Proceedings of the World Congress on Engineering and Computer Science 2014 Vol I
WCECS 2014, 22-24 October, 2014, San Francisco, USA

Parameter Estimation for Inventory of Load Models in Electric Power Systems

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***Abstract*—This paper presents an approach to characterize power system loads through estimation of contributions from individual load types. In contrast to methods that fit one aggregate model to observed load behavior, this approach estimates the inventory of separate components that compose the total power consumption. Common static and dynamic models are used to represent components of the load, and parameter estimation is used to determine the amount each load contributes to the cumulative consumption. Trajectory sensitivities form the basis of the parameter estimation algorithm and give insight into which parameters are well-conditioned for estimation. Parameters of interest are contributions to total load and initial conditions for dynamic loads. Results are presented for simulated data to demonstrate the feasibility of the approach.**

***Index Terms*—electric power systems, load modeling, simulation, trajectory sensitivities, parameter estimation.**

I. Introduction

As power system models and simulations used for planning and stability studies become more advanced, higher fidelity models of all power system components are needed. Loads are particularly difficult to describe due to their diverse composition and variation in time, yet their importance to voltage stability and transient behavior has been recognized [1]–[4]. The approaches to load modeling can be categorized as either measurement-based or component-based [4]. Measurement-based approaches utilize data collected from a substation or feeder to develop a model that matches observed behavior (see, for example, [5]–[7]). Component-based approaches combine known models of all devices that make up the load (see, for example, [2], [4]). A combination of the approaches, known as identification of load inventory, has been developed where measurements are used to estimate the fraction (percentage) that each different device within a load contributes to the aggregate power consumed [8], [9].

The focus of this paper is development of a load inventory model where parameter estimation is used to determine the amount each component contributes to the total power consumption. Parameter estimation is achieved via a Gauss-Newton method based upon trajectory sensitivities (see [10]) which computes parameters that best fit simulated model responses to simulated measurements on a single phase. The results indicate which load contributions are well-conditioned for estimation and that those parameters can be accurately estimated in the presence of measurement error. Initial conditions of the dynamic states in the load models are difficult to identify; however, the load contribution coefficients are identifiable.

Manuscript submitted on July 13, 2014.
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II. Load Modeling

A wide variety of load models exist to mathematically represent the power consumed by a load and its dependencies on voltage, frequency, type and composition. Three common mathematical models for loads in power systems are presented below and utilized in the proposed approach. In all cases, it is assumed that powers and voltages are normalized by base values such that their units are in per unit (p.u.) [11].

A. ZIP

A polynomial model is commonly used to represent loads and capture their voltage dependency. The average and reactive powers of the load are written as a sum of constant impedance (Z), constant current (I) and constant power (P), and referred to as the ZIP model [3], [11].

$$P = P_0(K_{1p}\left(\frac{V}{V_0}\right)^2 + K_{2p}\frac{V}{V_0} + K_{3p}) \qquad (1)$$

$$Q = Q_0(K_{1q}\left(\frac{V}{V_0}\right)^2 + K_{2q}\frac{V}{V_0} + K_{3q}) \qquad (2)$$

where P, Q are the average and reactive power consumed by the load, respectively, P_0, Q_0 represent the nominal average and reactive power of the load, respectively, V is the magnitude of the sinusoidal voltage at the bus to which the load is connected, V_0 is the magnitude of the nominal voltage at the bus, and coefficients K_{1p}, K_{2p}, K_{3p}, K_{1q}, K_{2q} and K_{3q} define the proportion of each component of the model. Coefficients for many load types have been experimentally determined and reported [2], [4], [5].

B. Exponential Recovery

The power profile is defined by

$$P = \frac{x_p}{T_p} + P_0\left(\frac{V}{V_0}\right)^{\alpha_t} \qquad (3)$$

$$Q = \frac{x_q}{T_q} + Q_0\left(\frac{V}{V_0}\right)^{\beta_t} \qquad (4)$$

where P, Q are the average and reactive power consumed by the load, respectively, P_0, Q_0 are the nominal average and reactive power, respectively, V is the magnitude of the sinusoidal voltage at the bus to which the load is connected, and V_0 is the magnitude of the nominal voltage at the bus. Parameters T_p and T_q are the average and reactive load recovery time constants, respectively, α_s and β_s are the steady-state dependence of average and reactive powers on voltage, respectively, and α_t and β_t are the transient dependence of average and reactive powers on voltage, respectively. The parameters govern the behavior of the load model, and are generally fit to measurements. States x_p and x_q are average

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

and reactive power recovery, respectively, and are governed by the differential equations [7], [12], [13]:

$$\dot{x}_p = \frac{-x_p}{T_p} + P_0\left(\left(\frac{V}{V_0}\right)^{\alpha_s} - \left(\frac{V}{V_0}\right)^{\alpha_t}\right) \quad (5)$$

$$\dot{x}_q = \frac{-x_q}{T_q} + Q_0\left(\left(\frac{V}{V_0}\right)^{\beta_s} - \left(\frac{V}{V_0}\right)^{\beta_t}\right). \quad (6)$$

Parameters for different load types have been experimentally determined and reported [7], [13].

