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New Jacobi Elliptic Function Solutions for Coupled KdV-mKdV Equation

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***Abstract*—A generalized $(G^{'}/G)$-expansion method is used to search for the exact traveling wave solutions of the coupled KdV-mKdV equation. As a result, some new Jacobi elliptic function solutions are obtained. It is shown that the method is straightforward, concise, effective, and can be used for many other nonlinear evolution equations in mathematical physics.**

***Index Terms*—generalized $(G^{'}/G)$-expansion method; the coupled KdV-mKdV equation; Jacobi elliptic function solutions**

I. Introduction

Seeking the traveling wave solutions of nonlinear evolution equations(NLEEs) has been an interesting and hot topic in mathematics physics for a long time. Many effective methods to construct traveling wave solutions of NLEEs have been established [1-11]. However, no method can be used for finding all solutions for all types of NLEEs. Recently, the $(G^{'}/G)$-expansion method [12] has become popular in the research community, and the initial idea has been refined by many studies [13-17]. It is shown that the $(G^{'}/G)$-expansion method is very effective, and many nonlinear equations have been successfully solved.

In this paper, some exact solutions of the coupled KdV-mKdV equation which are expressed by the Jacobi elliptic function are obtained by using the $(G^{'}/G)$-expansion method.

A. Description of the generalized $(G^{'}/G)$-expansion method

Assume that the nonlinear partial differential equation

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \tag{1}$$

where F is a polynomial in its arguments. The main steps of the generalized $(G^{'}/G)$-expansion method are descripted as follows.

Step 1. Seeking traveling wave solutions of (1) by taking $u(x,t) = u(\xi)$, $\xi = x - ct$, and transforming (1) to the ordinary differential equation(ODE)

$$F(u, u', -cu', c^2u'', -cu'', u'', \cdots) = 0. \tag{2}$$

Step 2.Looking for its solution $(u(\xi))$ in the polynomial form

$$u(\xi) = a_0 + \sum_{i=1}^{m} a_i \left(\frac{f'}{f}\right)^i, \tag{3}$$

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where $a_0, a_i (i = 1, 2, \cdots, m)$ are constants which will be determined later, $f = f(\xi)$ is the solution of the auxiliary LODE

$$f^{'2} = Pf^4 + Qf^2 + R, \tag{4}$$

where P, Q and R are constants.

Step 3. Determining the parameter m by balancing the highest order nonlinear term and the highest order partial derivative of u in (2).

Step 4. Substituting (3) and (4) into (2), and setting all the coefficients of all terms with the same powers of $(f^{'}/f)^k (k = 1, 2, \cdots)$ to zero. Then a system of nonlinear algebraic equations (NAEs) with respect to the parameters $c, a_0, a_i (i = 1, 2, \cdots, m)$ is obtained. By solving the NAEs if available, those parameters can be determined explicitly.

Step 5. Assuming that the constants $c, a_0, a_i (i = 1, 2, \cdots, m)$ can be obtained by solving the algebraic equations in Step 4, and substituting these constants and the known general solutions into (3). Then the explicit solutions of (1) can be obtained immediately.

B. Applications of method

In this section, we apply the $(G^{'}/G)$-expansion method to seek the exact solutions of the coupled KdV-mKdV equation [18,19] as follows:

$$u_t + \alpha u u_x + \beta u^2 u_x + u_{xxx} = 0, \tag{5}$$

where α and β are two constant parameters. Let $u(x,t) = u(\xi), \xi = x - ct$ in (5). Then

$$-cu^{'} + \alpha u u^{'} + \beta u^2 u^{'} + u^{'''} = 0, \tag{6}$$

where c is a constant which will be determined later. By integrating both sides of (6) with respect to ξ ,

$$-cu + \frac{\alpha}{2}u^2 + \frac{\beta}{3}u^3 + u^{''} = 0. \tag{7}$$

Then $m = 1$ by balancing u^3 and $u^{''}$ in (7). According to (3) and (4) we have

$$u(\xi) = a_0 + a_1 \left(\frac{f^{'}}{f}\right), \tag{8}$$

$$u^{''}(\xi) = 2a_1 \left(\frac{f^{'}}{f}\right)\left(\left(\frac{f^{'}}{f}\right)^2 - Q\right). \tag{9}$$

