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Capacity of a Bio-inspired Communication System

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***Abstract*—Bio-inspired communication (BC) is the next generation of communications at nano-scale, especially, in fluid media. Recent researches on BC still remains limited, questions on how to build a communication system from a bio-physical environment, or how to modulate information in such a environment are really challenging. Further, geometric shape of medium (free space, half space, box...) also affects ways what we approach to build a BC system. In this paper, we build a BC system from a half space with an absorbing boundary at receiver. We then express how to signify logical bits by using nano-scale particles, derive the channel capacity, and finally show numerical results.**

***Index Terms*—Nano-network, Brownian motion, capacity.**

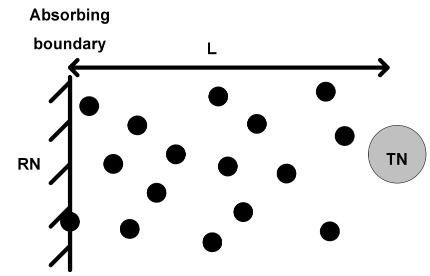

Fig. 1. DbBC with an absorbing boundary at RN (x=0).

I. Introduction

Bio-inspired communication is a new interdisciplinary field that spans many different research areas including nano, bio and communication technologies [1], [2]. BC is inspired by observing communication mechanisms in biological systems, e.g., communication among cells. Compared to traditional communication, BC has some outstanding advantages such as low power consumption, ability to communicate with nanomachines in biophysical systems [2]. For a BC system, the information exchange is performed at nano-scale by nanomachines that can be biological devices (e.g., proteins, cells) or artificial devices. These devices must be capable of performing tasks such as sensing, counting... In relative fields, basic capabilities of nanomachines have been studied, some initial results reveal that nanomachines can be engineered to perform above-mentioned tasks for future applications [3], [4].

A BC system has its own characteristics, hence there is a need to investigate the nature of the propagation. From perspective of communication, a simple BC system contains components: A transmitter nanomachine (TN) encoded the information onto particles and releases them into the medium; the motion of the particles follows some biological and physical laws that depend on the properties of the medium; and once the particles reach a receiver nanomachine (RN), they will be decoded. Such a BC system is called a diffusion-based BC (DbBC). A remarkable point is that the motion of particles will be modeled according to the biological characteristics of the medium. For example, if the medium between TN and RN is only fluid (Figure 1), the motion of particles can be modeled as Brownian motion[1] that describes the random motion of particles suspended in fluid medium.

Up to now, question on how to build a BC system from existing medium is really challenging. Some methods to implement a BC system can be listed here: Based on the free diffusion of particles in the fluid medium (e.g., blood stream) [5]–[7]; based on utilizing gap junctions between cells to allow particles to go through from the departure cell to destination cell [8]; finally, based on using molecular railways as a way to direct particles to specific place [9].

Recent works have been aiming to build physical models of a BC system in half space [5], [6] and mathematical expressions involved, e.g., capacity [6], modulation techniques [10]. However, these researches still remain limited and the theoretical development of a DbBC is in *infancy*. In our work, we consider is a diffusive fluid medium with an absorbing boundary at RN. The mechanism of the DbBC is as follows: TN encodes information onto particles and then releases them into medium. Because the medium is diffusive fluid, particles tend to diffuse far away from TN and obey diffusion laws (in stable medium, Fick's law is used to describe the motion of diffusing particles such as particles, atoms) [3]. Finally, particles reach the receive RN. Thus, there is a need to understand the nature of diffusion process, we study and express it in this paper. Moreover, we derive the channel capacity of a BC system based on such a diffusive fluid medium.

The rest of the paper is organized as follows. In Section II, we describe a biophysical model and present its essential aspects. In Section III, we propose a binary channel based on the biophysical model and then derive the capacity of the channel. In Section IV, numerical results are provided. Finally, conclusions are given in Section V.