C. Induction Motor

The induction motor's voltages of interest are $Ve^{j\theta} = V\cos(\theta) + jV\sin\theta$ at the stator terminal and $V'e^{j\theta'} = v_d' + jv_q'$ at the voltage behind transient reactance. The stator current is $I = i_d + ji_q = \frac{Ve^{j\theta} - V'e^{j\theta'}}{R_s + jX_s'}$ where $X_s' = X_s + \frac{X_r X_m}{X_r + X_m}$ is the transient reactance. Additional parameters are stator resistance and leakage reactance, R_s and X_s, respectively, magnetizing reactance, X_m, and rotor resistance and leakage reactance, R_r, X_r, respectively.

The voltage $V'e^{j\theta'} = v_d' + jv_q'$ has real and imaginary parts governed by the differential equations [11], [14]

$$\frac{dv_d'}{dt} = -\frac{R_r}{X_r + X_m}\left[v_d' + \left(\frac{X_m^2}{X_r + X_m}\right)i_q\right] + sv_q' \quad (7)$$

$$\frac{dv_q'}{dt} = -\frac{R_r}{X_r + X_m}\left[v_q' - \left(\frac{X_m^2}{X_r + X_m}\right)i_d\right] - sv_d' \quad (8)$$

$$\frac{ds}{dt} = \frac{1}{2H}\left(T_{mo}(1-s)^2 - T_e\right) \quad (9)$$

where $s = \frac{\omega_s - \omega_r}{\omega_s}$ is slip, ω_r is rotor speed, ω_s is angular velocity of the stator field, T_{mo} is a load torque constant, $T_e = v_d' i_d + v_q' i_q$ is electromagnetic torque, and H is the motor and motor load inertia.

The average power P and reactive power Q consumed by the motor are then given by

$$\begin{aligned} P &= \mathrm{Re}(Ve^{j\theta}I^*) \\ &= \frac{1}{R_s^2 + X_s'^2}\big(R_s(V^2 + V\cos(\theta)v_d' - V\sin(\theta)v_q') \\ &\quad -X_s'(V\cos(\theta)v_q' - V\sin(\theta)v_d')\big) \quad (10) \\ Q &= \mathrm{Im}(Ve^{j\theta}I^*) \\ &= \frac{1}{R_s^2 + X_s'^2}\big(R_s(V\cos(\theta)v_q' - V\sin(\theta)v_d') \\ &\quad +X_s'(V^2 - V\cos(\theta)v_d' - V\sin(\theta)v_q')\big) \quad (11) \end{aligned}$$

where $(\cdot)^*$ denotes complex conjugate of the complex quantity, $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote the real and imaginary parts of a complex number, respectively. Parameters for different load types have been experimentally determined and reported [11], [15], [16].

D. Load Inventory Concept

Load behavior will be modeled by using an aggregation of ZIP, exponential recovery and induction motor models from above with appropriate parameters selected for each to represent the load's components. This is shown conceptually in Figure 1 with the total complex power consumed $P_L + jQ_L$ the sum of the power consumed by all the individual load elements. The parameters of interest will be the contribution

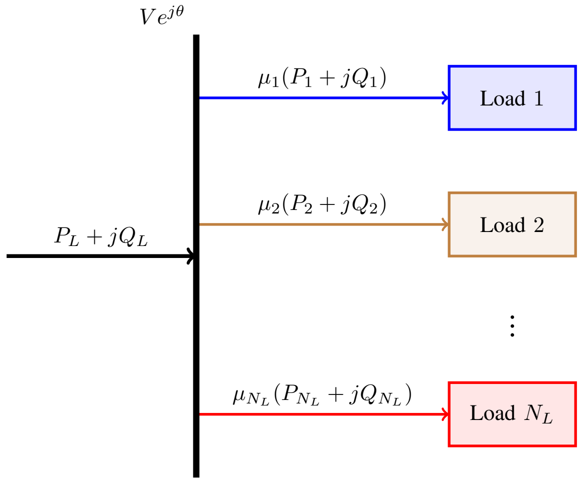

Fig. 1. Concept of load inventory

coefficients μ_i for $i = 1, 2, \ldots, N_L$ where N_L is the number of unique types of loads assumed to be connected to the bus as well as the initial conditions needed for each dynamic model.

