Modelling the Impact of International Travellers on the Trend of HIV Epidemic

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Abstract - In this paper, we propose mathematical model to study the impact of international travellers (tourists) on the spread of HIV and AIDS epidemic in compartmental differential equation models. Tourism is international travellers who stay outside their country for a period of days. Simulation studies of reproduction number are used to demonstrate new insights on the spread of HIV disease. The results indicate that with the constant flow of tourism into a country, the disease status remain epidemic whiles without tourism the status changes from epidemic to endemic. If the direct flow of tourism into the population is restricted, the persistent spread of the disease can be minimised.

Index Terms - HIV/AIDS, SIA model, Tourism, Stability, Simulation.

I. INTRODUCTION

Tourism has become the most universally accepted, supported and lucrative tool for big business. It is a successful business operation, generating considerable revenue and publicity for many countries worldwide. World Tourism Organization (UNWTO) defines tourism as the activities of persons travelling to and staying in places outside their usual environment for not more than one consecutive year for leisure, business and other purposes [1]. It is estimated that 50% of travellers engage in sexual practices while abroad [2]. For example, in 2004, Cable News Network (CNN) news agency reported that 16,000 to 20,000 people are estimated be child sex victims in Mexico, largely in border, urban, and tourist areas [3]. In United Kingdom a study was conducted involving 258 heterosexual travellers who attended the Hospital for Tropical Diseases in London. The rate at which foreigner acquired HIV infection was 33.2% as compared to the rate of 1.8% of locally acquired HIV infection [4]. Many travellers travel for affordability of sexual services to their travelling destination [5] and sex partners [6] who travelled to Thailand for sex tourism [7, 8]. For instance, in Europe, they have been an increase of new AIDS cases among immigrants [9]. This is no different from Asia, India and Africa, migrant labourers, long-distance truck drivers, and commercial sex workers HIV epidemics travel to other countries for green pasture. Specifically, travellers may be controlled or restricted in one way or other due to the movement of people coming overseas may not justified.

II. MODEL DESCRIPTION AND ANALYSIS

The model describes three components of comprising the number of susceptible \( S(t) \), infected (HIV) \( I(t) \), and AIDS cases who are showing AIDS symptoms \( A(t) \) at time \( t \). It is assumed that, at any time, there are new recruits of international tourists, which means the movement from one country to another \( T_s \), \( T_i \) and \( T_a \) who are sexually active enter the susceptible, HIV and AIDS respectively. It is assumed that new recruits of birth rate \( \omega b \) who enter the HIV class. Proportions \( \omega \) of these individuals are assumed to be infected of HIV (categorised in the \( I \) class) and the complementary proportion \((1 - \omega)b\), are susceptible (and move to the susceptible class \( S \)). The natural death rate is assumed to be the same proportion to the number in each class, \( \mu > 0 \). AIDS patients have an additional disease-induced mortality rate, \( \varepsilon > 0 \). We assumed that \( c_s \) is the rate at which an individual acquires new sexual partners, where subscript \( s \) represents with tourism and without tourism respectively. We further assumed that \( c_i \) will be 0.5% of tourists individuals. We also assumed that \( \delta \) is a fraction of tourists who left the population and \( \rho \) is period of stay the individual is permitted to stay in the country (i.e. stay/ max. period of stays allowed).

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The probability of transmission from an individual in category $S(t)$ to infected HIV individuals in category $I(t)$ is $\beta$. The description of the model is shown in Figure 1.

**Figure 1:** Flow of tourism on HIV/AIDS model

Based on the above assumptions and the description of the model with addition of demographics (birth and death), it consists of three nonlinear ordinary equations:

\[
\frac{dS}{dt} = (1 - \omega)bS + T_s - \beta kS - (\mu + \rho\delta)S \quad (1)
\]

\[
\frac{dI}{dt} = T_s + abI + \beta kS - (\mu + \sigma + \rho\delta)I \quad (2)
\]

\[
\frac{dA}{dt} = T_a + abA - (\mu + \epsilon + \rho\delta)A \quad (3)
\]

where $\epsilon = 0.5\%$.

These equations express the rate of change of the components $\left( \frac{d}{dt} \right)$ as a sum of the sources minus the sinks.

From (1)-(3), the total population is given by $N = S + I + A$, where $T = T_s + T_a + T_s$.