II. Biophysical Model

We assume that TN is a source of identical particles and transmits particles into the fluid medium. After being transmitted from TN, particles diffuse in the medium obeying Fick's second law that describes the distribution density $p(x,t)$ of any particle with respect to position and time

$$\frac{\partial p(x,t)}{\partial x} = D\frac{\partial^2 p(x,t)}{\partial x^2} \tag{1}$$

where D $\left(\text{length}^2/\text{time}\right)$ is diffusion constant.

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[1]The Brownian motion is named after Robert Brown who observed the irregular motion of pollen particles floating in water. The Brownian motion has been well known in biology, physics.

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At first, we consider the motion of particles in full space $(-\infty, +\infty)$ without any boundary condition. Thus, we consider the natural conditions that are defined as follows

$$\left\{ \begin{array}{l} p(x,0) = \delta(x) \\ p(\pm\infty, t) = 0 \end{array} \right. . \tag{2}$$

Let $p_\bullet(x,t)$ be the solution of equation (1) with the natural conditions. It is given by $p_\bullet(x,t) = \left(1/\sqrt{4\pi Dt}\right) \exp\left(-x^2/(4Dt)\right)$. Derivation of $p_\bullet(x,t)$ is shown in more detail in appendix B. This solution is only basic one whereby we can find out solutions to eq. (1) in more complex cases.

In our problem, we take boundary conditions into consideration, i.e., we consider the motion of particles in half space (Fig. 1 is provided as an illustration). This is to say that whenever particles reach the absorbing boundary (e.g., the membrane of the cell), they will be absorbed. Let $x_0 = -L$ be TN's position and 0 be RN's location. Mathematically, absorbing boundary conditions are defined as

$$\left\{ \begin{array}{l} p(+\infty, t) = 0 \\ p(0,t) = 0 \\ p(x,0) = \delta(x) \end{array} \right. . \tag{3}$$

Let $p_\star(x,t)$ be the solution of equation (1) with the absorbing boundary conditions. Then $p_\star(x,t)$ is given by

$$p_\star(x,t) = p_\bullet(x+L,t) - p_\bullet(x-L,t). \tag{4}$$

Derivation of $p_\star(x,t)$ is shown in more detail in appendix C.

A. The first passage time.

In researches on diffusion with absorbing boundary condition, one of the most important aspects is the first passage/hitting time that is defined as the duration of time that a particle moves from the origin TN to the absorbing boundary RN for the first time [11], [12]:

$$\mathcal{T} = \inf\left\{t \geq 0 : X(t) \leq b(t)\right\} \tag{5}$$

where $X(t)$ is a diffusion process described as a continuous time random walk (CTRW) and $b(t)$ is an absorbing boundary (in our case, $b(t) = 0$). The distribution of $\mathcal{T}$ will be derived in this subsection.

- Note: This subsection presents shortly how to derive the distribution of $\mathcal{T}$ from the solution $p_\star(x,t)$ of the equation (1) **with** the Dirichlet condition. To understand how to obtain $p_\star(x,t)$, please see Appendix B and Appendix C sequentially.

Let S_1 denote the event that a particle is absorbed at the boundary **before** time t. Let S_2 denote the event that a particle still remains diffusing in the medium **until** time t. We have $Pr\left\{S_1\right\} + Pr\left\{S_2\right\} = 1$. Let $F_{\mathcal{T}}(t)$ be the CDF of $\mathcal{T}$, it is obviously the probability that a particle reaches the absorbing boundary **before** time t. Thus, we have the relation

$$F_{\mathcal{T}}(t) = Pr\left\{S_1\right\} = 1 - Pr\left\{S_2\right\} \tag{6}$$

In addition, the probability $Pr\left\{S_2\right\}$ is given by

$$Pr\left\{S_2\right\} = \int_{-\infty}^{0} p_\star(x,t)dx \tag{7}$$

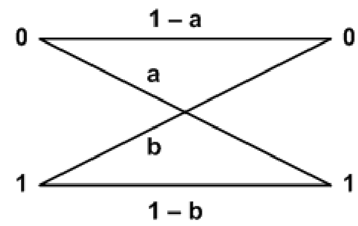

Fig. 2. Binary channel with the dependence of probabilities a and b upon the first passage time.

where $p_\star(x,t)$ is defined in (44), Appendix C. We also like to remind that the region of diffusion is half-space $x \leq 0$ with absorbing boundary at $x = 0$. So the integral is calculated over $(-\infty, 0]$.