In summary, the aggregate load with average power P_L and reactive power Q_L will be the fractional sum of each individual type of model. Each model represents a candidate load that might exist in the inventory and its fractional contribution indicated by μ_i is the parameter of interest for determining the inventory of loads. Total (aggregate) complex power is the sum of all load powers

$$\begin{aligned} P_L + jQ_L &= \mu_1(P_1 + jQ_1) + \mu_2(P_2 + jQ_2) \\ &\quad + \cdots + \mu_{N_L}(P_{N_L} + jQ_{N_L}). \quad (12) \end{aligned}$$

III. Trajectory Sensitivities

Differential-algebraic models are often utilized to represent the dynamic behavior of electric power systems [17]. When all loads are taken at a bus for the load inventory approach, the combined models presented above for loads can be represented in a manner similar to that of the broader power system:

$$\dot{x} = f(x, V) \quad (13)$$

$$y = g(x, V, \mu). \quad (14)$$

Here x is the vector of dynamic states (e.g., x_p, x_q, v_d', v_q', s for dynamic loads) that satisfy the differential equations (13), $y = [P_L,\ Q_L]^T$ is the 2×1 vector of total average and reactive powers consumed by the aggregate load, V is the magnitude of the voltage at the bus (treated as an input) and μ is the vector of contributions $\mu_{i=1,2,\ldots,N_L}$ taken as parameters.

Trajectory sensitivities provide a means to quantify the effect of small changes in parameters and/or initial conditions on a dynamic system's trajectory. Trajectory sensitivities will be utilized to guide the choice of how the parameters (here taken to be contributions μ and initial conditions x_0) should be altered to "best" match simulated trajectories to measurements of average power and reactive power consumed by the aggregate load. Following the presentation of trajectory sensitivities in [10], [17], flows of x and y that describe the

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

response of (13), (14) are defined as

$$x(t) = \phi_x(\lambda, V, t) \quad (15)$$
$$y(t) = g(\phi_x(\lambda, V, t), \lambda, V, t) = \phi_y(\lambda, V, t) \quad (16)$$

where $x(t)$ satisfies (13), all parameters of interest are combined into the vector $\lambda = [x_0,\ \mu]^T$ of dimension $M \times 1$, and flows show the dependence of the trajectories on parameters λ (here initial conditions and fractional contributions), input V and time t. To obtain the sensitivity of the trajectories to small changes in the parameters $\Delta\lambda$, a Taylor series expansion of (15), (16) can be formed. Neglecting higher order terms in the expansion yields

$$\Delta x(t) = \phi_x(\lambda + \Delta\lambda, V, t) - \phi_x(\lambda, V, t) \quad (17)$$
$$\approx \frac{\partial \phi_x(\lambda, V, t)}{\partial \lambda}\Delta\lambda \equiv x_\lambda(t)\Delta\lambda \quad (18)$$
$$\Delta y(t) = \phi_y(\lambda + \Delta\lambda, V, t) - \phi_y(\lambda, V, t) \quad (19)$$
$$\approx \frac{\partial \phi_y(\lambda, V, t)}{\partial \lambda}\Delta\lambda \equiv y_\lambda(t)\Delta\lambda. \quad (20)$$

The time-varying partial derivatives x_λ and y_λ are known as the trajectory sensitivities, and can be obtained by differentiating (13) and (14) with respect to λ. This differentiation gives

$$\dot{x}_\lambda = f_x(t)x_\lambda \quad (21)$$
$$y_\lambda = g_x(t)x_\lambda + g_\lambda(t) \quad (22)$$

where $f_x \equiv \frac{\partial f}{\partial x}$, $g_x \equiv \frac{\partial g}{\partial x}$ and $g_\lambda \equiv \frac{\partial g}{\partial \lambda}$ are time-varying Jacobian matrices, and $f_\lambda \equiv \frac{\partial f}{\partial \lambda} = 0$. Along the trajectory of the aggregate loads described by (13), (14) the trajectory sensitivities will evolve according to the linear, time-varying differential equations (21), (22). Initial conditions for x_λ are obtained from (15) by noting $x(t_0) = \phi_x(\lambda, V, t_0) = x_0$ such that $x_\lambda(t_0) = I$ where I is the identity matrix [10]. Initial conditions for y_λ follow directly from the algebraic equation (22) to yield $y_\lambda(t_0) = g_x(t_0) + g_\lambda(t_0)$.

