On the Performance of Sequential and Parallel Algorithm for Solving the Forward Problem of the Two-Dimensional Impedance Equation

A. Bucio R. *IAENG, Member*, A. Hernandez-Becerril *IAENG, Member*,
C. M. A. Robles G. *IAENG, Member* and M. P. Ramirez T. *IAENG, Member*.

***Abstract*—We study the performance of the numerical method for solving the forward problem of the two-dimensional Impedance Equation [15]. This numerical method is based upon the elements of the modern Pseudoanalytic Function Theory. Considering divide and conquer technique for constructing the parallel algorithm, some sub process can be taking advantage by processing them independently, we parallelize some processes of the numerical method through CUDA technology, obtaining considerable reduction of the temporal complexity. The processing time of the posed method is evaluated comparing with its sequential version and the speed-up rate of the parallel algorithm respect to the sequential one posed in [15]. The collection of experiments are displayed for illustrating its effectiveness.**

***Index Terms*—Algorithm, CUDA, Electrical Impedance Tomography, Pseudoanalytic Functions Theory, Parrallel Processing, Vekua Equation.**

I. Introduction

THE Pseudoanalytic Function Theory [3] has been just found to be an important tool for modern Mathematical Physics, and in sequel, for different branches of Engineering. Perhaps, beyond the original expectations that its main creators, professors L. Bers [3] and I. Vekua [20], could have foreseen at the time they first published the foundations of the theory.

The elements of the modern Pseudoanalytic Function Theory [12], have been considered as an important tool in Applied Mathematics and Theoretical Physics (see *e.g.* [4], [12] and [18]). This tool has been successfully applied for solving the forward problem of the two-dimensional Impedance Equation [15]

$$\text{div}\,(\sigma\ \text{grad}\ u) = 0, \tag{1}$$

where σ is the conductivity and u is the electric potential. It is possible to approach the general solution of (1) in asymptotic form, through the Taylor series in formal powers [3].

The detection of the relation between (1) in the plane, and the Vekua equation [20], by V. Kravchenko [13], and shortly after by K. Astala and L. Päivärinta [1], opened a complete new path for constructing numerical solutions of the forward problem corresponding to (1), based upon the modern Pseudoanalytic Function Theory [3].

The most important fact of caring about efficiently solving the forward problem for (1), is to use this problem as an approach to the solution of the Electrical Impedance Tomography Problem (also called inverse problem). This problem was widely exposed in plenty works, among which [21] is one of the most important. In this sense, the results posed in [13], and subsequently rediscovered in [1], are indeed very significant, because they allowed to find out the rink for approaching the general solution of the two-dimensional Impedance Equation.

The main contribution of this work is to analyse the performance of the numerical method and its sequential algorithm posed in [15] versus the parallel algorithm to be presented in this work. Employing this numerical method we propose the design of a parallel algorithm using *divide and conquer* technique [14] with CUDA technology, this will help us to obtain an adequate balance between the computational cost and accuracy. Then we examine some specific examples, in order to illustrate a comparison of the performance and effectiveness between the sequential and parallel method. The conclusions contain the arguments that justify the viability of employing this numerical method as an approach for employing it in medical image [21] this is for solving the Electrical Impedance Tomography Problem [6]. This work is organized as follows: In the second Section , we will explain the mathematical tools necessaries for the construction of the numerical method and algorithm for the forward problem of the two-dimensional Impedance Equation. In Section 3, we describe both algorithms. Then we will show some experimental results of the implementation of both algorithms employing different conductivity functions and finally the conclusions of this work.