Finally, the CDF of $\mathcal{T}$ is given by

$$F_{\mathcal{T}}(t) = 1 - \int_{-\infty}^{0} p_\star(x,t)dx = \operatorname{erfc}\left(\frac{L}{\sqrt{4Dt}}\right) \tag{8}$$

and then by taking the first order derivative of $F_{\mathcal{T}}(t)$, we obtain the PDF as follows

$$f_{\mathcal{T}}(t) = \frac{\partial}{\partial t} F_{\mathcal{T}}(t) = \frac{L}{\sqrt{4\pi Dt^3}} e^{-L^2/(4Dt)}. \tag{9}$$

III. Binary channel based on diffusion process

We consider a binary channel sketched in Figure 2.

A. Preliminaries

- Let M_i^j be a discrete variable denoting the number of particles transmitted at the beginning of the i^{th} and get absorbed in the j^{th} slot.
- Let $M_\uparrow^j$ be the total number of particles absorbed in the j^{th} slot, it is given by

$$M_\uparrow^j = \sum_{i=1}^{j} M_i^j = \sum_{i<j} M_i^j + M_j^j \cdot \tag{10}$$

  Obviously, we have $M_i^j \subseteq M_\uparrow^j$.
- Let $M_j^\uparrow$ be the total number of particles transmitted at the beginning of the j^{th} slot, it is given by

$$M_j^\uparrow = M_j^j + \sum_{i>j} M_j^i = \left\{ \begin{array}{ll} 0, & \text{bit } 0 \\ n \text{ (particles)}, & \text{bit } 1 \end{array} \right. . \tag{11}$$

- Let p_{ij} be the probability that a particle from the i^{th} slot gets absorbed in the j^{th} slot, it is given by

$$p_{ij} = F_{\mathcal{T}}((j-i+1)T_d) - F_{\mathcal{T}}((j-i)T_d) \cdot \tag{12}$$

- Let $\mathcal{P}_{M_i^j|M_i^\uparrow}\left(m_i^j \middle| m_i^\uparrow\right)$ be the probability that m_i^j out of $m_i^\uparrow$ particles get absorbed. Due to $M_i^\uparrow = \{0, n\}$, we have two cases
  - If $M_i^\uparrow = m_i^\uparrow = 0$, then $m_i^j = 0$ and

$$\mathcal{P}_{M_i^j|M_i^\uparrow}\left(m_i^j \,|0\right) = \left\{ \begin{array}{ll} 1, & m_i^j = 0 \\ 0, & \text{otherwise} \end{array} \right. . \tag{13}$$

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– If $M_i^\uparrow = m_i^\uparrow = n$, then $m_i^j \in \{0, 1, \ldots, n\}$ and

$$\mathcal{P}_{M_i^j|M_i^\uparrow}\left(m_i^j\,|n\right) = \left(\begin{array}{c} n \\ m_i^j \end{array}\right) p_{ij}^{m_i^j}\left(1-p_{ij}\right)^{n-m_i^j}. \tag{14}$$

B. Mutual Information

The *time synchronization* between TN and RN is assumed to be perfect. At TN and RN, time is slotted by T_d. Particles are always transmitted at the beginning of time slots $\{1, 2, \ldots, \infty\}$. At RN, we determine a threshold $\eta < n$ so that if the number of received particles $M_\uparrow^j = m_\uparrow^j < \eta$, we have bit 0. In contrast, we have bit 1.