The $N \times M$ sensitivity matrix for the ith output y^i is now defined by taking sensitivities (22) at discrete times $t_{k=0,1,\ldots,N-1}$

$$S^i(\lambda^j) = \begin{bmatrix} y^i_\lambda(t_0) \\ y^i_\lambda(t_1) \\ \vdots \\ y^i_\lambda(t_{N-1}) \end{bmatrix} \quad (23)$$

where N is the number of discrete values of time at which values are taken from the trajectory sensitivity (22) found by numerically solving the coupled system (13), (14), (21), (22); M is the number of parameters in λ; λ^j is the particular set of values for parameters used when computing the trajectory and associated sensitivities; and for this particular application $y^1_\lambda = P_{L_\lambda}$, $y^2_\lambda = Q_{L_\lambda}$ and the complete sensitivity matrix $S(\lambda^j) = \begin{bmatrix} S^1(\lambda^j) \\ S^2(\lambda^j) \end{bmatrix}$ is of dimension $2N \times M$.

As an example of computing the equations that will be solved for trajectory sensitivities, assume the total power is given by (12) and P_1, Q_1 are consumed by a load represented as the exponential recovery model's powers (3), (4). The sensitivities of P_L, Q_L to parameter μ_1 are

$$\frac{\partial P_L}{\partial \mu_1} \equiv P_{L_{\mu 1}} = P_1 = \frac{x_p}{T_p} + P_0\left(\frac{V}{V_0}\right)^{\alpha_t} \quad (24)$$
$$\frac{\partial Q_L}{\partial \mu_1} \equiv Q_{L_{\mu 1}} = Q_1 = \frac{x_q}{T_q} + Q_0\left(\frac{V}{V_0}\right)^{\beta_t} \quad (25)$$

where the states x_p, x_q will come from numerical solution of (5), (6) and V will be a specified input. The sensitivities of P_L, Q_L to the initial conditions $x_p(0)$, $x_q(0)$ are

$$\frac{\partial P_L}{\partial x_{p(0)}} \equiv P_{L_{xp(0)}} = \mu_1 \frac{1}{T_p}\frac{\partial x_p}{\partial x_{p(0)}} = \mu_1 \frac{1}{T_p} x_{p_{xp(0)}} \quad (26)$$
$$\frac{\partial P_L}{\partial x_{q(0)}} \equiv P_{L_{xq(0)}} = 0 \quad (27)$$
$$\frac{\partial Q_L}{\partial x_{p(0)}} \equiv Q_{L_{xp(0)}} = 0 \quad (28)$$
$$\frac{\partial Q_L}{\partial x_{q(0)}} \equiv Q_{L_{xq(0)}} = \mu_1 \frac{1}{T_q}\frac{\partial x_q}{\partial x_{q(0)}} = \mu_1 \frac{1}{T_q} x_{q_{xq(0)}} \quad (29)$$

where the sensitivities $x_{p_{xp(0)}} \equiv \frac{\partial x_p}{\partial x_p(0)}$, $x_{q_{xq(0)}} \equiv \frac{\partial x_q}{\partial x_q(0)}$ will be the solution to the following differential equations that govern the trajectory sensitivities found by differentiating (5), (6) with respect to the initial conditions $x_p(0)$, $x_q(0)$

$$\frac{\partial}{\partial x_{p(0)}}\dot{x}_p \equiv \dot{x}_{p_{xp(0)}} = -\frac{1}{T_p}\frac{\partial x_p}{\partial x_p(0)} = -\frac{1}{T_p} x_{p_{xp(0)}} \quad (30)$$
$$\frac{\partial}{\partial x_{q(0)}}\dot{x}_p \equiv \dot{x}_{p_{xq(0)}} = 0 \quad (31)$$
$$\frac{\partial}{\partial x_{p(0)}}\dot{x}_q \equiv \dot{x}_{q_{xp(0)}} = 0 \quad (32)$$
$$\frac{\partial}{\partial x_{q(0)}}\dot{x}_q \equiv \dot{x}_{q_{xq(0)}} = -\frac{1}{T_q}\frac{\partial x_q}{\partial x_q(0)} = -\frac{1}{T_q} x_{q_{xq(0)}}. \quad (33)$$

Note the order of the derivatives (with respect to parameter and time) were interchanged in the process.