II. Elements of Pseudoanalytic Function Theory and its relation with the Electrical Impedance Equation.

Let us consider the two-dimensional case of the Electrical Impedance Equation:

$$\text{div}\,(\sigma\ \text{grad}\ u) = 0,$$

where u is the electric potential and σ is a separable-variables non-vanishing function within a bounded domain Ω, with boundary Γ, such that:

$$\sigma = \sigma_1(x) \cdot \sigma_2(y). \tag{2}$$

Introducing the following notations:

$$W = \sqrt{\sigma}\partial_x u - i\sqrt{\sigma}\partial_y u, \quad p = \sqrt{\frac{\sigma_2(y)}{\sigma_1(x)}}, \tag{3}$$

A. Bucio R. is with the National Polytechnique Institute, UPIITA, Mexico, ari.bucio@gmail.com

A. Hernandez-Becerril, C. M. A. Robles G. and M. P. Ramirez T. are with Postgraduate Section of Mechanical Engineering School, Instituto Politecnico Nacional Mexico City, Mexico., croblesg1101@alumno.ipn.mx

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where $i^2 = -1$, $\partial_x = \frac{\partial}{\partial x}$ and $\partial_y = \frac{\partial}{\partial y}$; the equation (1) can be rewritten and transformed into a Vekua equation [20] of the form:

$$\partial_{\overline{z}} W - \frac{\partial_{\overline{z}} p}{p} \overline{W} = 0, \tag{4}$$

where $\partial_{\overline{z}} = \partial_x + i\partial_y$, and $\overline{W}$ represents the complex conjugation of W: $\overline{W} = \mathrm{Re}W - i\mathrm{Im}W$.

The general solution of the equation (4) can be expressed in terms of the Taylor series in formal powers [3]:

$$W = \sum_{n=0}^{\infty} Z^{(n)}(a_n, z_0; z). \tag{5}$$

This is a generalization of the classical postulates of Complex Analysis, that was mainly developed by L. Bers [3]. The following paragraphs contain a condensed description of the material that will be needed for our further discussions. The reader can find a complete and detailed explanation of these postulates in [3] and [12].

A. Taylor Series in Formal Powers.

The formal power $Z_0^{(0)}(a_0, z_0; z)$ with complex constant coefficient a_0, center at z_0, depending upon z, formal exponent 0, and corresponding to the generating pair (F_0, G_0); is expressed as:

$$Z_0^{(0)}(a_0, z_0; z) = \lambda_0 F_0 + \mu_0 G_0;$$

where λ_0 and μ_0 are real constants such that

$$\lambda_0 F_0(z_0) + \mu_0 G_0(z_0) = a_0;$$

and

$$F_0 = p, \quad G_0 = \frac{i}{p}.$$

where p possesses the form (3).

The formal powers with higher formal exponents n, are defined by the recursive integral expressions:

$$Z_{j_0}^{(n)}(a_n, z_0; z) = n \int_{z_0}^{z} Z_{j_1}^{(n-1)}(a_0, z_0; z) d_{(F_{j_0}, G_{j_0})} z. \tag{6}$$

where $j_0 = 0, 1$ and $j_1 = 1, 0$. This is, if $j_0 = 0$ then $j_1 = 1$, and if $j_0 = 1$ then $j_1 = 0$. The integral expressions in the right-hand side of (6) are what can be considered antiderivatives in the sense of Bers [3]:

$$\int_{z_0}^{z} Z_{j_1}^{(n-1)}(a_0, z_0; z) d_{(F_{j_0}, G_{j_0})} z =$$

$$= G_{j_0} \mathrm{Re} \int_{\Lambda} F_{j_0}^{*} Z_{j_1}^{(n-1)}(a_0, z_0; z) dz +$$

$$+ F_{j_0} \mathrm{Re} \int_{\Lambda} G_{j_0}^{*} Z_{j_1}^{(n-1)}(a_0, z_0; z) dz.$$

Here, Λ is a rectifiable curve going from z_0 upto z, and:

$$F_1 = \sqrt{\sigma}, \quad G_1 = \frac{i}{\sqrt{\sigma}}.$$

whereas

$$F_{j_0}^{*} = -iF_{j_0}, \quad G_{j_0}^{*} = -iG_{j_0},$$

as well

$$F_{j_1}^{*} = -iF_{j_1}, \quad G_{j_1}^{*} = -iG_{j_1}.$$

A detailed description of the construction of the formal powers can be found in [3] and [12]. Here we will only remark the two fundamental properties for this work.