Let $Q_\uparrow^j = \sum_{i<j} M_i^j \left| m_1^\uparrow, \ldots, m_{j-1}^\uparrow \right.$ be the number of "redundant" particles absorbed in j^{th} **given** they are transmitted from 1^{st} to $(j-1)^{th}$ slots. Thus, $Q_\uparrow^j = \{0, 1, \ldots, \sum_{i<j} m_i^\uparrow\}$ and the PMF of $Q_\uparrow^j$ is given by

$$\mathcal{P}_{Q_\uparrow^j}\left(q_\uparrow^j\right) = \bigotimes_{i<j} \mathcal{P}_{M_i^j|M_i^\uparrow}\left(m_i^j\left|m_i^\uparrow\right.\right) \tag{15}$$

where $m_i^\uparrow \in \{0, n\}$ and $m_i^j \leq m_i^\uparrow$.

Given that $M_i^\uparrow = m_i^\uparrow$ where $i < j$ is known, we make the connection between physical channel and binary channel as follows:

$$\begin{aligned} a &= \mathbb{P}\left\{M_\uparrow^j \geq \eta \left| m_j^\uparrow = 0\right.\right\} \\ &= \mathbb{P}\left\{M_\uparrow^j \geq \eta \left| m_1^\uparrow, \ldots, m_{j-1}^\uparrow; m_j^\uparrow = 0\right.\right\} \\ &= \mathbb{P}\left\{Q_\uparrow^j + m_j^j \geq \eta \left| m_j^j = 0\right.\right\} \\ &= \mathbb{P}\left\{Q_\uparrow^j \geq \eta\right\} \end{aligned} \tag{16}$$

$$\begin{aligned} b &= \mathbb{P}\left\{M_\uparrow^j < \eta \left| m_j^\uparrow = n\right.\right\} \\ &= \mathbb{P}\left\{M_\uparrow^j < \eta \left| m_1^\uparrow, \ldots, m_{j-1}^\uparrow; m_j^\uparrow = n\right.\right\} \\ &= \mathbb{P}\left\{Q_\uparrow^j + m_j^j < \eta \left| m_j^j \in \{0, 1, \ldots, n\}\right.\right\} \\ &= \sum_{q_\uparrow^j=0}^{\eta-1} \mathcal{P}_{Q_\uparrow^j}(q_\uparrow^j) \sum_{m_j^j=0}^{\eta-1-q_\uparrow^j} \mathcal{P}_{M_j^j|M_j^\uparrow}\left(m_i^j\,|n\right) \end{aligned} \tag{17}$$

The mutual information $I_j(M_j^\uparrow, M_\uparrow^j)$ in nats/s/Hz is a function of $\mathbb{P}\left\{M_j^\uparrow = n\right\} = \varphi$ and is given by

$$\begin{aligned} & I_j(M_j^\uparrow, M_\uparrow^j) = I_j(\varphi) \\ &= en\left(M_\uparrow^j\right) - (1-\varphi)en\left(M_\uparrow^j\,|0\right) - \varphi en\left(M_\uparrow^j\,|n\right) \\ &= -\left[(1-a) - \varphi(1-a-b)\right]\log\left[(1-a) - \varphi(1-a-b)\right] \\ &\quad - \left[a + \varphi(1-a-b)\right]\log\left[a + \varphi(1-a-b)\right] \\ &\quad + (1-\varphi)\left[(1-a)\log(1-a) + a\log a\right] \\ &\quad + \varphi\left[(1-b)\log(1-b) + b\log b\right] \end{aligned} \tag{18}$$

where

$$\begin{aligned} en\left(M_\uparrow^j\right) &= -\mathbb{P}\left\{M_\uparrow^j \geq \eta\right\}\log\mathbb{P}\left\{M_\uparrow^j \geq \eta\right\} \\ &\quad - \mathbb{P}\left\{M_\uparrow^j < \eta\right\}\log\mathbb{P}\left\{M_\uparrow^j < \eta\right\} \end{aligned} \tag{19}$$

$$en\left(M_j^\uparrow\,|0\right) = -(1-a)\log(1-a) - a\log a \triangleq en(a) \tag{20}$$

$$en\left(M_j^\uparrow\,|n\right) = -(1-b)\log(1-b) - b\log b \triangleq en(b) \tag{21}$$

are respectively entropy and conditional entropies.