With the aid of Maple, substituting (8) and (9) into (7), the left-hand side of (7) becomes a polynomial in $(f^{'}/f)$ and ξ. Setting their coefficients to zero yields a system of algebraic equations in a_0, a_1, c, β. Solving these over-determined algebraic equations, we get the following result:

$$a_0 = \frac{12Q}{\alpha}, a_1 = \frac{12\sqrt{Q}}{\alpha}, c = 4Q, \beta = -\frac{\alpha^2}{24Q}. \quad (10)$$

With the aid of the appendix [20] and from the formal solution (10), we get the following set of exact solutions of (5).

Case 1. Choosing $P = m^2, Q = -(1+m^2), R = 1$, and $f(\xi) = sn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_1 &= -\tfrac{12}{\alpha}\left((1+m^2) - \sqrt{-1-m^2}cs(\xi)dn(\xi)\right), \\ \xi &= x + 4(1+m^2)t. \end{aligned} \quad (11)$$

Case 2. Choosing $P = m^2, Q = -(1+m^2), R = 1$, and $f(\xi) = cd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_2 = &\tfrac{12}{\alpha}\left((1-m^2)\sqrt{-1-m^2}sd(\xi)nc(\xi)\right) \\ &-\tfrac{12}{\alpha}(1+m^2), \\ \xi = &x + 4(1+m^2)t. \end{aligned} \quad (12)$$

Case 3. Choosing $P = -m^2, Q = 2m^2 - 1, R = 1 - m^2$, and $f(\xi) = cn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_3 &= \tfrac{12}{\alpha}\left((2m^2-1) - \sqrt{2m^2-1}dc(\xi)sn(\xi)\right), \\ \xi &= x - 4(2m^2-1)t. \end{aligned} \quad (13)$$

Case 4. Choosing $P = -1, Q = 2 - m^2, R = m^2 - 1$, and $f(\xi) = dn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_4 &= \tfrac{12}{\alpha}\left((2-m^2) - m^2\sqrt{2-m^2}cd(\xi)sn(\xi)\right), \\ \xi &= x - 4(2-m^2)t. \end{aligned} \quad (14)$$

Case 5. Choosing $P = 1, Q = -(1+m^2), R = m^2$, and $f(\xi) = ns(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_5 &= -\tfrac{12}{\alpha}\left((1+m^2) + \sqrt{-1-m^2}cs(\xi)dn(\xi)\right), \\ \xi &= x + 4(1+m^2)t. \end{aligned} \quad (15)$$

Case 6. Choosing $P = 1, Q = -(1+m^2), R = m^2$, and $f(\xi) = dc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_6 = &\tfrac{12}{\alpha}\left((1-m^2)\sqrt{-1-m^2}sc(\xi)nd(\xi)\right) \\ &-\tfrac{12}{\alpha}(1+m^2), \\ \xi = &x + 4(1+m^2)t. \end{aligned} \quad (16)$$

Case 7. Choosing $P = 1 - m^2, Q = 2m^2 - 1, R = -m^2$, and $f(\xi) = nc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_7 &= \tfrac{12}{\alpha}\left((2m^2-1) + \sqrt{2m^2-1}dc(\xi)sn(\xi)\right), \\ \xi &= x - 4(2m^2-1)t. \end{aligned} \quad (17)$$

Case 8. Choosing $P = m^2 - 1, Q = 2 - m^2, R = -1$, and $f(\xi) = nd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_8 &= \tfrac{12}{\alpha}\left((2-m^2) + m^2\sqrt{2-m^2}cd(\xi)sn(\xi)\right), \\ \xi &= x - 4(2-m^2)t. \end{aligned} \quad (18)$$

Case 9. Choosing $P = 1 - m^2, Q = 2 - m^2, R = 1$, and $f(\xi) = sc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_9 &= \tfrac{12}{\alpha}\left((2-m^2) + \sqrt{2-m^2}dc(\xi)ns(\xi)\right), \\ \xi &= x - 4(2-m^2)t. \end{aligned} \quad (19)$$