1)
$$\lim_{z \to z_0} Z^{(n)}(a_n, z_0; z) = a_n (z - z_0)^n. \tag{7}$$

2) Let $a_n = a_n' + i a_n''$, where a_n' and a_n'' are both real constants. Thus

$$Z^{(n)}(a_n, z_0; z) =$$
$$= a_n' Z^{(n)}(1, z_0; z) + a_n'' Z^{(n)}(i, z_0; z). \tag{8}$$

The absence of the sub index j_0 and j_1 indicates that the properties are valid for all formal powers.

Notice the last statement establishes that any formal power $Z^{(n)}(a_n, z_0; z)$ can be approached by the linear combination of $Z^{(n)}(1, z_0; z)$ and $Z^{(n)}(i, z_0; z)$, thus the numerical calculations shall be exclusively performed to approach these two classes of formal powers.

On the other hand, in [9] is shown the proof of completeness of the set:

$$\left\{ \mathrm{Re}\, Z^{(n)}(1, z_0; z)|_{\Gamma}, \mathrm{Re}\, Z^{(n)}(i, z_0; z)|_{\Gamma} \right\}_{n=0}^{\infty}, \tag{9}$$

for approaching solutions of the forward Dirichlet boundary value problem corresponding to the equation (1).

This is, given a non-vanishing function σ within a bounded domain Ω, with boundary Γ, any boundary condition $u|_{\Gamma}$ can be approached by the linear combination of the elements belonging to (9), that are the real parts of the formal powers with coefficients 1 and i, valued at the points belonging to the boundary Γ:

$$u|_{\Gamma} = \sum_{n=0}^{\infty} c_n^{(1)} \mathrm{Re}\, Z^{(n)}(1, z_0; z)|_{\Gamma} +$$
$$+ \sum_{n=0}^{\infty} c_n^{(i)} \mathrm{Re}\, Z^{(n)}(i, z_0; z)|_{\Gamma}, \tag{10}$$

where $c_n^{(1)}$ and $c_n^{(i)}$ are all real constant coefficients.

A final statement is in place before studying the numerical method for approaching the elements of (9).

Employing the conjecture posed in [18], we can analyse a variety of conductivity functions and domains in [17]. This conjecture establishes that any function σ, fully defined within a domain Ω, can be considered at every single point, a special separable-variables function for which $j_0 = j_1$, thus it can be employed for numerically approaching the elements of (9).

B. Numerical approaching of the formal powers.

This Section is dedicated to briefly explain the numerical method for approaching integral expressions corresponding to (6). Enclosing the modern Pseudoanalytic Function Theory studied in [12], to the classical results of [3], we employ the numerical method posed in [15], that is an improved numerical method based onto a previous proposal [5]. The most important advantages of the numerical method posed in [15] unlike [5] is that possesses a higher degree of accuracy and numerical stability. In this case we will only expand procedure of the constructing the formal powers

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with coefficients $a_n = 1$, since there is not any important methodological variation when considering $a_n = i$.

Therefore, we will consider a collection of $K+1$ points

$$\{r\,[k]\}\,,\ k = 0, 1, ..., K;$$

equidistantly located in a closed interval $[0, 1]$. If the interval $[0, 1]$ coincides with a radius R of the unit circle, whose center is $z_0 = 0$, with some specific angle $\theta \in [0, 2\pi)$, we will obtain a collection of points in the plane:

$$\{(x[k], y[k])\}\,,\ k = 0, 1, ...K;$$

constructed as follows

$$x[k] = r[k]\cos(\theta);\ \ y[k] = r[k]\sin(\theta).$$

The formal powers over such radius R can be approached employing the recursive expressions:

$$\begin{aligned} Z^{(n+1)}[k] &= AF[k]\cdot \\ \cdot\mathrm{Re}\sum_{q=0}^{k}&\left(G^*[q]Z^{(n)}[q] + G^*[q+1]Z^{(n)}[q+1]\right)\Delta z[q] + \\ &+\ AG[k]\cdot \\ \cdot\mathrm{Re}\sum_{q=0}^{k}&\left(F^*[q]Z^{(n)}[q] + F^*[q+1]Z^{(n)}[q+1]\right)\Delta z[q], \end{aligned} \tag{11}$$

where

$$\Delta z[q] = x[q+1] - x[q] + i\,(y[q+1] - y[q])\,,$$

and A is a factor that ensures the numerical stability of the method (for additional information see [5] and [15]).

