Controller Performance Assessment of Servomechanisms for Nonlinear Process Control Systems

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Abstract—**This paper aims at assessing the setpoint tracking performance of proportional-integral-derivative (PID) controllers for nonlinear single-input-single-output (SISO) process loops. A comparison is made between the actual system output and an artificial process output response derived from nonlinear system identification and a user defined closed loop transient specification. Nonlinear system identification is achieved by fitting routine operating closed loop data to nonlinear autoregressive with exogenous input (NARX) models to describe a closed loop process model and the servo model.**

Once the nonlinear models are established they are linearized to corresponding autoregressive with exogenous input (ARX) models where they are incorporated into a controller performance strategy. The framework will allow for control practitioners to assess the current controller setpoint tracking performance for general nonlinear systems from a transient specification point of view. Simulation studies are given to validate the efficacy of the performance assessment procedure and demonstrate that it is an effective tool when setpoint tracking is of general interest.

Index Terms — **Performance assessment, PID control, nonlinear process, setpoint tracking**

I. INTRODUCTION

CONTROLLER performance assessment (CPA) is concerned with the design of analytical tools that are used to evaluate the performance of process control loops. The primary objective of CPA is to ensure that control systems operate at their full potential, and also indicate when a controller design is unsatisfactory. Many industrial control loops suffer from performance problems, possibly due to improper controller tuning, inadequate control structures, final control element deficiencies, oscillations and unmeasured disturbances. Modern process industry requires control loops to operate within acceptable limits in order to ensure safety and reduce product wastage. CPA for

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linear systems is a well established field with notable works conducted by [1]-[6]. Several software packages are commercially available such as ABB® Optimize^{IT} Loop Performance Manager, Honeywell® Loop Scout and Metso Automation® Loop Browser [7]. These usually contain several different metrics to indicate the quality of the controller performance and to aid in diagnosis of controller problems. Probably the most popular benchmark is the Harris index [2] which compares the current process output to the output that would have occurred if some *theoretical optimal controller* has been applied to the process. This metric can be the most indicative measure of the health of a loop which is generally not readily apparent from casual observation of a process loop trend. Most commercial packages employ the Harris index [2], whose work methodically showed how CPA can be achieved by utilizing linear time series modeling and minimum variance control (MVC). The method is powerful in that CPA is realized by merely fitting the closed loop process output with additive disturbance data to a linear time series model [3]. Only the process loop dead time must be known in order to compute the performance index. The achievable theoretical minimum variance which is derived from the time series model is then compared to the actual closed loop output variance. The method is practical and easy to implement but is only applicable for the linear case [8], [9].

Thus far the majority of research conducted in the field of CPA utilizes linear time models in determining suitable indices for CPA [8]. In practice however, industrial control loops invariably include nonlinearities from the control valve, sensor behavior, or inherent qualities in the process itself. Within this context, these nonlinearities must be taken into account at the design stage in order to improve controller ruggedness and also to ensure accurate performance benchmarking measures.

It is well known that most process loops are nonlinear to some extent and can be modeled sufficiently well using linear time series models [3]. However, some systems exhibiting higher degrees of nonlinearity may be more difficult to model due to the existence of intrinsic complexities and a non-Gaussian output [10]. In such cases, the closed loop process dynamics and disturbance models cannot be well characterized by either its impulse response or its equivalent time series model. It has been shown by [9] that traditional linear performance indices incorrectly yield biased performance benchmark measures in the presence of valve nonlinearity.

A survey of literature reveals that researchers investigating nonlinear systems fall into one of two groups [8]. The first group focused on the diagnosis of a common specific nonlinearity, namely valve stiction [9]; the second group tried to establish the minimum variance performance lower bound [8]. However, many outstanding issues still remain open for further research on nonlinear systems. The focus of this work is the development of a CPA tool for performance evaluation of PID controllers for setpoint tracking of nonlinear processes. To the authors knowledge this has not been previously considered in the literature.