We substitute $N$ and $I$ into (4) and (5).

\[
\frac{dN}{dt} = (1 - \omega)bN + 2abI - (1 - \omega)bA + T_s - \mu N - \rho\delta N - \sigma I - \epsilon A \quad (4)
\]

\[
\frac{dI}{dt} = T_s + abI + \beta k(N - I - A) - (\mu + \sigma + \rho\delta)I \quad (5)
\]

\[
\frac{dA}{dt} = T_a + abA - (\mu + \epsilon + \rho\delta)A \quad (6)
\]

\[N(0) = N_0 > 0, \quad I(0) = I_0 \geq 0, \quad A(0) = A_0 \geq 0.\]

The derivatives of (4)-(6) imply that the model is well posed for $N > 0$. We assumed that all the dependant variables and parameters of the model are non-negative. In order to solve these equations, we now analyse models (4)-(6) using the stability theory of differential equations.

**III. EQUILIBRIUM AND STABILITY ANALYSIS**

In this section, we analyse the stability of the equilibrium points of the models (4)-(6).

**A. Equilibrium of the model**

The model does not show a disease free equilibrium due to the direct inflow of tourism into all the three population compartment levels. There would be endemic equilibrium $E^* = (N^*, I^*, A^*)$.

\[ (1 - \omega)b - (\mu + \rho\delta)N^* + (2ab - \beta k)I^* - (1 - \omega)b - \epsilon)A^* + T_s = 0 \quad (7) \]

\[ T_s + abI + \beta k(N^* - I^* - A^*) - (\mu + \sigma + \rho\delta)I^* = 0 \quad (8) \]

\[ T_a + abA - (\mu + \epsilon + \rho\delta)A^* = 0 \quad (9) \]

By solving (7)-(9) simultaneously we get,

\[ I^* = \frac{1}{(1 - \omega)b - (\mu + \rho\delta)}(T_s + \epsilon I^*) \quad (10) \]

\[ A^* = \frac{1}{(1 - \omega)b - (\mu + \rho\delta)}(T_a + \epsilon A^*) \quad (11) \]

which are all positive when

\[ N < \frac{(1 - \omega)b - (\mu + \rho\delta)}{(1 - \omega)b - (\mu + \rho\delta)} \]

**B. Equilibrium of the model without tourism**

To look for steady states we solve the system (1)-(3) by substituting $T_s = 0$.

\[ (1 - \omega)bS - \beta kIS - \mu S = 0 \quad (12) \]

\[ abI + \beta kI - (\mu + \sigma + \sigma)I = 0 \quad (13) \]

\[ ab - (\mu + \epsilon)A = 0 \quad (14) \]

where $\epsilon = 1 - 0.5\%$.

From (13) and (14) we get either respectively,

\[ I = 0 \quad \text{or} \quad S = \frac{(\mu + \sigma + \sigma) - ab}{\beta c} \quad (15) \]

and

\[ A = \frac{\alpha d}{\mu + \epsilon} \quad (16) \]

At disease free equilibrium $E_0 = (N, 0, 0)$ with absent of disease when $I = A = 0$. To find the endemic equilibrium, we substitute (15) and (16) into (17)

\[ N, I, A, - N \quad (17) \]

Thus,

\[ I_s = \frac{(\mu + \epsilon)\beta}{\beta k} \left[ \beta k N - (\mu + \epsilon + \sigma) + ab \right] \quad (18) \]

\[ A_s = \frac{\alpha (\mu + \epsilon) \beta}{\beta k} \left[ \beta k N - (\mu + \epsilon + \sigma) + ab \right] \quad (19) \]

This leads to a unique endemic equilibrium given by

\[ E_s = \frac{\beta k}{\beta k} \left[ \beta k N - (\mu + \epsilon + \sigma) + ab \right] \quad (20) \]

**IV. LOCAL STABILITY AND EQUILIBRIUM POINTS**

To determine whether the disease continues to spread, we need to find the stability of the disease free equilibrium point. We have already observed that the disease cannot persist if $R_0 < 0$. The condition $R_0 < 0$ suffices the local as well as the global stability of the disease free equilibrium. Thus the disease will not continue to spread.
if $R_0 < 0$. Now, to determine whether the disease will continue to spread, we need to study the dynamics for $R_0 > 1$. If the disease persists in the system ($R_0 > 1$), then either the system will be stable around the interior equilibrium or there may exist periodic attractor. Moreover, in order to determine all these possibilities, we will use the above conditions to find various reproduction numbers in their various scenarios. For all time $t > 0$, all the parameters in the model system (4)-(6) are assumed to be non-negative. The model has a disease-free equilibrium $E_0 = (N, 0, 0)$, where there is no disease at the initial stages, this implies that $N = \frac{T_i}{\mu + \rho \delta - (1 - \omega) \mu}$, when $I(0) = I_0 \geq 0$ and $A(0) = A_0 \geq 0$.