The differential equations (30), (33) that govern the trajectory sensitivities are solved numerically in parallel with the differential equations (5), (6) that govern the model such that trajectories are obtained for both. Calculations of sensitivities for all load models to all parameters are given in [18].

IV. Parameter Estimation

Parameter estimation will be cast as a nonlinear least squares problem and solved using a Gauss-Newton iterative procedure [17]. Measurements of average and reactive power consumed by a load during a disturbance will be used to estimate the unknown model parameters (here contributions and initial conditions). The aim of parameter estimation is to determine parameter values that achieve the closest match between the measured samples and the model's simulated trajectory. Let measurements of the total average power P_L and reactive power Q_L (denoted by P_{L_m} and Q_{L_m}, respectively) consumed by the aggregate load be given by the appended sequences of N measurements

$$\vec{y}_m = \begin{bmatrix} [P_{L_m}(t_0),\ P_{L_m}(t_1),\ \ldots,\ P_{L_m}(t_{N-1})]^T \\ [Q_{L_m}(t_0),\ Q_{L_m}(t_1),\ \ldots,\ Q_{L_m}(t_{N-1})]^T \end{bmatrix} \quad (34)$$

with the corresponding simulated trajectory from numerically solving (13), (14) given by

$$\vec{y} = \begin{bmatrix} [P_L(t_0),\ P_L(t_1),\ \ldots,\ P_L(t_{N-1})]^T \\ [Q_L(t_0),\ Q_L(t_1),\ \ldots,\ Q_L(t_{N-1})]^T \end{bmatrix} \quad (35)$$

to get corresponding values at times $t_{k=0,1,2,\ldots,N-1}$. The mismatch between the measurements and corresponding model's trajectory can be written in vector form as

$$\vec{e}(\lambda) = \vec{y}(\lambda) - \vec{y}_m \tag{36}$$

where the notation is used to show the dependence of the trajectory, and correspondingly the error, on the parameters λ. The vectors $\vec{e}$, $\vec{y}$ and $\vec{y}_m$ will be of dimension $2N \times 1$ as N measurements of P_L and Q_L are assumed. The best match between model and measurement is obtained by varying the parameters so as to minimize the error vector $\vec{e}(\lambda)$ by some measure. One common measure is the two-norm (sum of squares) of the error vectors expressed as a cost function

$$C(\lambda) = \frac{1}{2}\left\|\vec{e}(\lambda)\right\|_2^2 = \frac{1}{2}\sum_{k=0}^{2N-1} e_k(\lambda)^2. \tag{37}$$

The error $e_k(\lambda)$ is the kth element of $\vec{e}$ and can be linearly approximated via a Taylor series expansion about an initial value of $\lambda = \lambda^j$ which yields

$$\begin{aligned} e_k(\lambda) &\approx e_k(\lambda^j) + \frac{\partial e_k(\lambda^j)}{\partial \lambda}(\lambda - \lambda^j) && (38)\\ &= e_k(\lambda^j) + y_{\lambda_k}(\lambda^j)\Delta\lambda && (39)\end{aligned}$$

where $\frac{\partial e_k(\lambda^j)}{\partial \lambda} = \frac{\partial y_k(\lambda^j)}{\partial \lambda} = y_{\lambda_k}(\lambda^j)$ since the measurements y_{m_k} are independent of λ and the definition $\Delta\lambda = \lambda - \lambda^j$ is utilized. The new value of λ^j denoted λ^{j+1} will be chosen to minimize the following cost function with linearized error.

$$\begin{aligned} C(\lambda) &= \frac{1}{2}\sum_{k=0}^{2N-1}\left(e_k(\lambda^j) + y_{\lambda_k}(\lambda^j)\Delta\lambda\right)^2 \\ &= \frac{1}{2}\sum_{k=0}^{2N-1}\left(e_k(\lambda^j) + S_k(\lambda^j)\Delta\lambda\right)^2 \\ &= \frac{1}{2}\left\|\vec{e}(\lambda^j) + S(\lambda^j)\Delta\lambda\right\|_2^2 && (40)\end{aligned}$$

where S_k is the kth row of the sensitivity matrix S.