The work of [11] proposed the use of lower bounds using integral of the absolute error (IAE) based on internal model control principles for setpoint tracking CPA. However, the processes considered are linear time invariant and complete knowledge of the open loop process model must be known. Open loop models are often difficult to obtain under normal operating plant conditions thus from a practical perspective the method may be at a disadvantage. In [12], controller setpoint tracking performance evaluation was proposed based on dimensionless settling time and dimensionless IAE. A simple first order plus dead time transfer function was used to develop the performance benchmark. The performance evaluation is limited to three performance classes namely: *High Performance, Excessively Sluggish* and *Poorly Tuned* and may be inflexible when different performance targets are expected from individual controllers. In this work we recommend a *user defined settling time* as a performance benchmark. An outline of the paper is now given. Section 2 provides a description of the nonlinear system considered in the study. Section 3 outlines the methodology for the proposed performance benchmark. Section 4 shows how the method can be applied within a pragmatic context by use of routine operating data in a simulation example. Section 5 concludes the study and provides recommendations for furthering the work.

II. System Description

A. Nonlinear feedback control loop

Fig. 1 is the nonlinear SISO control system considered in this study. The controlled process may be a general nonlinear system which can be adequately represented by a discrete time model (1):

$$y(t) = f\left[y(t-1),...,y(t-n_y),u(t-1),...,u(t-n_u)\right] + d(t) \qquad (1)$$

From Fig. 1, $y(t)$ is the process output, $u(t)$ is the controller output, n_y and n_u indicate the number of output and input delay respectively. The system inputs are a unit step input represented by $r(t)$ and disturbance given by $d(t)$. The effects of sensors noise and stochastic dynamic disturbances affecting the closed loop system are lumped into the aforementioned disturbance input. The nonlinear function $f(.)$ may be represented by a wavelet network or neural network. In this paper artificial neural networks are used since they are universal approximators and have received considerable attention in the field of system identification and controller design [13].
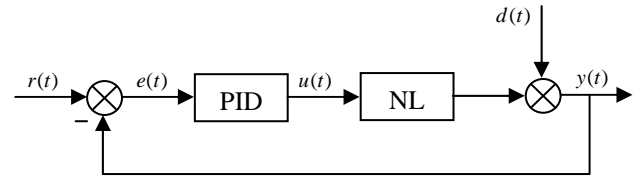

Fig. 1. Nonlinear feedback control loop.

The process model can be written in terms of a deterministic NARX model:

$$y_{NARX}(t) = NARX(\Phi(t)) \qquad (2)$$

where, the regression vector is defined as:

$$\Phi(t) = [y(t),u(t)]^T$$
$$= [y(t-1),...y(t-n_y),u(t-1),...,u(t-n_u)]^T \qquad (3)$$

The controller output $u(t)$ is a function of PID parameters and is represented by:

$$u(t) = k_c\left[e(t) + \frac{1}{\tau_i}\int e(t)dt + \tau_d\frac{de(t)}{dt}\right] \qquad (4)$$

where k_c , τ_i and τ_d represents the proportional gain, integral time constant and derivative time constant respectively. The control loop error is computed as:

$$e(t) = r(t) - y(t) \qquad (5)$$

A digital velocity form of the PID controller given in (4) can be written as:

$$\Delta u(z) = k_c\left[(1-z^{-1})e(z) + \frac{\Delta t}{2\tau_i}(1+z^{-1})e(z) + \frac{\tau_d}{\Delta t}(1-2z^{-1}+z^{-2})e(z)\right] \qquad (6)$$

where z^{-1} represents the backshift operator and the integral action of (6) is computed using trapezoidal approximation.

B. Linearization of nonlinear system

The concept of linearization of a nonlinear model around an operating point is well established and often used in control designs [14]. However a linearized model is only valid in the local neighborhood of the operating point and may provide poor approximations at other operating regions. Nevertheless it provides a means of analyzing the nonlinear system within a linear framework for which there exists a large knowledge base. In addition, computational complexities are thus avoided when using the linear approach. Given the nonlinear feedback control loop shown in Fig. 1, the approximate closed loop model within the desired operating region is represented as:

$$y_{ARX}(t)A(z) = B(z)u(t-n_k) + E(t) \qquad (7)$$

where,

$$A_{uy}(z) = 1 + a_1 z^{-1} + \ldots + a_{n_a} z^{-n_a} \qquad (8)$$

$$B_{uy}(z) = b_1 z^{-1} + b_2 z^{-1} + \ldots + b_{n_b} z^{-n_b + 1} \qquad (9)$$

The modeling error is given by $E(t)$, with n_a and n_b representing the order of the output and inputs terms, respectively. n_k denotes the system sample delay in which the input affects the output. The best approximate linear model within a range of input values is computed by solving the cost function (10) in the mean square sense:

$$\min J = \min_{A(z), B(z)} \left[\frac{1}{n} \sum_{i=1}^{n} (y_{NARX_i} - y_{ARX_i})^2 \right] \qquad (10)$$

Similarly the servo model may be computed using the process output response and the desired reference trajectory as the modeling input vector. Using the ARX model obtained from (7), an artificial process output can be derived based on different PID control designs. The derivation of the simulated process output and controller benchmarking is shown in the following section.