Case 10. Choosing $P = -m^2(1-m^2), Q = 2m^2-1, R = 1$, and $f(\xi) = sd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{10} &= \tfrac{12}{\alpha}\left((2m^2-1) + \sqrt{2m^2-1}cd(\xi)ns(\xi)\right), \\ \xi &= x - 4(2m^2-1)t. \end{aligned} \quad (20)$$

Case 11. Choosing $P = 1, Q = 2 - m^2, R = 1 - m^2$, and $f(\xi) = cs(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{11} &= \tfrac{12}{\alpha}\left((2-m^2) + \sqrt{2-m^2}ds(\xi)nc(\xi)\right), \\ \xi &= x - 4(2-m^2)t. \end{aligned} \quad (21)$$

Case 12. Choosing $P = 1, Q = 2m^2-1, R = -m^2(1-m^2)$, and $f(\xi) = ds(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{12} &= \tfrac{12}{\alpha}\left((2m^2-1) - \sqrt{2m^2-1}cs(\xi)nd(\xi)\right), \\ \xi &= x - 4(2m^2-1)t. \end{aligned} \quad (22)$$

Case 13. Choosing $P = 1/4, Q = (1-2m^2)/2, R = 1/4$, and $f(\xi) = ns(\xi) \pm cs(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{13} &= \tfrac{12}{\alpha}\left(\left(\tfrac{1}{2} - m^2\right) - \sqrt{\tfrac{1}{2} - m^2}ds(\xi)\right), \\ \xi &= x - 2(1-2m^2)t. \end{aligned} \quad (23)$$

Case 14. Choosing $P = (1-m^2)/4, Q = (1+m^2)/2, R = (1-m^2)/4$, and $f(\xi) = nc(\xi) \pm sc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{14} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}\left(1+m^2\right) + \sqrt{\tfrac{1}{2}\left(1+m^2\right)}dc(\xi)\right), \\ \xi &= x - 2(1+m^2)t. \end{aligned} \quad (24)$$

Case 15. Choosing $P = 1/4, Q = (m^2-2)/2, R = m^2/4$, and $f(\xi) = ns(\xi) \pm ds(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{15} &= \tfrac{12}{\alpha}\left(\left(\tfrac{1}{2}m^2 - 1\right) - \sqrt{\tfrac{1}{2}m^2 - 1}cs(\xi)\right), \\ \xi &= x - 2(m^2-2)t. \end{aligned} \quad (25)$$

Case 16. Choosing $P = m^2/4, Q = (m^2-2)/2, R = m^2/4$, and $f(\xi) = sn(\xi) \pm icn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{16} = &\tfrac{12}{\alpha}\left(\sqrt{\tfrac{1}{2}m^2 - 1}\tfrac{dn(\xi)(cn(\xi)-isn(\xi))}{sn(\xi)+icn(\xi)}\right) \\ &+\tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2 - 1\right), \\ \xi = &x - 2(m^2-2)t. \end{aligned} \quad (26)$$

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Case 17. Choosing $P = m^2/4, Q = (m^2-2)/2, R = m^2/4$, and $f(\xi) = \sqrt{m^2-1}sd(\xi) \pm cd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{17} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2-1\right) + \tfrac{12}{\alpha}\sqrt{\tfrac{1}{2}m^2-1} \\ &\cdot\left(\tfrac{\sqrt{m^2-1}cn(\xi)+\left(m^2-1\right)sn(\xi)}{dn(\xi)\left(\sqrt{m^2-1}sn(\xi)+cn(\xi)\right)}\right), \\ \xi &= x-2(m^2-2)t. \end{aligned} \tag{27}$$

Case 18. Choosing $P = 1/4, Q = (1-2m^2)/2, R = 1/4$, and $f(\xi) = mcd(\xi) \pm i\sqrt{1-m^2}nd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{18} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}-m^2\right) + \tfrac{12}{\alpha}\sqrt{\tfrac{1}{2}-m^2} \\ &\cdot\left(\tfrac{msn(\xi)\left(-1+m^2+i\sqrt{1-m^2}mcn(\xi)\right)}{dn(\xi)\left(mcn(\xi)+i\sqrt{1-m^2}\right)}\right), \\ \xi &= x-2(1-2m^2)t. \end{aligned} \tag{28}$$

