GA Based Selective Harmonic Elimination for Multilevel Inverter with Reduced Number of Switches

Hulusi Karaca, Enes Bektaş

Abstract—Multilevel inverters have been used in the power applications to get low total harmonic distortion (THD) in medium or high voltage levels. In order to get low THD in the output voltage, several techniques have been applied to multilevel inverter. One of these techniques is Selective Harmonic Elimination (SHE) that has an extensive research area in the field of power electronics. It is also an alternative to usual PWM techniques and includes nonlinear equations of stepped voltage waveform. In this paper, multilevel inverter with reduced number of switches which enables a reduction in the system cost has been proposed, and solution of SHE equations have been optimized by Genetic Algorithm (GA). The results of simulation and analysis have apparently proved that proposed GA based SHE technique eliminates desired harmonic order.

Index Terms—Genetic algorithm, minimum number of switches, multilevel inverter, selective harmonic elimination, total harmonic distortion

I. INTRODUCTION

In recent years, power converters applications in industry have increased due to the developments in semiconductor technology and the different pulse width modulation techniques. Especially, in high power and medium voltage applications, multilevel inverter topology has come up with a choice because power switches which withstand the medium voltage have not been developed yet [1].

Multilevel inverters generating output voltage in the form of stepped wave was presented in 1975, firstly. This form of the inverter has included the series connection of H-bridge. After that, the structures of the diode clamped multilevel inverter and the capacitor clamped multilevel inverter have appeared, respectively [1], [2].

Multilevel inverter has outstanding performances such as lower dv/dt for good electromagnetic compatibility, reduced voltage stress on the switches, input current with low distortion, more importantly having lower total harmonic distortion than traditional two-level inverter in output voltage and currents [1], [3], [4]. By applying the different techniques aiming at the elimination of harmonics, the THD of multilevel inverter’s output voltage can be decreased. One of the most important techniques is Selective Harmonic Elimination (SHE) method that can eliminate the desired any harmonics. At the first stage, the nonlinear equations of output voltage harmonics including trigonometric terms must be obtained. In the second stage, which is the basic purpose of SHE technique, switching angles can be acquired with solving the harmonic equations. These equations change according to the number of harmonics’ order desired to eliminate and the requested output voltage value [3], [6].

The usual methods to obtain switching angles are iterative methods such as Newton-Raphson [5] and Neural Networks using Newton-Raphson method. The iterative methods are sensible to the initial value and divergence problems are probably to arise, also the optimal switching angles may not be produced for the minimum THD [7]. These methods provide more complex solution than Genetic Algorithm due to the necessity of different look-up tables for each modulation indexes. The GA is a type of artificial intelligence approaches. The GA gives the optimum solution of the harmonic equations in contrast to the iterative methods [9], [10].

Multilevel inverters have several disadvantages. The most significant drawback is the necessity of great number of switches. In addition, the drivers of each level module must be isolated from each other. As THD decreases with rising level, the hardware of multilevel inverter needs more isolated power source. That leads the increasing expense of inverter according to conventional inverter. Thus, to realize the lower cost applications, it is very important to reduce the number of power semiconductor switches and drivers [1], [10]. In this paper, multilevel inverter with reduced number of switches using the isolated DC sources has been proposed. SHE equations of the stepped output voltage have been solved by genetic algorithm software which eliminates desired harmonics order without the use of GA toolbox of MATLAB. The results of simulation have clearly proved that the proposed SHE technique for multilevel inverter with reduced number of switches can eliminate the desired harmonics’ order.

II. SUGGESTED MULTILEVEL INVERTER TOPOLOGY

The proposed cascaded multilevel inverter consists of level module units having half-bridge topology with two switches. Each half-bridge is connected isolated DC power supply as shown in Fig. 1. The overall structure of suggested topology includes two basic parts. The first part is the level...
module units, which produce DC voltage levels. The second is the H-bridge generating positive and negative stepped output voltage. Regarding isolated DC voltage sources’ number is k, also the number of level-modules, the maximum and minimum rates of level’s output voltage are

\[ V_{o,\text{min}} = 0 \]
\[ V_{o,\text{max}} = kV_{dc} \]

The multilevel inverter is known as symmetric multilevel if all voltage sources equal to \( V_{dc} \) [2]. The number of output voltage levels in symmetric multilevel inverter is

\[ N_{\text{level}} = 2k + 1 \]

Maximum and minimum rates of load voltage are

\[ V_{L,\text{min}} = -kV_{dc} \]
\[ V_{L,\text{max}} = +kV_{dc} \]

The number of switches is \( 4k \) in H-bridge inverter topology. But in the suggested topology, this value is \( 2k + 4 \). When compared usual cascaded multilevel inverter and suggested topology in terms of switches’ number, suggested topology is more advantageous in multilevel inverter with more than level number 5 (7, 9, …).