III. CONTROLLER PERFORMANCE ASSESSMENT FOR NONLINEAR SYSTEMS

A. Derivation of the artificial process output

It has been shown by [15] that an achievable PI control performance assessment for linear systems can be conducted based on routine operating data. In their approach a closed loop servo model of the process loop under investigation is identified from closed loop experimental data. Generation of the excited closed loop process output is obtained by acceptable setpoint step changes made while the feedback loop is closed. This is a compelling approach from a practical perspective since the feedback loop is not broken. An obvious advantage of this approach is when dealing with open loop unstable processes and during normal operating plant conditions when manual mode of the control loop is not permitted. Furthermore, in most process plants there is minimal opportunity for the control practitioner to perform such tests for open loop model identification. A review of the methodology is thus provided for linear time invariant systems. Considering a disturbance transfer function ($D=d(t)/a_t$) process driven by white noise a_t, the expression for the process output $y(t)$ is given by:

$$y(t) = \left[\frac{D}{1 + G_{p\_OL} G_c} \right] a_t \qquad (11)$$

Where, G_{p_OL} and G_c represents the open loop process model and the current controller transfer function respectively. Now, if the current controller G_c is replaced by a new controller G_c* then the new output $y_{(t)}*$ is given by the following equation:

$$y(t)* = \left[\frac{D}{1 + G_{p\_OL} G_c *} \right] a_t \qquad (12)$$

Equations (11) and (12) can now be used to derive the following expression:

$$\frac{y(t)*}{y(t)} = \left[\frac{1 + G_{p\_OL} G_c}{1 + G_{p\_OL} G_c *} \right] \qquad (13)$$

From (13), the r.h.s represents a filter [15] which gives an artificial closed loop data series ($y_{(t)}*$) when the current process output data ($y_{(t)}$) is passed through it. Now from the negative feedback control loop the open loop model is represented as:

$$G_{p\_OL} = \frac{G_{p\_CL}}{(1 - G_{p\_CL}) G_c} \qquad (14)$$

Where G_{p_CL} is the closed loop servo model. Substitution of (14) into (13) yields the expression:

$$\frac{y(t)*}{y(t)} = \left[\frac{1 + \dfrac{G_{p\_CL}}{(1 - G_{p\_CL})}}{1 + \dfrac{G_{p\_CL} G_c *}{(1 - G_{p\_CL}) G_c}} \right] = \frac{G_c}{(1 - G_{p\_CL}) G_c + G_{p\_CL} G_c *} \qquad (15)$$

Based on the expression given by (15), the artificial process output ($y(t)*$) can be used in a CPA framework. It should be noted that the disturbance transfer function (D) is not required. Only routine operating closed loop data, such as the process output ($y(t)$), controller output ($u(t)$) and reference trajectory ($r(t)$) is needed. Once a suitable nonlinear model has been identified, the controller parameters can be directly estimated from the linearized model. Determination of the new controller parameters is provided in the subsequent section.

B. PID controller tuning based on ARX model

PID controllers are commonly used in industry since they have shown to be versatile and robust in many industrial process control applications. The reasons may be due to its simple mathematical structure which can be easily understood. Even under difficult process conditions it may perform sufficiently well when compared to more elaborate designs such as model based controllers. In a survey of status conducted by Desborough and Miller [16] of industrial controllers, it was reported that in a typical chemical plant 98% of the controllers were of the PID family. This situation is unlikely to change in the foreseeable future because advanced control implementation requires well-tuned PID controllers in the lower level [17]. Although there have been advances made in the assessment of control loop performance using MVC as a benchmark [3], the metric may be regarded as overly optimistic since a large number of industrial controller belong to the PID family [15]. Furthermore MVC does not account for the large control efforts required to produce minimal variance of the controlled variable which in practice may cause damage to the final control element within a shorter period of time. Therefore there is considerable incentive to develop more realistic CPA tools for restricted structure controllers

of the PID type.