Case 19. Choosing $P = 1/4, Q = (1-2m^2)/2, R = 1/4$, and $f(\xi) = msn(\xi) \pm idn(\xi)nd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{19} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}-m^2\right) + \tfrac{12}{\alpha}\sqrt{\tfrac{1}{2}-m^2} \\ &\cdot\left(\tfrac{mcn(\xi)(dn(\xi)-imsn(\xi))}{msn(\xi)+idn(\xi)}\right), \\ \xi &= x-2(1-2m^2)t. \end{aligned} \tag{29}$$

Case 20. Choosing $P = 1/4, Q = (1-2m^2)/2, R = 1/4$, and $f(\xi) = \sqrt{m^2-1}sc(\xi) \pm idc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{20} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}-m^2\right) + \tfrac{12}{\alpha}\sqrt{\tfrac{1}{2}-m^2} \\ &\cdot\left(\tfrac{\sqrt{m^2-1}dn(\xi)-isn(\xi)\left(m^2-1\right)}{cn(\xi)\left(\sqrt{m^2-1}sn(\xi)+idn(\xi)\right)}\right), \\ \xi &= x-2(1-2m^2)t. \end{aligned} \tag{30}$$

Case 21. Choosing $P = (m^2-1)/4, Q = (m^2+1)/2, R = (m^2-1)/4$, and $f(\xi) = msd(\xi) \pm nd(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{21} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}\left(1+m^2\right) + m\sqrt{\tfrac{1}{2}\left(1+m^2\right)}cd(\xi)\right), \\ \xi &= x-2(1+m^2)t. \end{aligned} \tag{31}$$

Case 22. Choosing $P = m^2/4, Q = (m^2-2)/2, R = 1/4$, and $f(\xi) = sn(\xi)/(1 \pm dn(\xi))$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{22} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2-1+\sqrt{\tfrac{1}{2}m^2-1}cs(\xi)\right), \\ \xi &= x-2(m^2-2)t. \end{aligned} \tag{32}$$

Case 23. Choosing $P = -1/4, Q = (m^2+1)/2, R = \left(1-m^2\right)^2/4$, and $f(\xi) = mcn(\xi) \pm dn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{23} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}(m^2+1) - m\sqrt{\tfrac{1}{2}(m^2+1)}sn(\xi)\right), \\ \xi &= x-2(m^2+1)t. \end{aligned} \tag{33}$$

Case 24. Choosing $P = \left(1-m^2\right)^2/4, Q = (m^2+1)/2, R = 1/4$, and $f(\xi) = ds(\xi) \pm cs(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{24} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}(m^2+1) - \sqrt{\tfrac{1}{2}(m^2+1)}ns(\xi)\right), \\ \xi &= x-2(m^2+1)t. \end{aligned} \tag{34}$$

Case 25. Choosing $P = 1/4, Q = (m^2-2)/2, R = m^2/4$, and $f(\xi) = dc(\xi) \pm \sqrt{1-m^2}nc(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{25} &= \tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2-1\right) + \tfrac{12}{\alpha}\sqrt{\tfrac{1}{2}m^2-1} \\ &\cdot\left(\tfrac{sn(\xi)\left(1-m^2+\sqrt{1-m^2}dn(\xi)\right)}{cn(\xi)\left(dn(\xi)+\sqrt{1-m^2}\right)}\right), \\ \xi &= x-2(m^2-2)t. \end{aligned} \tag{35}$$

Case 26. Choosing $R = m^2Q^2/\left(m^2+1\right)^2P, Q < 0, P > 0$, and $f(\xi) = \sqrt{-m^2Q/(m^2+1)P}\ sn(\sqrt{-Q/(m^2+1)}\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{26} &= \tfrac{12}{\alpha}Q + \tfrac{12}{\alpha}\sqrt{Q}\sqrt{\tfrac{-Q}{1+m^2}}cs\left(\sqrt{\tfrac{-Q}{1+m^2}}\xi\right) \\ &.dn\left(\sqrt{\tfrac{-Q}{1+m^2}}\xi\right), \\ \xi &= x-4Qt. \end{aligned} \tag{36}$$

