Macroeconomic Analysis and Parametric Control of Economies of the Customs Union Countries Based on the Single Global Multi-Country Model

Abdykappar A. Ashimov, Yuriy V. Borovskiy, Nikolay Yu. Borovskiy

***Abstract*— The paper describes a proposed global multi-country model, related to a class of computable general equilibrium models. There are given results of parametric identification and extended verification of the model and the results of macroeconomic analysis as well. There are described setting and solving of parametric control problems on evaluation of economic policy tools at the level of single countries of the economic union based on the model. It has been shown that the coordinated problem solution for estimating optimal values of the tools of countries of the regional economic union has an advantage in comparison with the corresponding specific problems.**

***Index Terms*— CGE model, economic growth, parametric control, parametric identification.**

I. Introduction

CONSTANTLY rolling waves of the global financial and economic crisis and the growing interdependence of economies show that the ability to maintain a stable growth of each individual country lies in the integration of its economic potential with other countries within the regional union. For Kazakhstan, Russia and Belarus such opportunity associated with their participation in the Customs Union since 2010.

Since the world has a large number of factors that have a huge impact on economic development, the problem of determining the optimal coordinated economic policies of the Eurasian Economic Union (EEU) countries is far from trivial. The solution of this problem requires, first of all, macroeconomic analysis and forecasting current economic processes on the basis of their adequate and science-based descriptions. It should be noted that the present state of the theory of integration is still not giving a clear positive solutions to such problems.

The solution of this problem can be obtained by using an approach of the parametric control theory based on verified global multi-country mathematical models in the class of computable general equilibrium models [1]. Application of this theory enables to solve the problems of scenario macroeconomic analysis and making optimal (in some sense) recommendations for effective economic policy in terms of exposure of economies to the internal and external effects.

This paper is devoted to solution of macroeconomic analysis problems and estimation of optimal values of economic policy tools at the level of the countries of the regional economic union taking for example three countries of the Customs Union. The mentioned estimation is made based on the elaborated global computable general equilibrium model (CGE model) and the parametric control theory [1].

Application of the proposed CGE model (hereinafter the Model) differs from existing results by the following:

- values of all its exogenous and endogenous variables (economic indicators) for the identification period reproduce corresponding statistical values, the Model's structure does not change in the forecasting period compared to one in the identification period;
- calculation of equilibrium values of endogenous variables in the nonlinear Model is made without model linearization;
- the Model describes the government sector, which incorporates an expanded representation of monetary and fiscal policies;
- the Model describes investments into fixed assets by producers and the government of both own country and other regions.
- there are proposed five approaches for the extended verification – testing the calibrated Model for the possibility of practical application.

II. Brief Description of the Model

The model describes a global economy in accordance with the following general prerequisites. World economy is presented by functioning of interacting economies of the following seven Regions (countries), numerated by index i: 1 – Kazakhstan, 2 – Russia, 3 – Belarus, 4 – Armenia, 5 – Kyrgyzstan, 6 – European Union (as one country), 7 – the Rest of the World (as one country). The first three countries in the list form the Customs Union (CU), Armenia and Kyrgyzstan are the candidates for entering this union. The constructed CGE model describes a behavior and interaction of stated below economic agents of the mentioned Regions in the framework of the CU agreements effective from 2010.

Manuscript received July 22, 2014; revised August 13, 2014.

All authors are with the Kazakh National Technical University, 22 Satpayev St., 050013, Almaty City, Republic of Kazakhstan.
A. A. Ashimov (phone: +7(727)292-0344; e-mail: ashimov37@mail.ru).
Yu. V. Borovskiy (e-mail: yuborovskiy@gmail.com).
N. Yu. Borovskiy (e-mail: nborowski86@gmail.com).

Economic agents of the Model and their main functions, as well as a brief description of the markets in the model are stated below. Hereinafter $i = 1, \dots, 7$ is a number of the corresponding Region.

Agent – Aggregate Producers (AP) of the Region *i*: Produce intermediate, consumer, investment products for domestic consumption, and also export products for other Regions; Consume (domestic and imported) intermediate and investment products, and also labor; pay taxes to Government; define demands for loans and deposits of legal entities.

Agent – Households of the Region *i*: Offer labor for AP of the Region i; Consume domestic and imported consumer products; Pay taxes and compulsory pension contributions to Government and receive from it subsidies; Define demands for loans and deposits of individuals.