It has been shown by Acara *et. al* [18] that PID parameters can be directly estimated from the ARX model given in (7). The following direct relationships between the PID parameters and the ARX model [18] with $n_a = n_b = 2$ are given as:

$$k_c = \frac{-T_n(a_1 + a_2)}{T_{s_{des}}(b_1 + b_2)} \tag{16}$$

$$\tau_i = \frac{-T_n(a_1 + a_2)}{1 + (a_1 + a_2)} \tag{17}$$

$$\tau_d = \frac{T_n a_2}{(a_1 + 2a_2)} \tag{18}$$

Where, Ts_{des} is the desired closed loop settling time and T_n is the sampling rate. The main advantage of using the ARX based tuning rules is that the PID parameters can be generated quickly. This is not the case as with other restricted structure CPA methods which require an optimization function to be solved for determination of PID tuning parameters [19]. Furthermore, a single user defined parameter (Ts_{des}) can be used to characterize the desired closed loop response for each individual process control loop.

C. Proposed methodology

In many practical cases the desired performance characteristics of control systems are specified in terms of time domain qualities. These qualities are given in terms of transient response to a unit step input namely; rise time (*Tr*), peak time (*Tp*), maximum overshoot (*POS*) and settling time (*Ts*). If we choose to specify a certain value for the settling time then this would invariably alter the transient response of the control system. The settling time relates to the largest time constant of the control system and is the time required for the response curve to reach and stay within a range (usually 2% or 5%) of the final value. By specifying a desired closed loop settling one can incorporate this into a performance benchmark which is tailored to the specific requirements of the setpoint tracking capabilities of the controller. Fig. 2 illustrates the procedure used for estimating current controller performance.
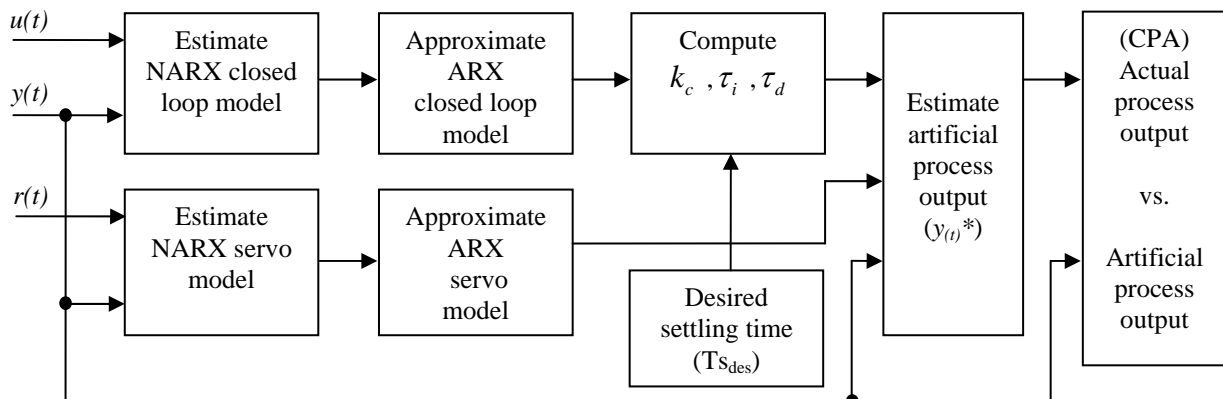
Using logged closed loop data from a setpoint step change experiment, nonlinear models are obtained for the closed loop process and a servo model. The models are then linearized around the desired operation point. PID parameters are computed directly from the closed loop ARX model and used in conjunction with the servo model to estimate the artificial closed loop response of the system to the excitation provided by the initial step change.

Comparative analysis based on transient response characteristics of the actual process output versus the simulated process output can be conducted and a conclusion concerning the current controller performance based on user defined settling time can be made.

IV. ILLUSTRATIVE EXAMPLE

A. Preliminaries for the simulation

In this section a simulation example is provided to demonstrate the methodology outlined in this paper. The simulation was conducted in MATLAB (R2010a) SIMULINK® with 400 samples collected at a sampling rate of 1 second. A reference trajectory *r(t)* is injected into the system with a unit step change occurring at t=100. The desired settling time for the closed loop process is $Ts_{des} = 40$ seconds.