Minimizing the cost function of linearized error (40) can now be performed via the Gauss-Newton method [17]. The process starts with an initial guess λ^0 for parameter values and then parameters are updated according to iterations of the following two steps.

$$\begin{aligned} S(\lambda^j)^T S(\lambda^j)\Delta\lambda^{j+1} &= -S(\lambda^j)^T e(\lambda^j) && (41)\\ \lambda^{j+1} &= \lambda^j + \alpha^{j+1}\Delta\lambda^{j+1} && (42)\end{aligned}$$

where S is the trajectory sensitivity matrix defined in (23), α^{j+1} is a scalar that determines step size, and iterations stop when $\Delta\lambda^{j+1}$ is sufficiently small. The resulting parameter values λ^{j+1} will be a local minimum for the cost function (37) due to the linearization and will be dependent on the initial guess λ^0. An additional note is that the parameter estimation process breaks down if $S^T S$ is ill-conditioned, i.e., nearly singular. This leads to the concept of identifiability and quantification of parametric effects [17], [19]. The invertibility of $S^T S$ can be investigated through its singular values and condition number, eigenvalues, or magnitude of sensitivities over a trajectory via the 2-norm. The less-rigorous, 2-norm will be utilized here to gain insight into the condition of $S^T S$. The 2-norm will be defined for the sensitivity of the ith output y^i to the jth parameter λ_j summed over discrete times t_k as

$$\left\|S^{ij}\right\|_2^2 = \sum_{k=0}^{N-1} S^{ij}(t_k, \lambda)^2. \tag{43}$$

The size of the values computed via (43) will give an indication of the effect of parameters on the trajectory, and in turn give guidance as to which parameters can be estimated.

V. Example of Application via Simulation

The parameter estimation approach described above was implemented through simulation to estimate load contributions and initial conditions. Five loads, each represented by one of five models, were taken as connected to a bus as shown in Figure 1 with magnitude V of the bus's phasor voltage taken to be the input to the models, and average and reactive powers P_L, Q_L, respectively, taken to be the (measurable) outputs. Load 1 was an exponential recovery model, load 2 was the model of a residential induction motor, load 3 was a model of a small industrial induction motor, load 4 was a model of a large industrial induction motor, and load 5 was a ZIP model. That made the $M = 16$ unknown parameters

$$\begin{aligned}\lambda = \big[\, & x_{p1}(0), x_{q1}(0), v'_{d2}(0), v'_{q2}(0), s_2(0), v'_{d3}(0), v'_{q3}(0), \\ & s_3(0), v'_{d4}(0), v'_{q4}(0), s_4(0), \mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \,\big]^T \quad (44)\end{aligned}$$

which includes initial conditions for all the states in the individual models as well as the fractional contributions of each load model to the aggregate power consumed. For the five loads considered, the total load is given by

$$\begin{aligned} P_L + jQ_L &= \mu_1(P_1 + jQ_1) + \mu_2(P_2 + jQ_2) \\ &\quad + \mu_3(P_3 + jQ_3) + \mu_4(P_4 + jQ_4) \\ &\quad + \mu_5(P_5 + jQ_5) && (45)\end{aligned}$$

with $N_L = 5$; $\mu_{i=1,2,\ldots,5}$ the fractional contributions of each load model; P_1, Q_1 given by (3), (4); P_2, Q_2 given by (10), (11); P_3, Q_3 given by (10), (11); P_4, Q_4 given by (10), (11); and P_5, Q_5 given by (1), (2). Representative values for parameters used in each model were taken from [4], [13], [16] and are given in Table I, and the nominal bus voltage is taken to be $V_0 = 1$p.u.

TABLE I
Parameter values for models of loads

Load	Model and Parameters							
1	Exponential Recovery							
	P_0	T_P	α_s	α_t	Q_0	T_q	β_s	β_t
	1.25	60	0	2	0.5	60	0	2
2	Induction Motor - Residential							
	R_s	X_s	X_m	R_r	X_r	H	T_0	
	0.077	0.107	2.22	0.079	0.098	0.74	0.46	
3	Induction Motor - Small Industrial							
	R_s	X_s	X_m	R_r	X_r	H	T_0	
	0.031	0.1	3.2	0.018	0.18	0.7	0.6	
4	Induction Motor - Large Industrial							
	R_s	X_s	X_m	R_r	X_r	H	T_0	
	0.013	0.067	3.8	0.009	0.17	1.5	0.8	
5	ZIP							
	P_0	K_{1p}	K_{2p}	K_{3p}	Q_0	K_{1q}	K_{2q}	K_{3q}
	1.0	0.15	0.6	0.25	0.7	0.05	-0.05	1.0

A. Results when no error in measurements

For study in simulation, a 3% decrease in the magnitude of the bus's voltage V was taken to be the input. The voltage was dropped from its nominal value of 1p.u. to 0.97p.u. at 50 seconds. With the initial conditions and load contributions at specified values, the simulation was run to generate synthetic, "measured" data to represent data that might be collected by a voltage disturbance monitor or phasor measurement unit. A sampling rate of 10Hz was used to record the total average and reactive powers consumed by the five loads. Nominal values for parameters are given in Table II, and plots of the aggregate power consumption P_L and Q_L are given in Figure 2 as the solid lines.

