(1+r\right)\left(F_1 - E_1\left[F_2\right]\right)\theta_1 + E_1\left[F_2\right]Q \end{aligned}$$

$$\begin{aligned} Var_1\left(W_3\right) &= \theta_1^2\left(1+r\right)^2 Var_1\left(F_2\right) \\ &\quad - 2\theta_1 Q\left(1+r\right) Var_1\left(F_2\right) + Q^2 Var_1\left(F_2\right) \end{aligned}$$

Then the necessary and sufficient conditions for the optimal θ_1 are

$$\begin{aligned} &\left(1+r\right)\left(\mathrm{F}_1 - \mathrm{E}_1\left[\mathrm{F}_2\right]\right) \\ &+\lambda\left(\theta_1\left(1+r\right)^2 \mathrm{Var}_1\left(\mathrm{F}_2\right) - Q\left(1+r\right)\mathrm{Var}_1\left(\mathrm{F}_2\right)\right) = 0 \end{aligned}$$

When there is no arbitrage opportunity, i.e., $F_1 = E_1\left[F_2\right]$, we have

$$\theta_1^* = \frac{Q}{1+r}.$$

The corresponding utility of the terminal wealth with the optimal financial hedging strategy is

$$\begin{aligned} \max_{Q\ge 0,\theta_1,\theta_2} E_1\left[U\left(W_3\right)\right] &= E_1\left[W_3\right] \\ &= \left(1+r\right)^2 W_0 + \left(p + F_1 - \left(1+r\right)^2 S_1\right)Q \end{aligned}$$

IV. Exponential Utility Criterion

This section describes the case in which the firm employs an exponential utility function on the terminal wealth, i.e.,

$$U\left(W_3\right) = -\exp\left(-\rho W_3\right)$$

where $\rho > 0$ represents the firm's risk sensitivity. A large ρ implies the firm has a more risk-averse attitude. Note that $U'\left(W_3\right) > 0$ and $U''\left(W_3\right) < 0$. Therefore, the firm's objective is to maximize the expected value of a strictly concave utility function of the terminal wealth, i.e.,

$$\max_{\theta_1,\theta_2} E_1\left[-\exp\left(-\rho W_3\right)\right] \qquad (2)$$

Theorem 2: The optimal hedging strategy with the exponential utility is

$$\theta_1^* = \frac{Q}{1+r},\ \theta_2^* = Q.$$

The corresponding maximal utility of the terminal wealth is given as

$$\max_{Q\ge 0,\theta_1,\theta_2} E_1\left[-\exp\left(-\rho W_3\right)\right] = -\exp\left(-\rho V_0\left(Q\right)\right)$$

where

$$V_0\left(Q\right) = \left(1+r\right)^2 W_0 + \left(p + F_1 - \left(1+r\right)^2 S_1\right)Q$$

The results indicate that the volatility of the utility of the terminal wealth can be perfectly hedged by the proposed strategy as all the terms in the optimal utility are deterministic.

Proof: Similar to the case with mean-variance utility, this problem can be solved backwardly by using the following two steps, i.e., the firm chooses θ_1 and θ_2 sequentially. Apparently, these choices, at $t=1$ and $t=2$ respectively, are interdependent. Specifically, we have the following two sub-problems.

At $t=2$,

$$\max_{\theta_2} E_2\left[-\exp\left(-\rho W_3\right)\right] \text{ given } \theta_1 \tag{3}$$

At $t=1$,

$$\max_{Q\geq 0,\theta_1} E_1\left[-\exp\left(-\rho W_3\right)\right] \text{ given } \theta_2 \text{ will solve (1).} \tag{4}$$

In view of the strict concavity of the utility, the necessary and sufficient conditions for problem (3) are:

$$E_2\left[\rho\exp\left(-\rho W_3\right)\left(F_2-F_3\right)\right]=0 \tag{5}$$

Or, equivalently

$$F_2 E_2\left[\exp\left(-\rho W_3\right)\right]=E_2\left[\exp\left(-\rho W_3\right)F_3\right].$$