B. Case study

Consider a nonlinear dynamic system which can be represented by a second order Volterra series as [8]:

$$y_t = 0.2u_{t-3} + 0.3u_{t-4} + u_{t-5} + 0.8u_{t-3}^2 + 0.8u_{t-3}u_{t-4} - 0.7u_{t-4}^2 - 0.5u_{t-5}^2 - 0.5u_{t-3}u_{t-5} + D \tag{19}$$

Where the disturbance transfer function (*D*) is given as:

$$D = \frac{a_t}{1 - 1.6z^{-1} + 0.8z^{-2}} \tag{20}$$

The white noise sequence (a_t) has zero mean and a variance of 10^{-5}. A proportional-integral (PI) controller with parameters $k_c = 0.2$ and $\tau_i = 0.67$ is used to control the simulated process.

Fig. 2. Proposed methodology for nonlinear setpoint tracking performance assessment. *u(t)* – controller output, *y(t)*- process output, *r(t)* - setpoint

C. Simulation results for the case study

Fig 3. shows the closed loop response to a unit step input. It is evident that the response has large overshoots and undershoots which inevitably leads to a longer settling time. Using the methodology outlined in the previous section, the response of the nonlinear closed loop process model and servo model are identified and presented in Fig. 4 and Fig. 5 respectively. A NARX (2,2,1) is fitted to both models giving a 88.66% and 74.95% fit to the closed loop model and servo model respectively. The corresponding nonlinear models are linearized within the operating region [0 1] using the MATLAB *linapp* function which gives the following ARX models:

$$A_{uy}(z) = 1 - 0.8334z^{-1} + 0.3255z^{-2}$$
$$B_{uy}(z) = -0.1334z^{-1} + 0.8185z^{-2}$$
(21)

$$A_{ry}(z) = 1 - 1.322z^{-1} + 0.7552z^{-2}$$
$$B_{ry}(z) = -0.009225z^{-1} + 0.4652z^{-2}$$
(22)

Using the tuning rules (16) and (17), the PI controller parameters are calculated as:

$$k_c = 0.0741$$
(23)

$$\tau_i = 1.032$$
(24)

The artificial closed loop response which is obtained from (15) is illustrated in Fig. 6. Comparisons between the transient specifications of the fictitious process output (*y_estimated*) and the actual process output (*y_actual*) are listed in Table 1. In addition, the transient response conditions for the process output (*y_new*) with the new PI parameters are included. Fig 7. shows original process output versus new process output based on the controller parameter recommendations of the CPA methodology.

D. Discussion of results

From the transient response specifications given in Table 1 it is evident that artificial process output derived from the proposed methodology results in faster settling time. This implies that the existing PI controller has potential for improvement based on the desired settling time (Ts_{des}). Applying the new controller parameters derived from the proposed methodology results in significantly smaller percentage overshoot. The tradeoff for this improvement in percentage overshoot is the longer rise time and time to peak. Marginal errors are observed between the theoretical estimated process output and the new process output which

can be attributed to modeling errors. An improved IAE is observed with the new PI parameters given by the proposed methodology.

V. CONCLUSION

Although the method has the benefit of yielding PID parameters that lead to desired closed loop transient specifications, care must be taken when selecting a suitable value for the desired settling time as the tuning algorithm is sensitive to this factor. In addition, selection of short settling times in which the process is not capable of will result in large proportional gains, which may lead to instability when applied. Sampling rate must be chosen such that it makes it compatible to a wide range of process time constants where process dynamics are adequately captured. As with most methods that rely on process models, the CPA methodology will not function well when there are considerable modeling errors. Furthermore an excitation in the form of a step change is necessary to capture the servo nonlinear model dynamics. Although the proposed method yields encouraging results, the presented example is based on simulation. Additional work is required to test the methodology on real nonlinear process control loops.

REFERENCES

[1] T. Hägglund, "A control loop performance monitor," *Control Engineering Practice*, vol.3, no. 11, pp. 1543–1551, 1995.
[2] T. J. Harris, "Assessment of Control Loop Performance," *Canadian Journal of Chemical Engineering*, vol. 67, pp. 856–861, Oct. 1989.
[3] T. J. Harris, C. T. Seppala and L. D. Desborough, "A review of performance monitoring and assessment techniques for univariate and multivariate control systems," *Journal of Process Control*, vol. 9, pp. 1–17, 1999.
[4] B. Huang, S. L. Shah and E. K. Kwok, "Good, Bad or Optimal? Performance assessment of Multivariable Processes," *Automatica*, vol.33, pp. 1175–1183, 1997
[5] M. J. Grimble, "Controller performance benchmarking and tuning using generalized minimum variance control," *Automatica*, vol.38, pp. 2111–2119, 2002.