Hereinafter, for briefness we will denote the operations described in (11) with the following form:

$$Z^{(n)}[k] = \mathcal{B}\left[Z^{(n-1)}[k]\right].$$

III. Development of the Algorithms

In previous works, first in [4] was presented a numerical method for constructing the so-called Taylor Series in formal powers and its application to the two-dimensional case of the electrical impedance equation (1), afterwards in [5] was posed the simplified method for numerically solve equation (1) and finally in [15] was presented the optimized numerical method and its sequential algorithm for solving equation (1).

Thus, in this work we will describe the most important characteristics of the algorithm presented in [15] and employing this algorithm as baseline we developed a parallel algorithm using CUDA technology. For making a clear description of both the sequential [15] and the parallel algorithms, in the next subsections we will summarize the programming details including the pseudo code of the algorithms. For constructing the solution for the forward problem of the two-dimensional impedance equation we employed the following parameters: N formal powers numerically approached, over S number of radii, and considering $K+1$ points per radius.

It is essential to remark that the numerical method was fully developed in *C*, employing as graphic card an NVIDIA ® GeForce ® GTX 770M GPU, this GPU posses 960 CUDA cores is important to mention that with the evolution of graphic cards will improve performance of the algorithms including the speed up of memory and number of cores. The processor Intel ® Core ™ i7-4700MQ CPU @2.40GHz 2.40GHz.

A. Sequential Algorithm

As a measure of the efficiency of an algorithm, usually are considered the resources consumed by the algorithm, as memory and time. This has been developed to obtain values that specify the evolution of the spending time and memory, that is, depending on the size of the input values. We will describe the algorithm posed in [15], this will give us the values for making a comparison between this sequential algorithm [15] an a new version for parallel processing.

In the Algorithm 1 to handle complex numbers in the numerical method detailed in previous Section II, we employed data type *struct* to manage real and imaginary part. If it observes the Algorithm 1 it can be seen that is conformed by a main process where the Taylor series in formal powers (11) are calculated. Then in function *Orthonormalization*, we apply the standard Gram-Schmidt Orthonormalization Process for obtaining the following set

$$\{u_s(l)\}_{s=0}^{2N},$$

where we obtain a linear independent system of $2N+1$ vectors. After the orthonormalization process, we proceed to approach the solution for the conductivity function σ within a bounded domain Ω, with boundary Γ, any boundary condition $u|_\Gamma$ as shown in equation (10). Finally we save the results obtained, as the orthogonal system, coefficients and the approach solution.

In order to test out the numerical steadiness and convergence of the algorithm we calculate the Lebesgue measure for introducing an error parameter $\mathcal{E}$:

$$\mathcal{E} = \left(\int_\Gamma \left(u_{\mathbf{c}}|_\Gamma - u_{app}\right)^2 dl\right)^{\frac{1}{2}}. \tag{12}$$

where u_{app} represents the approached solution. This will help us to compare with other works as [16] and verifying the numerical stability of the algorithms.

B. Parallel Algorithm

About constructing the parallel algorithm for solving the forward problem of the two-dimensional impedance equation we employed *divide and conquer* technique since is a powerful tool for solving conceptually problems. This technique consist on breaking the problem into sub-problems, and solving the original problem by combining the solution of those sub-problems. This technique is useful for parallel algorithms, because can be adapted for execution in multi-cores, this is, distinct sub-problems can be executed on different cores.