To show the choice of θ_2 is dependent upon θ_1, denote the solution to equation (3) as $\theta_2\left(\theta_1\right)$. By taking the first derivative with respect to θ_1, the condition for the optimal solution to equation (4) can be written as:

$$E_1\left[\rho\exp\left(-\rho W_3\right)\left[\begin{array}{l}\left(1+r\right)\left(F_1-F_2\right)\\ +\left(F_2-F_3\right)\dfrac{d\theta_2\left(\theta_1\right)}{d\theta_1}\end{array}\right]\right]=0$$

Due to the fact that $\theta_2\left(\theta_1\right)$ does not depend on the random variable F_3, we have

$$\begin{aligned}&E_1\left[\rho\exp\left(-\rho W_3\right)\left(F_2-F_3\right)\frac{d\theta_2\left(\theta_1\right)}{d\theta_1}\right]\\ &=E_1\left[\frac{d\theta_2\left(\theta_1\right)}{d\theta_1}E_2\left[\rho\exp\left(-\rho W_3\right)\left(F_2-F_3\right)\right]\right]\\ &=0\end{aligned}$$

Therefore, the first order condition to equation (4) can be further written as

$$F_1 E_1\left[\exp\left(-\rho W_3\right)\right]=E_1\left[\exp\left(-\rho W_3\right)F_2\right]$$

Let $\alpha_1=\left(1+r\right)\theta_1 \text{-} Q$ and $\alpha_2=\theta_2-Q$ represent the amounts the firm overhedges at $t=1$ and $t=2$, respectively. They can be taken as the firm's speculative decision variable. The decision on the position of futures contract is driven by partially stabilizing the contingent payment and partially speculating on an increased price.

Substituting α_1 and α_2 into the terminal wealth expression and by assuming that no basis risk assumption, we have

$$W_3=V_0\left(Q\right)+\left(1+r\right)\left(F_1-F_2\right)\alpha_1+\left(F_2-F_3\right)\alpha_2$$

where

$$V_0\left(Q\right)=\left(1+r\right)^2 W_0+\left(p+F_1-\left(1+r\right)^2 S_1\right)Q$$

It can be seen from the above equation that the choice of α_1 and α_2 are equivalent to the choice of θ_1 and θ_2, respectively.

From (5), if α_1^* and α_2^* are optimal, then

$$E_2\left[\rho\exp\left(-\rho\left(\begin{array}{l}V_0\left(Q\right)+\left(F_2-F_3\right)\alpha_2^*\\ +\left(1+r\right)\left(F_1-F_2\right)\alpha_1^*\end{array}\right)\right)\left(F_2-F_3\right)\right]=0$$

With the non-arbitrage argument we have $F_t=E_t\left[F_{t+1}\right]$.

By taking $\alpha_2=0$, the above equation can be rewritten as

$$\exp\left(-\rho\left(V_0\left(Q\right)+\left(1+r\right)\left(F_1-F_2\right)\alpha_1^*\right)\right)\left(F_2-E_2\left[F_3\right]\right)=0$$

Therefore, we have $\alpha_2^*=0$ from the above equations.

Consequently, the optimal solution for α_1 is

$$E_2\left[\rho\exp\left(-\rho\left(\begin{array}{c}V_0\left(Q\right)+\\ \left(1+r\right)\left(F_1-F_2\right)\alpha_1^*\end{array}\right)\right)\left(F_2-F_3\right)\right]=0$$

Similarly, as $F_1=E_1\left[F_2\right]$, $\alpha_1^*=0$

Therefore, the best futures positions at $t=1$ and $t=2$ are $\theta_1^*=\dfrac{Q}{1+r}$ and $\theta_2^*=Q$, respectively.

V. Concluding Remarks

In this study, a simple yet effective two-period hedging strategy is proposed for a risk-averse small-and-medium-sized firm to minimize the volatility of the utility with respect to the terminal wealth. The volatility arises from the contingent payment received from the buyer, which depends on the commodity spot price on the day when the order is satisfied. The optimal positions of the futures contracts are the same for the firm with respect to exponential utility and mean-variance utility. In other words, the firm's optimal hedging strategy is independent of these two risk preferences. This happens because the only uncertainty comes from the volatile commodity spot price which can be fully hedged in the proposed model. It is also worth noting that the strategy is quite general because it does not involve any specific models for price evolution of the futures contracts and the underlying commodity. Also, the results can be readily extended to a multi-period hedging model. For further study, other realistic issues such as transaction cost and basis risk might be taken into account.

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