TABLE I
TRANSIENT RESPONSE SPECIFICATIONS FOR THE SIMULATION

	y_actual	y_estimated	y_new
POS	130	1.2	6.6
Tp	106	235	234
Tr	1	15	11
Ts(5%)	163	127	146
IAE	19.4	-	13.8

POS= percentage overshoot, Tp= time taken to peak, Tr= time to rise, Ts = 5% settling time, IAE = Integral absolute of the error.
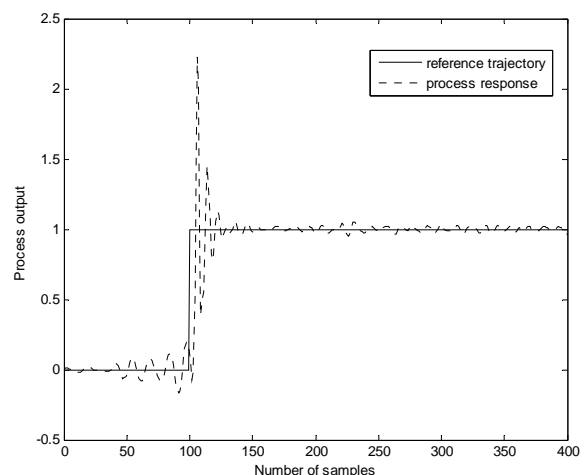

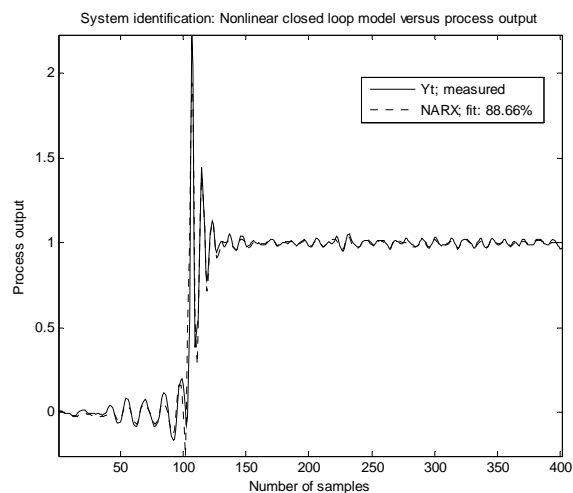Fig. 3. Process output response to a unit step input

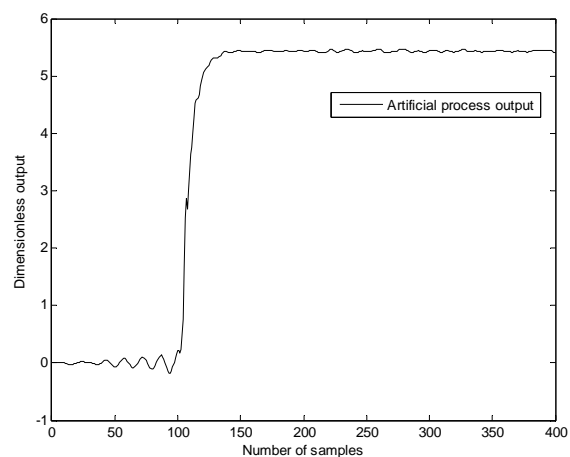Fig. 4. Nonlinear system identification, NARX model versus closed loop model

Fig. 6. Artificial process output generated from actual process output and new PI parameters.
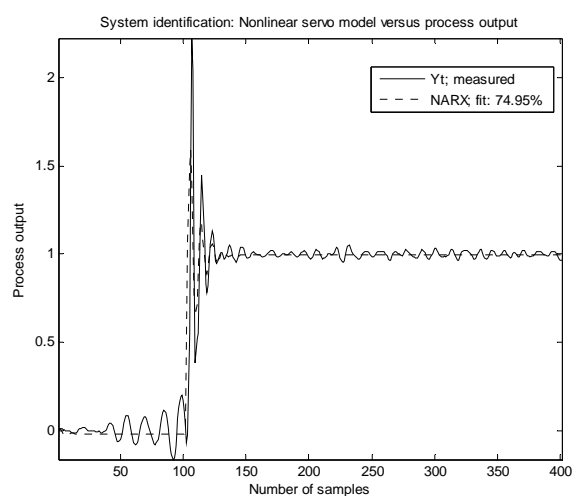