The simulation was then modified to run with initial guesses for parameter values, and the iterative Gauss-Newton process described by (41), (42) implemented to update the values of the parameters until they converged to within a specified tolerance. At each iteration, the system's model (13), (14) and trajectory sensitivities (21), (22) were numerically solved using Matlab's *ode15s()* solver for a new trajectory using updated values of the parameters. The results of the iterative process can be seen Figure 3 and show convergence of the load contributions to values used to create the synthetic measurements. The final, estimated values of all parameters (both initial conditions and load contributions) are given in Table II. The simulated trajectories for P_L and Q_L as the parameters are updated can be seen as the dashed lines in Figure 2.

TABLE II
VALUES, GUESSES AND ESTIMATES OF PARAMETERS

Load	Model and Estimated Parameters				
1	Exponential Recovery				
		$x_p(0)$	$x_q(0)$		μ_1
	value:	0.0010	0.0007		0.1000
	guess:	0.0025	0.0015		0.3000
	estimate:	0.0010	0.0007		0.1000
2	Induction Motor - Residential				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_2
	value:	0.8659	0.1439	0.0399	0.2000
	guess:	0.9000	0.1800	0.0550	0.3000
	estimate:	0.8659	0.1439	0.0399	0.2001
3	Induction Motor - Small Industrial				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_3
	value:	0.8842	0.0527	0.0120	0.2000
	guess:	0.9000	0.0750	0.0600	0.1000
	estimate:	0.8840	0.0527	0.0121	0.2001
4	Induction Motor - Large Industrial				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_4
	value:	0.9124	0.0308	0.0078	0.3000
	guess:	0.8900	0.5000	0.0150	0.2000
	estimate:	0.9125	0.0308	0.0078	0.2999
5	ZIP				
					μ_5
	value:				0.2000
	guess:				0.1000
	estimate:				0.2000

B. *Study of identifiability*

For the sample application above where no measurement error was introduced, the 2-norm of all sensitivities was calculated and shown in Table III. Of particular note is that the sensitivities of the average and reactive power are at least an order of magnitude larger for load contributions than

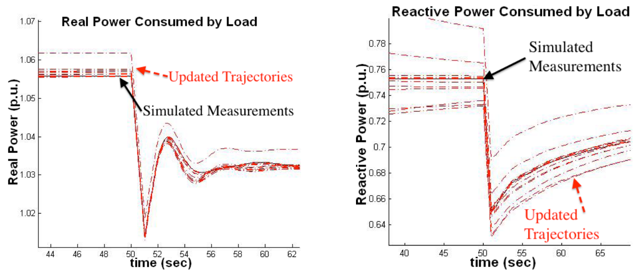

Fig. 2. Average power (left) and reactive power (right) consumed by aggregate load; simulated measurements are solid lines and trajectories for updated parameter estimates are dashed lines

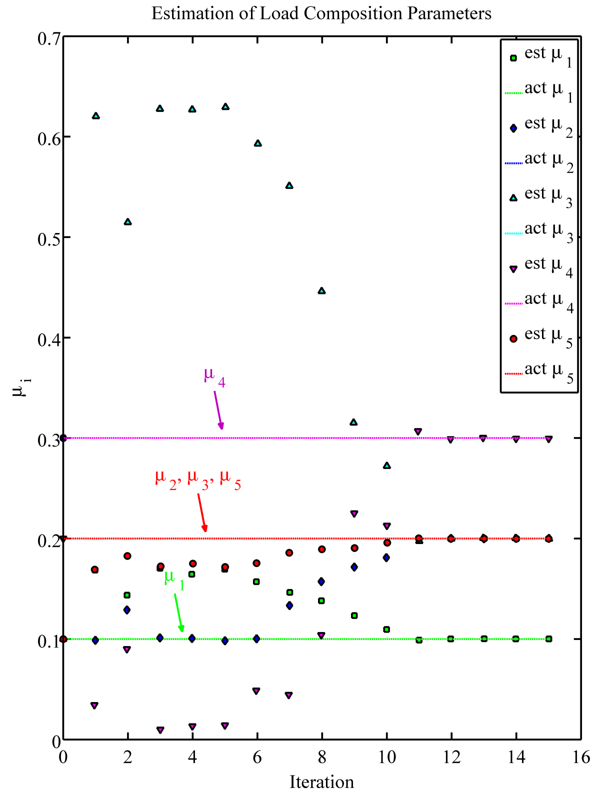

Fig. 3. Estimates of parameters (as markers/symbols) representing load contributions over iterations; dashed lines are actual values