Agent – Government of the Region *i*: Forms income and spending parts of the government budget of the Region i; Defines government demand for domestic and imported Consumer products; Subsidizes Households and AP transfers of the Region i; Forms National fund income and National fund spending. Governments of three countries of the Customs Union distribute jointly collected customs duties on import among these countries.

Agent – Banks of the Region *i*: Define refinancing interest rate, money holding, interest rates for deposits and loans in the Region i; Meet demands for loans and deposits of AP and households of the Region i.

Markets of the Model are to define the prices, at which obtains corresponding equalities of demands and supplies of products (including VAT) and of labor. The Model has seven markets of domestic intermediate products of each Region; seven markets of domestic consumer products of each Region; seven markets of domestic investment products of each Region; seven labor markets of each Region; 21 markets of export (import) products for each pair of Regions.

General view of the constructed Model is presented by the following system of relations [1], [2].

1) Subsystem of difference equations, linking values of variables $x_1(t)$ (output, fixed assets of agents-producers, account balances of agents in banks and others for the abovementioned Regions) for two successive years:

$$x_1(t+1) = f_1\big(x_1(t), x_2(t), x_3(t), u(t), a(t)\big),$$
$$x_1(t) = x_{1,0}. \quad (1)$$

Here $t = 0, 1, \dots, (n-1)$ – serial number of year, discrete time; $x(t) = (x_1(t), x_2(t), x_3(t)) \in R^m$ – vector of all endogenous variables of system, describing statuses of economies in the Regions;

$$x_k(t) \in X_k(t) \subset R^{m_k}, k = 1,2,3. \quad (2)$$

Here $m_1 + m_2 + m_3 = m$; $x_2(t)$ – demand and supply values of agents in all markets and others; $x_3(t)$ – different types of market prices.

$u(t) \in U(t) \subset R^q$ – vector function of controllable (adjustable) parameters. Coordinate values of this vector correspond to different government economic policy tools of mentioned Regions, for example, different tax rates, refinancing interest rates, money holdings, etc.;

$a(t) \in A \subset R^s$ – vector function of uncontrollable parameters (factors). Coordinate values of this vector describe different depending on time external and internal social and economic factors of Regions: prices for different kind of export and import productions, labor quantity, production function parameters, etc.

$X_1(t)$, $X_2(t)$, $X_3(t)$, $U(t)$ – compact sets with non-empty interiors; $X_k \in \bigcup_{t=1}^{n} X_k(t) \quad k = 1,2,3; \quad X \in \bigcup_{k=1}^{3} X_k$; $U \in \bigcup_{t=1}^{n} U(t)$; A – open connected set; $f_1: X \times U \times A \to R^{m_1}$ – continuous mapping.

2) Subsystem of algebraic equations, describing behavior and interaction of agents in different markets during chosen year, these equations assume expressing variables $x_2(t)$ via rest of endogenous variables for chosen exogenous functions $u(t)$ and $a(t)$:

$$x_2(t) = f_2\big(x_1(t), x_3(t), u(t), a(t)\big). \quad (3)$$

Here $f_2: X_1 \times X_3 \times U \times A \to R^{m_2}$ – continuous mapping.

3) Subsystem of recurrence relations for iterative solution of market prices' equilibrium values in all markets of the Model:

$$x_3(t)[Q+1] = f_3(x_2(t)[Q], x_3(t)[Q], u(t), a(t), L). \quad (4)$$

Here $Q = 0, 1, \dots$ – serial number of iteration; L – set of positive numbers (adjustable iteration constants; economic system faster obtains its equilibrium as their values decrease, however the risk of price shifting to the negative side increases simultaneously; $f_3: X_2 \times X_3 \times U \times A \times (0, +\infty)^{m_3} \to R^{m_3}$ – continuous mapping (joint with f_2 is contracting at fixed t, $x_1(t) \in X_1(t)$ and some fixed L). In this case the mappings f_2 and f_3 have the only fixed point, to which leads the iterative process (3), (4).

CGE model (1), (3), (4) at fixed values of exogenous functions $u(t)$ and $a(t)$ for each moment t defines values of endogenous variables $x(t)$, appropriate to demand and supply equilibrium prices in all markets of the model.