Case 1: Without tourism

In this scenario, we find the reproduction number for the entire model. We calculate the basic reproduction number $R_0$ using the approaches explained in details, e.g., in [11, 12]. Using the notations by van den Driessche and Watmough [13], we let $F_i(x)$ be the rate of appearance of new infections in compartment $i$, and let $V_j(x)$ represents the rate of infections from one compartment to another, for that we write the right-hand side of system (4)-(6) as $F - V$ with

$$F = \begin{pmatrix} 0 \\ \rho \delta N - \beta F N - A \end{pmatrix}$$

$$V = \begin{pmatrix} b_{NN} + \rho \delta N - (1 - \omega) b_{N} + b_{I} - 2ab_{I} + c_{I} + (1 - \omega) b_{A} + \rho \delta A \\ -a_{I} + \rho \delta A - b_{I} + \mu + \alpha + \rho \delta + \rho \delta A + \rho \delta \sigma + \rho \delta \alpha \\ 0 + \rho \delta \sigma + \rho \delta \alpha - \alpha - \rho \delta \sigma \end{pmatrix}$$

Then we consider the Jacobian matrices associated with $F$ and $V$:

$$J_F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta N & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$J_V = \begin{pmatrix} \mu + \rho \delta & b - 2ab + \sigma & (1 - \omega) b + \omega \\ -b + \rho \delta & \mu + \alpha + \rho \delta & 0 \\ 0 & 0 & \mu + \varepsilon + \rho \delta \end{pmatrix}$$

The basic reproduction number (4)-(6) is obtained as the spectral radius of the matrix $J_F \times J_V^{-1}$ at the disease-free equilibrium is

$$R_0 = \frac{\beta N}{\mu + \alpha + \sigma + \rho \delta - \omega b}$$

Case 2: With tourism

To find the reproduction number of the entire model (tourism) we substitute (25) into (24). This implies that

$$N = \frac{I_t}{\mu + \rho \delta - (1 - \omega) \mu}$$

and then

$$R_w = \frac{\beta c_{T_r}}{(\mu + \alpha + \sigma - \omega b)(\mu + \rho \delta + \omega b - b)}$$

V. EXPERIMENTAL AND RESULTS AND ANALYSIS

The first reported cases of HIV/AIDS in Malaysia were in 1986. Between 1986 and 2010 91,362 men, women and children have been notified infected with HIV while 12,943 have died of AIDS[14]. The majority of these HIV infected are found in adults aged 15 to 49. In 2010, 40% of new reported HIV cases were attributed to heterosexual intercourse, a dramatic increase from 27% in 2009. In the same year, there were 3652 and 904 new HIV infections and AIDS related deaths respectively in an average of 10 days daily[14], which corresponds to 95,014 individuals for the starting infected $I(0)$ compartment (HIV cases) and 13,847 individuals for the starting AIDS cases $A(0)$ compartment. In 2010, Malaysian population was 28,588,600 individuals[15]. Thus, there are 28,479,739 million foreign migrants who were permanent residents in Malaysia[16]. Each year there is an average increase of 1.5 million individuals who enter Malaysia. In 2010 year there was total number of 24,577,196 tourists in the country[17]. Based on the above information, the following sets of initial conditions are established, as presented in the Table 1. We solve the set of differential equations using the deSolve package in R-project.