Case 27. Choosing $R = (1-m^2)Q^2/\left(m^2-2\right)^2P, Q > 0, P < 0$, and $f(\xi) = \sqrt{-Q/(2-m^2)P}\ dn\left(\sqrt{Q/(2-m^2)}\xi\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{27} &= \tfrac{12}{\alpha}Q - \tfrac{12}{\alpha}\sqrt{Q}\sqrt{\tfrac{-Q}{m^2-2}}m^2cd\left(\sqrt{\tfrac{-Q}{m^2-2}}\xi\right) \\ &.sn\left(\sqrt{\tfrac{-Q}{m^2-2}}\xi\right), \\ \xi &= x-4Qt. \end{aligned} \tag{37}$$

Case 28. Choosing $R = m^2(m^2-1)Q^2/\left(2m^2-1\right)^2P, Q > 0, P < 0$, and $f(\xi) = \sqrt{-m^2Q/(2m^2-1)P}cn\left(\sqrt{Q/(2m^2-1)}\xi\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{28} &= \tfrac{12}{\alpha}Q - \tfrac{12}{\alpha}\sqrt{Q}\sqrt{\tfrac{Q}{2m^2-1}}dc\left(\sqrt{\tfrac{Q}{2m^2-1}}\xi\right) \\ &.sn\left(\sqrt{\tfrac{Q}{2m^2-1}}\xi\right), \\ \xi &= x-4Qt. \end{aligned} \tag{38}$$

Case 29. Choosing $P = 1, Q = 2-4m^2, R = 1$, and $f(\xi) = sn(\xi)dn(\xi)/cn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{29} &= \tfrac{12}{\alpha}\left(\tfrac{\sqrt{2-4m^2}\left(-2m^2sn^2(\xi)+m^2sn^4(\xi)+1\right)}{cn(\xi)sn(\xi)dn(\xi)}\right), \\ &+\tfrac{12}{\alpha}(2-4m^2) \\ \xi &= x-8(1-2m^2)t. \end{aligned} \tag{39}$$

Case 30. Choosing $P = m^4, Q = 2m^2-4, R = 1$, and $f(\xi) = sn(\xi)cn(\xi)/dn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{30} &= \tfrac{12}{\alpha}\left(\tfrac{\sqrt{2m^2-4}\left(m^2sn^4(\xi)-2sn^2(\xi)+1\right)}{dn(\xi)sn(\xi)cn(\xi)}\right), \\ &+\tfrac{12}{\alpha}(2m^2-4) \\ \xi &= x-8(m^2-2)t. \end{aligned} \tag{40}$$

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Case 31. Choosing $P=1, Q=2m^2+2, R=1-2m^2+m^4$, and $f(\xi)=cn(\xi)dn(\xi)/sn(\xi)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{31} &= \tfrac{12}{\alpha}\left(2(m^2+1)+\tfrac{\sqrt{2(m^2+1)}\left(m^2sn^4(\xi)-1\right)}{dn(\xi)sn(\xi)cn(\xi)}\right), \\ \xi &= x-8(m^2+1)t. \end{aligned} \quad (41)$$

Case 32. Choosing $P=A^2(m-1)^2/4, Q=(m^2+1)/2+3m, R=(m-1)^2/4A^2$, and $f(\xi)=cn(\xi)dn(\xi)/A\left(1+sn(\xi)\right)\left(1+msn(\xi)\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{32} &= \tfrac{12}{\alpha}\left(\tfrac{m^2sn^2(\xi)+msn^2(\xi)-m-1}{dn(\xi)cn(\xi)}\right) \\ &\quad \cdot\sqrt{\tfrac{1}{2}m^2+\tfrac{1}{2}+3m}+\tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2+\tfrac{1}{2}+3m\right), \\ \xi &= x-2(m^2+1+6m)t. \end{aligned} \quad (42)$$