III. Parametric Identification and Verification of the Model

Parametric identification (calibration) of the Model has been performed in three stages.

On the first stage, the parameters of multiplicative production functions which determine the values of gross output by the aggregate producers of each Region of the Model depending on factors of production (fixed assets, labor, intermediate products, and imported oil) were evaluated.

On the second stage, the Model parameters in the form of the values of its exogenous functions (except correcting coefficients) for the historical period (2000-2012) were taken based on observed statistical data for the Regions of the Model.

On the third stage, the values of all exogenous correcting coefficients for the model's corresponding equations for the period 2000-2012 were determined based on observed

ISBN: 978-988-19253-7-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

statistical data for exogenous and endogenous variables of the Model. Correcting coefficient ensures fulfillment of corresponding equation when using observed values of variables in it.

The evaluated Model accurately reproduces the statistical data of 936 its endogenous variables for the period 2000-2012. Basic calculation of the Model for the period 2000-2018 is made by forecasting exogenous functions and coefficients for 2013-2018.

Extended verification of the evaluated Model has been made by using the following five approaches.

1) **The stability indicator** characterizes the maximum number of percent by which might change the values of all endogenous variables of the model for the chosen value of time in comparison with the basic variant given the change in input parameters of the model within the ball with the one percent radius and the center at the point of input (exogenous) parameters of the model in relative values [3, 4].

In the experiments conducted, as input parameters were considered external prices, output shares and expense shares of AP, and others for 2000. As output parameters – GDP and CPI of all of the Regions of the Model for the chosen year.

To calculate the stability indicators, 19 mappings set by the Model $f_t: D \to E_t$ $(t = 2000, \dots, 2018$ is an estimated time) were chosen. They were determined in a domain D – some neighborhood of observed vector p_0 of exogenous factor values (p_0 – a vector of external prices, output shares, spending shares of AP, etc. for 2000). As ranges of E_t mappings were considered sets of vectors of all of the values of endogenous variables in the Model for the chosen year t. Estimation results for stability indicators at point p_0 for mappings f_t are presented in Table I. The values indicated in table do not exceed 9.57, which characterizes the stability of the Model when estimated from 2000 to 2018 as sufficiently high.

TABLE I
STABILITY INDICATORS OF THE MODEL

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Indicator	0.41	1.05	1.41	1.88	2.30	1.82	3.17	2.38	3.99	4.53
Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	
Indicator	2.92	4.66	4.20	5.00	4.16	8.32	5.76	7.33	9.57	

2) The verification of the model with **retroforecast** is made as following.

- With observed data for 2000-2010, there was created a version of the Model.

- Corresponding values of all endogenous macroeconomic indicators of the Model were calculated based on extrapolation of exogenous variables of the model for 2011-2012.

- The relative root-mean-square deviation of all calculated values for 2011-2012 from corresponding observed values was about 2.9%.

3) The verification of the Model was also made through estimation of **sensitivity coefficients** for values of endogenous variables of the Model (2001 to 2018) by its exogenous parameters. The calculation results are partly presented in Table II. The signs of coefficients in these tables are conform to main tenets of the macroeconomic theory.

4) Estimation of **Stability of smooth mappings** set by the Model in sense of [5]. The experiment results demonstrate an absence of singular points of abovementioned mappings f_t in their corresponding domains and their stability for all considered time values t.

5) **Comparison of the results** of mid-term forecast of the main macroeconomic indicators **with IMF forecast** is partly illustrated in Fig. 1.