<table>
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<tr>
<th>Parameters</th>
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<th>Sources</th>
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<tr>
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<td>$\rho$</td>
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<td>------</td>
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<tr>
<td>$\varepsilon$</td>
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</table>

The results of a numerical simulation based on literature sources and assumptions are shown in Figures 2 to 4. Figure 2 shows the distribution of susceptible, HIV and AIDS population without time during the flow of external tourism into the susceptible, HIV and AIDS stage as well as without tourism.
Figure 2: The dynamics of susceptible population

There was no constant flow of tourism into the susceptible population, the new born cases of infected individuals entered to the HIV population (dash line). This implies that more people are infected due direct inflow of tourism as compared to without tourism. For example, without tourism, there are 28.66324, 28.78804 and 29.04307 million people at 2013, 2015 and 2019 years, respectively. The disease still continues to remain endemic in the population at a high prevalence because the infected individuals continue to infect more susceptible individuals at $I_v = 0.6253$ million people and $A_v = 0.0914$ million people. In case of (red line) for instance, with tourism, with tourism, there are 8.022597, 7.224243 and 7.223329 million people at 2011, 2013 and 2019 years, respectively. The disease still continues to remain epidemic in the population at a high prevalence because the infected individuals continue to infect more susceptible individuals at $I_v = 23.616$ million people and $A_v = 7.388$ million people as compared to that of without tourism. Thus, the number of susceptible individuals decreases significantly as the prevalence continues to increase at constant number of susceptible people.

Figure 3: The dynamics of HIV population

As shown in Figure 3, without constant flow of tourism (as shown in blue line), at 2015, 2017 and 2019 years, the HIV population is 76141, 6979 and 6402 thousand people, respectively. Thus, the HIV cases will obtain it maximum equilibrium levels and it will start to decrease. With tourism flow, at 2013, 2015 and 2019 years, the HIV population is expected to increase to 783135, 783123, and 783120 thousand people, respectively. To minimise the spread of HIV, the movement rate of $\alpha$ must be increased to reduce the number of infected individuals who will move into the AIDS population. Thus, the movement rate of $\alpha$ is the major contributing factor of the spread of the disease.

Figure 4: The dynamics of AIDS population

As shown in Figure 4 there is slight increase from 2015 years to 2019 years, followed by a sharp increase to 2020 years with all the four different cases, with the number of AIDS patients of 7461, 9481, 11298 thousand people at years 2015, 2017, and 2019, respectively (case 1 with tourism and migration). In case 2 with tourism the number of AIDS patients of 767189 and 803884 thousand people at years 2015 and 2019 respectively. Finally in case 4 without tourism the number of AIDS patients of 74608, 9481 and 11298 thousand people at years 2015, 2017, and 2019, respectively which depicts that there is increase in AIDS case to HIV cases in (case 2 without tourism). To minimise the spread of AIDS, the movement rate of $\alpha$ must be increased to reduce the number of infected individuals who will transfer into the AIDS population. Thus, the movement rate of $\alpha$ and $\epsilon$ are the major contributing factor of the increase of the AIDS population.

VI. CONCLUSION

The main objective of this study was to formulate and analyse a deterministic mathematical model of how the impact of international travellers (tourism) has on the trend of HIV epidemic with human population via a system of nonlinear ordinary differential equations. Analytical results show that when, if $R_0 > 1$, then an epidemic of AIDS occurs, and if $R_0 < 1$, then the disease becomes endemic. From the analysis of the model, the reproduction number is used to determine whether the disease is instability of the infection-free steady state or stability state. We found that $R_0 = 0.1875$ for the case where there is a constant flow
of tourists, which indicates the HIV disease has become endemic whereas in case 2 the HIV disease was at the stability states with \( R_0 = 0.0239 \) indicating that the HIV disease has become endemic without tourism in the country. There are high increases of HIV disease in case 1. This shows that the constant flow of tourism into the country has much impact of spread of HIV disease in the country which makes it epidemic. The simulation results indicated that by controlling especially the constant of tourist into the population stages (Figures 3 and 4), the spread of the disease can be minimised significantly. Notwithstanding, the current analysis enables us to gain valuable insights and remains absolute an important step in theoretically analysing the HIV disease. The HIV cases would obtain it maximum equilibrium levels from 2015 to 2017 that will assist policy makers to put preventive measures in place to minimise the spread in case 2 (as shown with red line in Figure 3). To reduce the spread of the disease, the flow of tourism can be restricted. Further study is necessary to obtain realistic parameter values. Having those challenges in mind, these results require further investigation before being used as a guide for early treatment of infected individuals, but the results do suggest an emphasis on prevention policies that reduce tourism policy.

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