Taking the first derivative of $I_j(\varphi)$ w.r.t φ and putting it to zero, we obtain the extreme value of φ.

$$\frac{\partial I_j(\varphi)}{\partial\varphi} = 0 \Rightarrow \varphi = \frac{1 - a(\varsigma_j + 1)}{(1-a-b)(\varsigma_j + 1)} \tag{22}$$

where $\varsigma = \exp\left\{\frac{en(b) - en(a_j)}{1-a-b}\right\}$. Moreover, taking the second derivative of $I_j(\varphi)$ w.r.t φ, we have $\frac{\partial^2 I_j(\varphi)}{\partial\varphi^2} < 0$. Thus $I_j(\varphi)$ is a concave function of φ, the optimal value of φ is exactly the extreme value.

Substituting the optimal value of φ into (18), we obtain the channel capacity corresponding to the j^{th} slot

$$\begin{aligned} C_j &= \max_\varphi I_j(\varphi) \\ &= -\left(\frac{\varsigma}{\varsigma+1}\right)\log\left(\frac{\varsigma}{\varsigma+1}\right) - \left(\frac{1}{\varsigma+1}\right)\log\left(\frac{1}{\varsigma+1}\right) \\ &\quad - \frac{\left[\varsigma - b(\varsigma+1)\right]en(a) + \left[1 - a(\varsigma+1)\right]en(b)}{(1-a-b)(\varsigma+1)}. \end{aligned} \tag{23}$$

IV. Numerical Results

In this section, we provide an illustration of diffusion process and some numerical examples of the channel capacity.

Regarding the illustration of diffusion process, we use a CTRW to simulate the motion of particles suspended in fluid medium. The relation between CTRW and diffusion process (i.e., diffusion equation) is shown in appendix A.

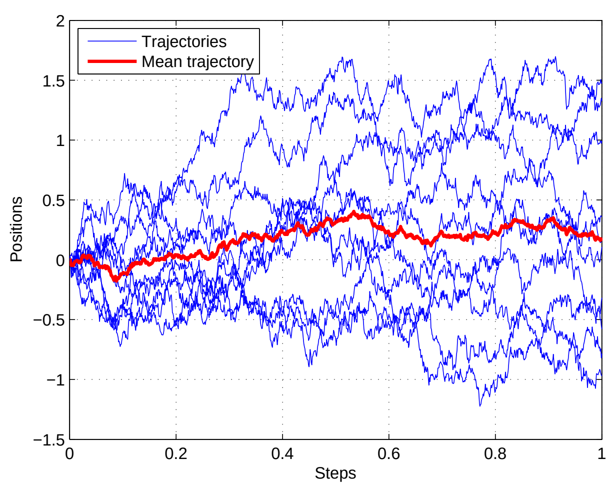

Fig. 3. 10 Brownian paths from time 0 to 1 with 1000 steps and their average.