TABLE II
SENSITIVITY COEFFICIENTS FOR VALUES OF SOME ENDOGENOUS VARIABLES FOR 2009 BY ITS EXOGENOUS PARAMETERS

Exogenous Parameter of corresponding Region for 2009	Region					
	Kazakhstan		Russia		Belarus	
	GDP	CPI	GDP	CPI	GDP	CPI
Price of non-oil export products	0.25	1.32	0.21	1.16	0.23	1.38
Price of imported consumer products from the rest of the world	-0.05	-0.74	-0.05	-0.63	-0.05	-0.67
Price of imported intermediate products from the rest of the world	0.00	-1.08	-0.01	-0.50	0.00	-0.88
Price of imported investment products from the rest of the world	-0.06	-0.70	-0.05	-0.56	-0.06	-0.65
Technological coefficient of gross output	1.11	0.78	1.07	0.74	1.24	0.85
Intermediate products' share in output	0.04	0.02	-0.02	-0.03	0.02	0.00
Consumer products' share in output	0.00	0.09	-0.01	-0.02	-0.02	0.00
Investment products' share in output	0.00	-0.01	0.01	0.02	0.05	0.01
Export products' share in output	0.02	0.01	0.01	-0.04	0.06	0.04
Intermediate products' share in AP consumption	0.00	0.03	0.02	0.00	-0.01	0.02
Investment products' share in AP consumption	-0.01	-0.01	0.01	-0.02	-0.01	0.03
Share of government spending in the state budget	0.18	0.40	0.23	0.66	0.40	0.64
Effective rate of CIT (corporate income tax)	-0.29	0.32	-0.46	0.33	-0.72	0.26
Refinancing rate shock	-0.21	-0.34	-0.12	-0.18	-0.21	-0.35
Monetary base shock	0.06	0.14	0.06	0.11	-0.01	0.10
World oil price	0.23	1.13	0.24	1.10	-0.07	-0.59

ISBN: 978-988-19253-7-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

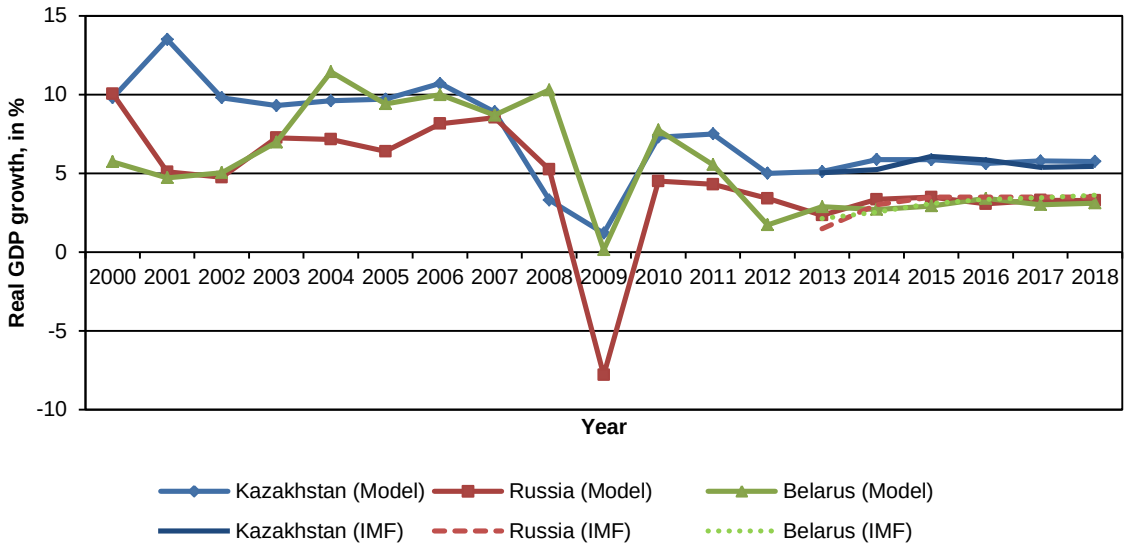

Fig. 1. Real GDP growth rates of three countries of the Customs Union

Stated above the verification results of the Model by five approaches show its acceptable adequacy.

IV. Macroeconomic Analysis of Causes Resulted in the 2009 Recession

One of the directions of macroeconomic analysis based on the Model aimed to determine reasons of macroeconomic events which were related to basic macroeconomic indicators of the Regions changing during the crisis period in 2009.

In the framework of solving this problem, there was evaluated the sensitivity of impact of the following external and internal factors (exogenous parameters including government policy tools for 2008-2009) for assigning GDP variables (Yg_i) and the consumer price index (P_i) for 2009:

1) prices on the countries' export products into the Rest of the world (Pex_i);

2) prices on various products imported to the countries from the Rest of the world ($PcIm_i$, $PzIm_i$, $PnIm_i$);

3) technological coefficients of the gross output production functions of the countries (Y_i);

4) the share of AP production of various products in the countries (Ez_i, Ec_i, En_i, Eex_i);

5) the share of AP consumption of various products in the countries (Oz_i, On_i);

6) the share of government consumption in government spending of the countries (G_i);

7) effective rates of CIT of the countries (T_i);

8) refinancing rates of the countries (Ref_i);

9) monetary base of the countries (DB_i);

10) world oil price ($Poil$).