In the design of the parallel algorithm for the forward problem of the two-dimensional impedance equation, we divide the sub-problems as shown in Algorithm 2, because it is well known that not all the process can be executed in parallel, but in one of the most significant functions of the algorithm can be applied, the computation of the values of coordinates and conductivities were obtained in different CUDA cores, is being calculating the values of every radii independently in the different cores. The main advantage of having this parallel algorithm is decreasing the time complexity considerably this while comparing with the Algorithm 1.

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Algorithm 1 Sequential Algorithm for solving the forward problem of the two-dimensional impedance equation

S (Number of radii)
N (Maximum number of Formal Powers)
$K+1$ (Number of points per radius)
while $s = 1 \rightarrow S$ **do**
  while $n = 1 \rightarrow N$ **do**
    while $q = 0 \rightarrow K$ **do**
      call Coordinates_Operations
      call Conductivities_Calculations
      call Generating_Pairs
      $Z^{(n)}[q] = \mathcal{B}\left[Z^{(n-1)}[q]\right]$
    end while
  end while
end while
call ORTHONORMALIZATION
call APPROACH_BOUNDARY_CONDITION
call LEBESGUE_MEASURE
save ORTHOGONAL_SYSTEM
save COEFFICIENTS

function ORTHONORMALIZATION
  (Classical Gram-Schmidt Orthonormalization Process)
end function
function APPROACH_BOUNDARY_CONDITION
  $\sum_{n=0}^{N}\left(\alpha_n u^{(n)}(1,0,z) + \beta_n u^{(n)}(i,0,z)\right)$
end function
function LEBESGUE_MEASURE
  $\mathcal{E} = \left(\int_\Gamma \left(u_{\mathbf{c}}|_\Gamma - u_{app}\right)^2 dl\right)^{\frac{1}{2}}$
end function

Algorithm 2 Parallel Algorithm for solving the forward problem of the two-dimensional impedance equation

S (Number of radii)
N (Maximum number of Formal Powers)
$K+1$ (Number of points per radius)
call NUMERICAL_BERS_OPERATIONS
call FORMAL_POWER_APPROACHER
call ORTHONORMALIZATION
call APPROACH_BOUNDARY_CONDITION
call LEBESGUE_MEASURE
save ORTHOGONAL_SYSTEM
save COEFFICIENTS

function NUMERICAL_BERS_OPERATIONS
for $q = 0 \rightarrow K$ **in parallel do**
  call Coordinates_Operations
  call Conductivities_Calculations
  call Generating_Pairs
end for
end in parallel
end function
function FORMAL_POWER_APPROACHER
for $s = 1 \rightarrow S$ **do**
  for $n = 1 \rightarrow N$ **do**
    for $q = 0 \rightarrow K$ **do**
      $Z^{(n)}[q] = \mathcal{B}\left[Z^{(n-1)}[q]\right]$
    end for
  end for
end for
end function
function ORTHONORMALIZATION
  (Classical Gram-Schmidt Orthonormalization Process)
end function
function APPROACH_BOUNDARY_CONDITION
  $\sum_{n=0}^{N}\left(\alpha_n u^{(n)}(1,0,z) + \beta_n u^{(n)}(i,0,z)\right)$
end function
function LEBESGUE_MEASURE
  $\mathcal{E} = \left(\int_\Gamma \left(u_{\mathbf{c}}|_\Gamma - u_{app}\right)^2 dl\right)^{\frac{1}{2}}$
end function

One of the most significant differences between sequential 1 and parallel Algorithm 2 is that for the parallel one we transformed the *structs* into arrays because while working with CUDA we use arrays that can be easily passed to the device global memory and accessed by each core. And one of the main advantages of this method is that we can greatly increase the parameters employed for the approach, the maximum number of radii and maximum number of points can be raised up to $100,000$ this is hard task for the sequential algorithm 1 posed in [15], since the time complexity is high. This advantage not only help us to approximate the solution with a greater convergence but also when considering the possibility of apply this numerical method and algorithm to the solution of the inverse or electrical impedance tomography problem.