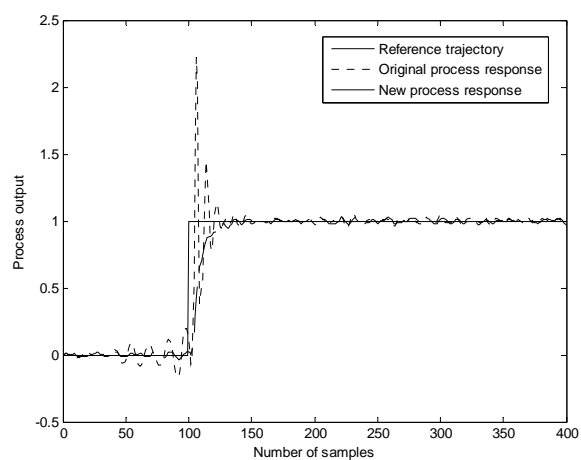Fig. 5. Nonlinear system identification, NARX model versus servo model

Fig. 7. Original process output versus new process output using PI settings obtained from (16) and (17).

[6] B. S. Ko and T. F. Edgar, "PID Control Performance Assessment: The Single-Loop Case," *American Institute of Chemical Engineers*, vol.50, no. 6, pp. 1211–1218, 2004.

[7] M. Jelali, "Control Performance Management in Industrial Automation," 1st ed., M. J. Grimble, M. A Johnson, Ed. London: Springer-Verlag, 2013, pp. 394–396.

[8] T. J. Harris, W. Yu, "Controller assessment for a class of non-linear systems," *Journal of Process Control*, vol. 17, pp. 607–619, 2007.

[9] W. Yu, D. I. Wilson and B. R. Young, "Nonlinear control performance assessment in the presence of valve stiction," *Journal of Process Control*, vol. 20, pp. 754–761, 2010.

[10] Z. Zhang, L. Hu and W. Shi, "Estimation of Benchmark Performance for Nonlinear Control Systems," in *Proc. American Control Conference*, San Francisco, CA, USA, 2011, pp. 5079–5084.

[11] Z. Yu, J. Wang, B. Huang and Z. Bi, "Performance assessment of PID control loops subject to setpoint changes," *Journal of Process Control*, vol. 21, pp. 1164–1171, 2011.

[12] A. P. Swanda and D. E. Seborg, "Controller Performance Assessment Based on Setpoint Response Data," in *Proc. American Control Conference*, San Diego, California, USA, 1999, pp. 3863–3867.

[13] K. S. Narendra and K. Parthasarathy, "Identification and Control of Dynamical Systems Using Neural Networks," *IEEE Transactions on Neural Networks*, vol.1, no. 1, pp. 4–27, 1990.

[14] J. Chen and T. C. Huang, "Applying neural networks to on-line update PID controllers for nonlinear process control," *International Journal of Process Control,* vol.14, pp. 211–230, 2004.

[15] M. Jain and S. Lakshminarayanan, "A Filter-Based Approach for Performance Assessment and Enhancement of SISO Control Systems," *Ind. Eng. Chem. Res.,* vol.44, pp. 8260-8276, 2005.

[16] L. D. Desborough and R. M. Miller, "Increasing Customer Value of Industrial Control Performance Monitoring-Honeywell's Experience," in *Proceedings of CPC VI*, Tucson, AZ, USA, 2001.

[17] P. Agrawal, S. Lakshminarayanan, "Tuning proportional-integral-derivative controllers using achievable performance indices", *Ind. Eng. Chem. Res.*, vol. 42, pp. 5576-5582, 2003.

[18] S. A. Arcara and P. Anderson, "Self Tuning of P-I-D Controller by Conversion of Discrete Time Model Identification Parameters," U.S. Patent 4 407 013, Sep 27, 1983.

[19] B .S. Ko and T. F. Edgar, "Assessment of achievable PI control performance for linear processes with dead time", *Proc. American Control Conf.*, Philadelphia, USA, 1998.