Analysis of calculated elasticity coefficients (Table II) shows that the impact of output shares (Ec_i, En_i, Eex_i) and consumption shares (Oz_i, On_i) on studied macroeconomic indicators is little enough.

Further, by using counterfactual scenario analysis the impact degree of indicated above factors on variables Yg_i and P_i was evaluated in accordance with the following algorithm.

1. Ten scenarios are calculated in which one j-parameter of the given list (except Ec_i, En_i, Eex_i, Oz_i, On_i) remains in 2008 and 2009 at the level of one in 2007, and the rest of indicators from the list are statistical. Corresponding increments of the variables Yg_i and P_i compared to baseline values are obtained: ΔYg_{ij} and ΔP_{ij}.

2. The scenario in which all mentioned ten parameters in 2008 and 2009 remain at the level of those in 2007 is calculated. Corresponding increments of the variables Yg_i and P_i compared to baseline values are ΔYg_i and ΔP_i.

3. Relationships of $\Delta Yg_{ij}/\Delta Yg_i$ and $\Delta P_{ij}/\Delta P_i$ (in %) are calculated. These relationships characterize the impact degree of corresponding factors on the decline of indicators.

It is worth to note that if the values of mentioned parameters for 2008-2009 remained at the level of those in 2007 the real GDP of Kazakhstan for 2009 would have been higher than the observed one by 11.8%, and CPI by 3.7%; GDP of the Russia – higher by 12.9%, CPI of the Russia – by 4.4%; GDP of the Belarus – higher by 24.6%, CPI of the Belarus lower by 3.9%.

Analysis of the Table III shows that state policy measures of the Republic of Kazakhstan in 2008-2009 were directed correct but not optimal (as the following results of parametric control indicate).

TABLE III
Impact Degrees of Parameters Fluctuations on Increments of the Variables in 2009 (in %)

Variable	Parameter											Total
	Pex_1	$PcIm_1$	$PzIm_1$	$PnIm_1$	Y_1	G_1	T_1	Ref_1	DB_1	$Poil$	Others	
Yg_1	-72,2	+1,7	+1,0	+1,2	-99,3	+74,8	+5,0	+66,6	+80,0	-79,8	-71,9	-100
P_1	-83,8	+4,5	+3,8	+5,6	-23,14	+22,3	-1,9	+35,8	+36,2	-71,7	-26,9	-100

ISBN: 978-988-19253-7-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

V. Setting and Solving the Parametric Control Problems Based on the Model

In the framework of estimation of optimal values of economic policy tools of the Regions in the Model for 2014-2018, there was defined and solved a number of problems on synthesis of the parametric control laws. Present non-formal settings for five such economic policy problems Pr_j $(j = 0,1,2,3,4)$, directed at economic growth, improvement of foreign trade indicators, decrease in government debt and decrease in disparities in regional development. In these problems the values of all uncontrollable exogenous variables in the Model correspond to baseline forecast of these variables.

Pr_j parametric control problems setting $(j = 0, \dots, 4)$. On the basis of the Model, to find the values of economic tools (effective CIT rates, shares of government consumption in spending of government budgets of three countries of the Customs Union) for 2014-2018 for each problem Pr_j, which provide maximum of criterion K_j at corresponding restrictions for indicators of macroeconomic policy coordination and the mentioned economic tools. Here:

K_1 – Normalized value of the CU countries' GDP per capita for 2014-2018, in USD;

K_2 – Normalized value of the government debt in the CU countries for 2014-2018, in mln USD (with minus sign);

K_3 – Normalized value of export from the CU countries for 2014-2018, in million USD;

K_4 – Normalized value of import into the CU countries for 2014-2018, in mln USD (with minus sign);

$K_0 = \sum_{j=1}^{5} a_j K_j$.

Here $a_j = 1$ $(j = 1, \dots, 5)$ – weight coefficients.

K_5 – convergence criterion – normalized sum of modules of differences in GDP rates, consumer price indices, and the ratios of the government budget deficit to GDP for each pair of the CU countries for 2014-2018 (with minus sign).