Regarding the channel capacity, we depict Fig. 4 and 5. In Fig. 4, we depict the channel capacity C_4 (i.e., the 4^{th} slot is

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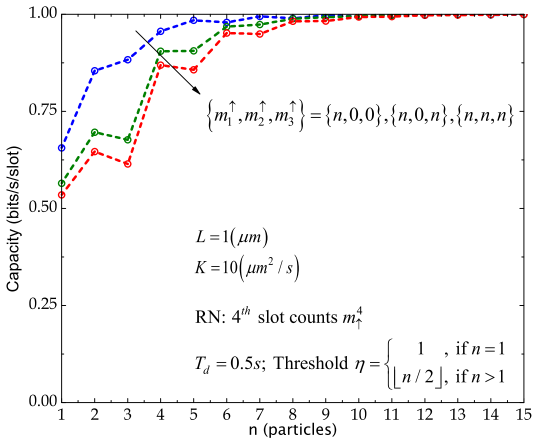

Fig. 4. The capacity is a function of n. Three out of eight cases of $\{m_1^\uparrow, m_2^\uparrow, m_3^\uparrow\}$ are exhibited for comparison.

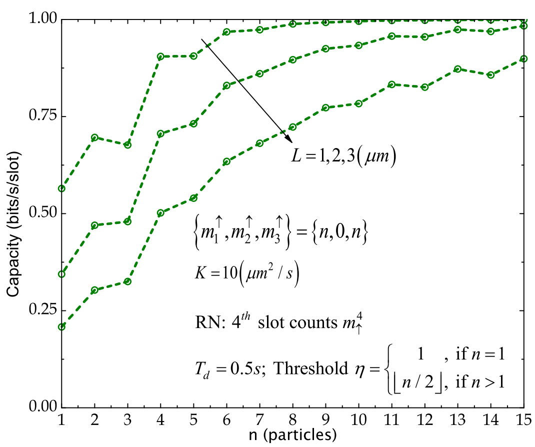

Fig. 5. The capacity is a function of the number of transmitted particles. The distance between TN and RN takes value 1, 2 and 3 μm in turn.

under consideration) w.r.t. the number of transmitted particles n given that the first three slots transmitted $\{m_1^\uparrow, m_2^\uparrow, m_3^\uparrow\}$. To examine the effect of previous slots on the channel capacity at the 4^{th} slot, the values of the stream $\{m_1^\uparrow, m_2^\uparrow, m_3^\uparrow\}$ are, respectively, assumed to be $\{n,0,0\}$, $\{n,0,n\}$, $\{n,n,n\}$. We see that the worst case is when all previous slots transmit bit 1. When the number of particles increases, the effect of $\{m_1^\uparrow, m_2^\uparrow, m_3^\uparrow\}$ on C_4 becomes trivial even in the worst case. In fig. 5, we also depict C_4 but let $\{m_1^\uparrow, m_2^\uparrow, m_3^\uparrow\} = \{n,0,n\}$ fixed and examine the effect of L. The result shows that C_4 decreases inversely with L.

V. Conclusions

In this paper, we propose a scheme for signifying logical bits and formulating the capacity of such a system. Numerical results show that we can achieve high channel capacity by using a large number of particles regardless of the transmission of previous slots and the increase of the distance L. In future work, other aspects of this system (e.g., the reliability) will be examined. Moreover, we are going to examine different models and propose suitable schemes.

Appendix

A. Continuous Time Random Walk and Diffusion Equation

1) Continuous Time Random Walk (CTRW): A CTRW is generated by a sequence of independent identically distributed (iid) random jumps $\Delta X_i = x_i - x_{i-1}$ and a sequence of iid positive random waiting (or stopping) times $\Delta T_i = t_i - t_{i-1}$ between two successive jumps.

In general, the jump length and the waiting time depend on each other, their distributions can be drawn from a joint probability density $\phi(x,t)$ as follows

$$\lambda(x) = \int_0^\infty \phi(x,t)dt \tag{24}$$

$$\psi(t) = \int_{-\infty}^\infty \phi(x,t)dx \tag{25}$$

Based on the above notion, the PDF $p(x,t)$ of a particle being in position x at time t is given by

$$p(x,t) = \delta(x)\Psi(t) + \int_0^t \int_{-\infty}^\infty p(x',t')\phi(x-x',t-t')dt'dx' \tag{26}$$

where $\delta(x)$ is the delta function, $\Psi(t) = \int_t^\infty \psi(t')dt' = 1 - \int_0^t \psi(t')dt'$ is the *survival probability* that the quantity x does *not* change value during the time interval $(0,t)$. We note that $\Psi(t)$ can also be understood as the probability that a particle only changes its position *after* instant t.