initial conditions. This indicates that the load contributions have a larger impact on the trajectory and in turn should be more readily identified through parameter estimation. When no measurement error was assumed, both initial conditions and load contributions were identified, but when random error was introduced (as discussed in the next section) the estimates of the initial conditions were inaccurate. The zero entries in the table imply that the load models' powers do not depend on that particular parameter.

As an additional check of identifiability for the parameters of interest, both the condition number and eigenvalues were computed for the matrix $S^T S$. The condition number was 2.281×10^{15} with sensitivities for initial conditions included which confirmed an ill-conditioned matrix and potential difficulty in estimating initial conditions. When sensitivities were removed from $S^T S$ leaving only those to load contributions, the condition number improved to 202.4 which indicated load contributions are identifiable. Eigenvalues of $S^T S$ were computed and further confirmed identifiable parameters as

ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE III
2-NORM OF TRAJECTORY SENSITIVITIES FOR PARAMETERS (LOAD CONTRIBUTIONS AND INITIAL CONDITIONS) OF INTEREST

Load	Model and 2-norm of Trajectory Sensitivities				
1	Exponential Recovery				
		$x_p(0)$	$x_q(0)$		μ_1
	$\|S_{P_L}\|_2^2$:	0.029	0		7.920
	$\|S_{Q_L}\|_2^2$:	0	0.027		15.37
2	Induction Motor - Residential				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_2
	$\|S_{P_L}\|_2^2$:	0.005	0.003	0.002	14.37
	$\|S_{Q_L}\|_2^2$:	0.002	0.001	0.001	13.59
3	Induction Motor - Small Industrial				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_3
	$\|S_{P_L}\|_2^2$:	0.001	0.003	0.002	1.116
	$\|S_{Q_L}\|_2^2$:	0.081	0.003	0.008	3.220
4	Induction Motor - Large Industrial				
		$v'_d(0)$	$v'_q(0)$	$s(0)$	μ_4
	$\|S_{P_L}\|_2^2$:	0.004	0.003	0.003	4.095
	$\|S_{Q_L}\|_2^2$:	0.009	0.005	0.007	4.725
5	ZIP				
					μ_5
	$\|S_{P_L}\|_2^2$:				75.97
	$\|S_{Q_L}\|_2^2$:				64.89

small eigenvalues were associated with the initial conditions and much larger eigenvalues were associated with the load contributions.

C. Results with 2% random error in measurements

The study described above was repeated, but this time with an error in each measurement achieved by adding a normally distributed random number to each measurement with mean 0 and standard deviation 2%. The method of estimating parameters worked well for the load contributions, but not the initial conditions. This is attributed to the issues with identifiability discussed above. Table IV shows the "true values", initial guesses and estimates of the parameters representing load contributions in this case.

TABLE IV
VALUES, GUESSES AND ESTIMATES OF PARAMETERS WHEN 2% RANDOM ERROR IN MEASUREMENTS

Load	Model and Estimated Parameters			
1	Exponential Recovery			
	μ_1	value	guess	estimate
		0.1000	0.3000	0.1231
2	Induction Motor - Residential			
	μ_2	value	guess	estimate
		0.2000	0.3000	0.1698
3	Induction Motor - Small Industrial			
	μ_3	value	guess	estimate
		0.2000	0.1000	0.1740
4	Induction Motor - Large Industrial			
	μ_4	value	guess	estimate
		0.3000	0.2000	0.3102
5	ZIP			
	μ_5	value	guess	estimate
		0.2000	0.1000	0.2272

VI. CONCLUSION

This paper presented an application of trajectory sensitivities and parameter estimation to estimate an aggregate load's composition. An example is presented using simulated data for five types of loads modeled by an exponential recovery model, three induction motor models with different parameters, and a ZIP model. Future work will be to incorporate and investigate additional models, and apply the approach to real data collected from a voltage disturbance monitor or phasor measurement unit.

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ISBN: 978-988-19252-0-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)