Solution results for the five problems by numerical method using the Nelder-Mead algorithm are presented in the following Table IV. It should be noted that the Model with the found optimal values of economic experiments was retested for the possibility of practical application using considered above in section 3 stages 1, 3, and 4. These verification results also demonstrated acceptable adequacy of found solutions to the parametric control problems in question.

TABLE IV
K_i Criteria Change Given the Solution of Five Parametric Control Problems

Problem	K_i criterion change (in %)			
	K_1	K_2	K_3	K_4
Pr_0	+3.87	-3.78	+6.95	-3.48
Pr_1	+4.55	-0.91	+3.92	+0.23
Pr_2	+0.37	-6.69	+0.21	-0.79
Pr_3	+3.06	-0.34	+8.14	+0.32
Pr_4	-0.23	-0.44	-0.11	-5.59

Analysis of the Table IV demonstrates an advantage of solving the general parametric control problem Pr_0 in comparison with solving specific problems Pr_j.

Some solution results of the problem Pr_0 at the level of individual countries of the Customs Union are presented further in Tables V and VI. Here K_{ji} are the components of the criterion $K_j, (j = 1, \dots, 4)$, corresponding to the country i $(i = 1,2,3)$.

TABLE V
K_{ji} Criterion Change Relatively to Basic Variant (in %) Given the Solution of the Problem Pr_0

Country	K_{ji} criterion change (in %)			
	K_{1i}	K_{2i}	K_{3i}	K_{4i}
Kazakhstan ($i = 1$)	+3.78	-3.71	+6.63	-3.36
Russia ($i = 2$)	+3.65	-3.84	+6.57	-3.20
Belarus ($i = 3$)	+3.71	-4.21	+6.40	-3.69

TABLE VI
Baseline and Optimal Values of Economic Policy Tools in Kazakhstan

Effective rate of the corporate income tax						
Year	2013	2014	2015	2016	2017	2018
Baseline	0.0795	0.0779	0.0763	0.0748	0.0733	0.0718
Optimal	0.0754	0.0752	0.0702	0.0581	0.0644	0.0553
Effective rate of the value added tax						
Year	2013	2014	2015	2016	2017	2018
Baseline	0.0061	0.0061	0.0060	0.0060	0.0059	0.0058
Optimal	0.0059	0.0059	0.0057	0.0048	0.0047	0.0043
Share of government consumption in government spending						
Year	2013	2014	2015	2016	2017	2018
Baseline	0.4691	0.4808	0.4928	0.5051	0.5178	0.5307
Optimal	0.4879	0.5082	0.5468	0.5768	0.5803	0.6445

Analysis of the Tables IV to VI shows high potentials of the parametric control approach for making recommendations on coordinated optimal state economic policies of the regional economic union's countries.

VI. Conclusion

1. A global multi-country computable general equilibrium model for macroeconomic analysis and recommendation-making in the sphere of economic policy has been proposed for the countries of the Customs Union.

2. The methods of extended verification of the calibrated Model have been demonstrated.

3. Effectiveness of parametric control theory's application for estimation of optimal values of economic policy tools has been shown.

4. Preference for solution of the coordinated estimation problem of values of economic tools at the level of the regional economic union in comparison with the corresponding specific problems has been illustrated.

5. Obtained results could be used for solving practical problems in economic policies of regional economic unions.

ISBN: 978-988-19253-7-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

REFERENCES

[1] A. A. Ashimov, B. T. Sultanov, Yu. V. Borovskiy, Zh. M. Adilov, D. A. Novikov, R. A. Alshanov, As. A. Ashimov, *Macroeconomic Analysis and Parametrical Control of a National Economy*. New York: Springer, 2013, p. 288.
[2] V. L. Makarov, A. R. Bakhtizin, and S. S. Sulashkin, *The Use of Computable Models in Public Administration*. Moscow: Scientific Expert, 2007, in Russian.
[3] C. Robinson, "Structural stability on manifolds with boundary," *Journal of Differential Equations*, vol. 3, 1980, pp. 1-11.
[4] A. I. Orlov, *Econometric. A textbook.* Moscow: Ekzamen, 2002.
[5] M. Golubitsky, V. Guillemin, *Stable Mappings and their Singularities*. New York, Heidelberg, Berlin: Springer-Verlag, 1973.

ISBN: 978-988-19253-7-4
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)