IV. Experimental Results

We will perform a characterization of the numerical method and algorithms, using as domain the unitary circle, and a variety of conductivity functions. This characterization employs the optimized method, first exposed in [15] and then in [16], using the Pseudoanalytic Function Theory, and taking into account that we can analyse any conductivity function, approaching the solution for the Dirichlet boundary value forward problem.

In this work, we will empathize performance of the time complexity in the sequential and parallel algorithms for comparing its effectiveness with the results obtained in [16] when analysing the unitary disk. We will use conductivity functions with exact representation, namely, we will examine exponential, Lorentzian and a geometrical distribution constructed with concentric circles.

For this experimental results, we will employ the Algorithm 1 and Algorithm 2 and making variations in the number of formal powers N, number of radii S and number of points K will be fixed both in 1000.

A. Exponential Conductivity Function Case

Considering a non-separable variables exponential conductivity function with the following form

$$\sigma = e^{\alpha xy}, \tag{13}$$

where α represents a coefficient that is used to change the behaviour of the function, for this case $\alpha = 5$, this conductivity function is represented in Figure 1. And the boundary condition imposed is given in the next expression:

$$u|_\Gamma = e^{-\alpha xy}. \tag{14}$$

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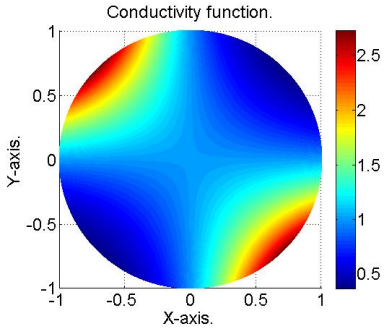

Fig. 1: Exponential Conductivity Function

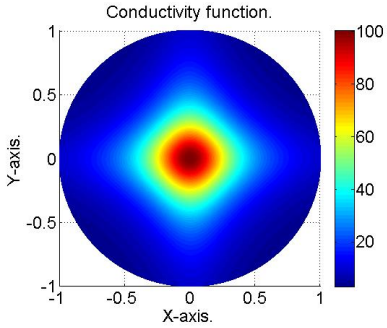

Fig. 2: Lorentzian Conductivity Function

TABLE 1: Brief relation of the time complexity in both algorithms with an exponential conductivity function

Formal Powers N	Time Sequential t (seconds)	Time Parallel t (seconds)
5	446.3	3.261
10	816.2	3.841
15	1187.342	4.615
20	1567.3	5.391
25	1924.43	6.215
30	2302.0	7.044
35	2688.15	7.794
40	3050.5	8.742

because it is an exact solution of (1), as appointed in [16].

As can be seen in Table 1 the difference in the execution of both algorithms it is remarkable that parallel algorithm has the advantage without loosing accuracy.

B. Lorentzian Conductivity Function Case

For this case we propose a conductivity function with the form:

$$\sigma = \left((x+d_x)^2 + L_c\right)^{-1} \cdot \left((y+d_y)^2 + L_c\right)^{-1}, \qquad (15)$$

where d_x and d_y represent the displacements over the x-axis and y-axis respectively, and L_c denotes a real constant. In Figure 2 is displayed this Lorentzian conductivity function.

We will imposed the boundary condition [18]:

$$u|_\Gamma = \frac{1}{3}(x+d_x)^3 + \frac{1}{3}(y+d_y)^3$$

$$+L_c(x+d_x+y+d_y); \qquad (16)$$

since it is an exact solution of (1). In this case we will develop the experiment with $L_c = 0.5$ and in 2 the results are shown with displacement in axis, but in 3 we displayed $x = 0.5$ and $y = 0.5$.

In this case we can observe that as it was expected the best results in time complexity are displayed while employing the parallel algorithm. In Table 2 we have no displacement in the conductivity function, but in Table 3 we displaced $x = 0.5$ and $y = 0.5$, this cases are important because it can be observed that in Table 3 the time complexity decrease in comparison with the one without displacement since there are more operations that can be executed independently.