Case 33. Choosing $P=A^2(m+1)^2/4, Q=(m^2+1)/2-3m, R=(m+1)^2/4A^2$, and $f(\xi)=cn(\xi)dn(\xi)/A\left(1+sn(\xi)\right)\left(1-msn(\xi)\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{33} &= \tfrac{12}{\alpha}\left(\tfrac{m^2sn^2(\xi)-msn^2(\xi)+m-1}{dn(\xi)cn(\xi)}\right) \\ &\quad \cdot\sqrt{\tfrac{1}{2}m^2+\tfrac{1}{2}-3m}+\tfrac{12}{\alpha}\left(\tfrac{1}{2}m^2+\tfrac{1}{2}-3m\right), \\ \xi &= x-2(m^2+1-6m)t. \end{aligned} \quad (43)$$

Case 34. Choosing $P=-4/m, Q=6m-m^2-1, R=-2m^3+m^4+m^2$, and $f(\xi)=mcn(\xi)dn(\xi)/\left(msn^2(\xi)+1\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{34} &= \tfrac{12}{\alpha}sn(\xi)\left(\tfrac{msn^2(\xi)(m+1)^2-m(m+2)-1}{dn(\xi)cn(\xi)(msn^2(\xi)+1)}\right) \\ &\quad .\sqrt{6m-m^2-1}+\tfrac{12}{\alpha}\left(6m-m^2-1\right), \\ \xi &= x-4(6m-m^2-1)t. \end{aligned} \quad (44)$$

Case 35. Choosing $P=4/m, Q=-6m-m^2-1, R=2m^3+m^4+m^2$, and $f(\xi)=mcn(\xi)dn(\xi)/\left(msn^2(\xi)-1\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{35} &= \tfrac{12}{\alpha}sn(\xi)\left(\tfrac{msn^2(\xi)(m-1)^2+m(m-2)+1}{dn(\xi)cn(\xi)(msn^2(\xi)-1)}\right) \\ &\quad .\sqrt{-6m-m^2-1}+\tfrac{12}{\alpha}\left(-6m-m^2-1\right), \\ \xi &= x+4(6m+m^2+1)t. \end{aligned} \quad (45)$$

Case 36. Choosing $P=-(m^2+2m+1)B^2, Q=2m^2+2, R=(2m-m^2-1)/B^2$, and $f(\xi)=\left(msn^2(\xi)-1\right)/B\left(msn^2(\xi)+1\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{36} &= \tfrac{12}{\alpha}\left(\tfrac{4m\sqrt{2(m^2+1)}sn(\xi)cn(\xi)dn(\xi)}{m^2sn^4(\xi)-1}\right) \\ &\quad +\tfrac{12}{\alpha}\left(2(m^2+1)\right), \\ \xi &= x-8(m^2+1)t. \end{aligned} \quad (46)$$

Case 37. Choosing $P=-(m^2-2m+1)B^2, Q=2m^2+2, R=-(2m+m^2+1)/B^2$, and $f(\xi)=\left(msn^2(\xi)+1\right)/B\left(msn^2(\xi)-1\right)$, we obtain the Jacobi elliptic function solution of (5)

$$\begin{aligned} u_{37} &= -\tfrac{12}{\alpha}\left(\tfrac{4m\sqrt{2(m^2+1)}sn(\xi)cn(\xi)dn(\xi)}{m^2sn^4(\xi)-1}\right) \\ &\quad +\tfrac{12}{\alpha}\left(2(m^2+1)\right), \\ \xi &= x-8(m^2+1)t. \end{aligned} \quad (47)$$

Remark 1. The validity of all the solutions which are obtained are verified.
Remark 2. In fact, there are more than three solutions compared to the latest related works [21] .

II. Conclusion

In this paper, some exact solutions of Jacobi elliptic function form from the coupled KdV-mKdV equation are derived. When the modulus of the Jacobi elliptic function $m\to 0$ or 1 , the corresponding solitary wave solutions and trigonometric function solutions are also obtained. It is shown that the $(G^{'}/G)$-expansion method provides a very effective and powerful tool for solving nonlinear equations in mathematical physics.

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