On the right hand side (RHS) of eq. (26), the first term implies the persistence of being in initial position $x_0 = 0$. While the second term (space-time convolution) relates $p(x,t)$ to the event that the particle just arrived at position $x' \in (-\infty,\infty)$ at instant $t' < t$, and then jumps to position x at instant t (after waiting time $t - t'$).

Applying Fourier - Laplace transform to both sides of the equation (26), we get

$$\begin{aligned} \hat{\tilde{p}}(k,s) &= \mathcal{F}_x\left\{\mathcal{L}_t\left\{p(x,t);s\right\};k\right\} \\ &= \tilde{\Psi}(s) + \hat{\tilde{p}}(k,s)\hat{\lambda}(k)\tilde{\psi}(s) \\ &= \left[1 - \tilde{\psi}(s)\right]/s + \hat{\tilde{p}}(k,s)\hat{\lambda}(k)\tilde{\psi}(s) \end{aligned} \tag{27}$$

where

$$\mathcal{L}_t\{p(x,t);s\} = \tilde{p}(x,s) = \int_0^\infty e^{-st}p(x,t)dt, \tag{28}$$

$$\mathcal{F}_x\{p(x,t);k\} = \hat{p}(k,t) = \int_{-\infty}^\infty e^{ikx}p(x,t)dx \tag{29}$$

denote the Laplace transform with respect to (wrt) time variable t and the Fourier transform wrt position variable x, respectively. After a simple manipulation, we obtain

$$\hat{\tilde{p}}(k,s) = \frac{1-\tilde{\psi}(s)}{s}\frac{1}{1-\hat{\lambda}(k)\tilde{\psi}(s)}. \tag{30}$$

According to basic theory of CTRW, we recall two following Lemmas:

Lemma 1. *If the PDF $\lambda(x)$ satisfies symmetric property $\int_{-\infty}^{-a}\lambda(x)dx = \int_a^\infty \lambda(x)dx$ for $a \geq 0$, then in Fourier space we have the asymptotic*

$$1 - \hat{\lambda}(k) \approx 0.5\langle\Delta x^2\rangle k^2 \text{ for } k \to 0. \tag{31}$$

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Lemma 2. *In Laplace space we have the asymptotic*

$$1-\tilde{\psi}(s) \approx \langle \Delta t \rangle s \text{ for } 0 < s \to 0. \tag{32}$$

Let us make smaller all random jumps ΔX_i by a factor $\rho > 0$, all random waiting times ΔT_i by a factor $\varrho > 0$. Consequently, we get a *scaled* CTRW that have $x_{\rho,n} = \rho x_n$ and $t_{\varrho,n} = \varrho t_n$. Thus, the PDF $\lambda_\rho(x)$ of the scaled jumps $\rho \Delta X_i$ and the PDF $\psi_\varrho(t)$ of the scaled waiting times $\varrho \Delta T_i$ are, respectively, given by

$$\lambda_\rho(x) = \rho^{-1}\lambda(x/\rho) \tag{33}$$
$$\psi_\varrho(t) = \varrho^{-1}\psi(t/\varrho) \tag{34}$$

Using the scaling relations in (33)-(34) and the approximations in the Lemmas, we approximate the equation (30) to

$$\hat{\tilde{p}}(k,s) \approx \left(s + \frac{\mu\rho^2}{\nu\varrho}k^2\right)^{-1} \tag{35}$$

as $\rho \to 0$ and $\varrho \to 0$.