TABLE 2: Brief relation of the time complexity in both algorithms with a Lorentzian conductivity function with no displasment in the axis

Formal Powers N	Time Sequential t (seconds)	Time Parallel t (seconds)
5	448.2	3.209
10	813.57	3.59
15	1197.05	4.329
20	1569.81	5.109
25	1943.99	5.992
30	2317.1	6.728
35	2689.09	7.511
40	3026.92	8.35

TABLE 3: Brief relation of the time complexity in both algorithms with a Lorentzian conductivity function with displacement $x = 0.5$ and $= 0.5$ in the axis

Formal Powers N	Time Sequential t (seconds)	Time Parallel t (seconds)
5	450.34	3.199
10	824.81	3.631
15	1210.84	4.542
20	1564.96	5.33
25	1946.94	6.203
30	2320.72	7.026
35	2691.31	7.851
40	3069.09	8.706

C. Geometrical Conductivity: Concentric Circles

For this case, the conductivity function is composed as follows: one disk with radius $r_1 = 0.2$ representing $\sigma = 100$, the ring delimited by $r_2 = 0.4$ and r_1 possessing a conductivity $\sigma = 30$, another ring between $r_3 = 0.6$ and r_2 having $\sigma = 20$, whereas the one within $r_4 = 0.8$ and r_3 exhibits $\sigma = 15$. Finally, the remaining value within the boundary is $\sigma = 10$. The boundary condition is given by the following expression

$$u|_\Gamma = \frac{1}{3}(x^3 + y^3) + 0.5(x+y). \qquad (17)$$

The results are summarized in Table 4.

This case was selected because in [16] was presented as an interesting case for verifying the numerical stability and convergence of the method. In Table 4 we can verify again that with the parallel algorithm we obtained a significant decrease in the time complexity.

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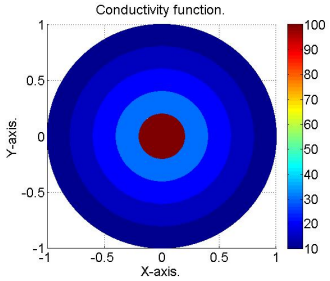

Fig. 3: Geometrical conductivity within a non-smooth domain. Combination of a disk and concentric rings.

TABLE 4: Brief relation of the time complexity in both algorithms with a geometrical conductivity distribution constructed with concentric circles

Formal Powers N	Time Sequential t (seconds)	Time Parallel t (seconds)
5	449.48	3.592
10	825.57	3.918
15	1199.95	4.687
20	1572.79	5.446
25	1928.5	6.061
30	2322.39	7.053
35	2653.13	7.846
40	3001.47	8.746

V. Conclusions

The construction of the numerical method and algorithms for approaching the solution for the forward problem of the two-dimensional impedance equation, is an important contribution to the Electrical Impedance Tomography theory. This is asserted by considering that the examples presented in this work, could well pose a difficult challenge if analysed with classical numerical methods, as the Finite Element variations are (see e.g. [9]).

In this sense, this numerical method and algorithms can be employed for analysing physical conductivity distributions, since no variations are needed for examining those cases when the exact mathematical expressions are unknown [18]. Furthermore, the results suggest that an adequately balance between the numerical accuracy, steadiness and the computational cost has been achieved. Even though that when using the parallel Algorithm 2 we have as advantage that we can obtain a better approximation to the solution because this algorithm give us the possibility of increasing significantly the parameters of number of points and number of radii. While employing the parallel Algorithm 2 we notice that time complexity reduced considerable, without provoking loss of precision or accuracy. Another important thing that was observed is that the time complexity is proportional to the complexity of the conductivity function.

The development of this parallel algorithm enable us to continue with the hard due of applying this algorithm in the solution of the Electrical Impedance Tomography problem, because this reduction of time will allow us to study more complex conductivity functions and analyse different domains (smooth and non-smooth). When applying this algorithm in medical image will be needed to have accuracy with low time complexity. Beside, many characteristics discussed along these pages might be subjects of further works.

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