2) The connection between CTRW and DE: Laplace transform for $\partial p(x,t)/\partial t$ is given by

$$\mathcal{L}_t\left\{\frac{\partial p(x,t)}{\partial t}; s\right\} = s\tilde{p}(x,s) \tag{36}$$

while Fourier transform for $\partial^2 f(x)/\partial x^2$ is given by

$$\mathcal{F}_x\left\{\frac{\partial^2 f(x)}{\partial x^2}; k\right\} = -k^2\hat{p}(k,t). \tag{37}$$

Thus, respectively applying Laplace transform wrt time variable t and Fourier transform wrt position variable x to the equation (1), we obtain

$$\hat{\tilde{p}}(k,s) = (s + Dk^2)^{-1}. \tag{38}$$

Comparing eq. (35) with eq. (38), we deduce that FDE is the limit of CTRW under the scaling relation $D = \left(\mu\rho^2\right)/\left(\nu\varrho\right)$. Consequently, the unit of diffusion constant K is $length^2/time$.

B. Diffusion equation with natural conditions

Considering natural conditions of a diffusion process

$$\begin{cases} p(\pm\infty, t) = 0 \\ p(x,0) = \delta(x) \end{cases}, \tag{39}$$

we formulate the solution $p_\bullet(x,t)$ of the DE (1) as follows:

Firstly, we rewrite eq. (38) as

$$\hat{\tilde{p}}_\bullet(k,s) = \frac{1}{s+Dk^2} \overset{(a)}{=} s^{-1}\sum_{l=0}^{\infty}\left(\frac{-Dk^2}{s}\right)^l \tag{40}$$

where the equality (a) is obtained from Taylor expansion around $Dk^2/s = 0$. Secondly, we apply the inverse Laplace transform to the above equation and obtain

$$\begin{aligned}\hat{p}_\bullet(k,t) &= \sum_{l=0}^{\infty}(-Dk^2)^l\mathcal{L}_s^{-1}\left\{s^{-(l+1)}; t\right\} \\ &\overset{(b)}{=} \sum_{l=0}^{\infty}\frac{(-Dk^2t)^l}{\Gamma(l+1)} \overset{(c)}{=} e^{-Dk^2t}.\end{aligned} \tag{41}$$

The equality (b) follows from that $\mathcal{L}_s^{-1}\{s^{-z};t\} = t^{z-1}/\Gamma(z)$; the equality (c) is the use of the definition of the Mittag-Leffler function. Finally, we apply the inverse Fourier transform to the above equation and obtain

$$p_\bullet(x,t) = \mathcal{F}_k^{-1}\left\{e^{-Dk^2t}; x\right\} = \frac{1}{\sqrt{4\pi Dt}}e^{-x^2/(4Dt)}. \tag{42}$$

C. Diffusion equation with absorbing boundary conditions

Given $p_\bullet(x,t)$ in (42), the solution $p_\star(x,t)$ can be found out through a so-called *image method* [2] [13]. For our problem, the region of diffusion is just half space $(-\infty, 0]$, thus the resulting probability $p_\star(x,t)$ will be the superposition of two component solutions (one caused by particle, the other caused by image antiparticle):

$$p_\star(x,t) = p_\bullet(x+L,t) + wp_\bullet(x-L,t) \tag{43}$$

where w is the weight. By substituting (43) into the conditions in (3), we find out $w = -1$. Finally, the probability $p_\star(x,t)$ in (43) is rewritten as

$$p_\star(x,t) = p_\bullet(x+L,t) - p_\bullet(x-L,t). \tag{44}$$

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[2] Image method: We consider a *real particle* initially at $x_0 = -L < 0$ and an *image antiparticle* initially at $-x_0 = L$. Both of them diffuse freely on $(-\infty, +\infty)$. According to Appendix B, the probabilities of real particle and image antiparticle are, respectively, $p_\bullet(x - x_0, t) = p_\bullet(x + L, t)$ and $p_\bullet(x + x_0, t) = p_\bullet(x - L, t)$.

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