In these types of inverter, in order to get sinusoidal output voltage, switching angles have been calculated analytically by (6) and (7) [1]. This technique aiming sinusoidal output voltage is named as SPWM (Sinusoidal Pulse Width Modulation).

\[ N_{\alpha} = k = \frac{N_{\text{level}} - 1}{2} \]
\[ \alpha_i = \sin^{-1}\left(\frac{2i-1}{N_{\text{level}} - 1}\right) \quad i = 1, 2, 3, ... k - 1, k \]

where, \( N_{\alpha} \) is the number of switching angles. In five-level inverter, for example, two switching angles with the condition \( \alpha_1 < \alpha_2 < \pi/2 \) have been calculated. It is quite apparent that other angles can be obtained from these angles because of the quarter wave symmetry as shown in Fig. 2. \( S_1 \) and \( S_2 \) is the passkey of the five-level inverter. Therefore, \( \alpha_1 \) and \( \alpha_2 \) is calculated for these switches respectively. Switches’ state, in the same module, cannot be simultaneously due to the fact that to elude short-circuit.

In Fig. 2, the switching angles can be seen for one cycle of output voltage wave form. Dashed line in the figure represents the voltage (shown in Fig. 1. as \( V_o \)) applied to full-bridge and it’s frequency is twofold of output voltage.

### III. Calculation of Switching Angles

The control of multilevel inverter is based on specifying the switching angles to synthesize a desired sinusoidal voltage waveform. In this paper, the stepped voltage waveform with odd-harmonic components is given in (8) for \( 2k+1 \) level inverter. Because of symmetric wave, output voltage has not included even-harmonic components. For equal dc sources, Fourier series expansion of the stepped output voltage is given as,

\[ V_o(\omega t) = \sum_{n=1,3,5,7,9, ...}^{\infty} \frac{4V_{dc}}{\pi n} \left[ \cos(n \alpha_k - 1) + \cos(n \alpha_k) \right] \sin \omega t \]

The desired effective value of output voltage, \( V_1 \), is controlled by using the modulation index and higher harmonic order desired to abolish have to been equal to zero. It is clear that so as to eliminate harmonic-order until the number of \( k - 1 \) and control modulation index, we need harmonic equations until the number of \( k \). Therefore, for example, a nine-level inverter may ensure the control of modulation index and elimination of three harmonic-orders [7, 12].

The modulation index is given by:

\[ M = \frac{V_1}{kV_{dc}} \]

where, \( V_1 \) is the first harmonic of the output voltage in the same time. For \( (2k + 1) \) level inverter, harmonic equations until the number of \( k \) is given in the following equations first of which is related to desired output voltage value \( (V_1) \).

Equation (10) changes according to harmonic-orders’ number desired to eliminate and level number of multilevel inverter.

![Fig 1. The proposed structure of multilevel inverter](image1)

![Fig 2. Output voltage wave form of 5-level inverter](image2)
These equations are transcendental and nonlinear. To handle this problem that consists of minimizing the harmonics, optimization techniques are applied.

\[
\cos(\alpha_1) + \cos(\alpha_2) + \cdots + \cos(\alpha_{k-1}) + \cos(\alpha_k) = M \frac{\pi}{4} \\
\cos(3\alpha_1) + \cos(3\alpha_2) + \cdots + \cos(3\alpha_{k-1}) + \cos(3\alpha_k) = 0 \\
\vdots \\
\cos((N_{level} - 4)\alpha_1) + \cdots + \cos((N_{level} - 4)\alpha_k) = 0 \\
\cos((N_{level} - 2)\alpha_1) + \cdots + \cos((N_{level} - 2)\alpha_k) = 0 
\]

(10)

Conventional techniques such as Newton-Raphson method can be applied to this problem. But this method may not give optimum solution of equations above. Also, it is time consuming and complicated in accordance with GA. Therefore, in this paper the solution of the SHE equations has been solved by GA. This method uses fitness function to optimize the solution. In the solution of SHE problem, the fitness function must be THD’s value given in (15).

\[
0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \pi/2 
\]

(12)

The population changes according to type of problem, size of population is selected as 500.

Selection is reproduction of the starting population. In this paper, tournament selection is used. In tournament selection, two chromosomes are chosen randomly to become pairs. One of these pairs is specified according to the penalty function which is acquired from constraint function and incorporated in the new generation. Penalty function evaluates closeness of the values of the chromosomes should be produced if there is any out of range of the constraint function. This range is between 0 and 0.01 in this study. This range determines precision of produced switching angles in multilevel inverter. The penalty function used in GA is given in (13).

\[
P(\alpha) = f(\alpha) + \sum_{k=1}^{4} r[g_k(\alpha)] 
\]

(13)

where, \(P(\alpha)\) is penalty function, \(r=500\), \(g_k\) are constraint functions. The process of selection can be summarized with three sections;

- If both pairs of chromosomes are in appropriate range, that is constraint functions are not between 0 and 0.01, chromosome with small penalty function is incorporated in the new generation.
- If one of the pairs of chromosomes is in the appropriate range, that is the result of penalty function is equal to zero, this chromosome is incorporated in the new generations.
- If both pairs of chromosomes are in the appropriate range, chromosome with small fitness function is incorporated in the new generation.

One of the most notable parts of GA is crossover. Crossover performs creation of new offspring by taking into account selected chromosomes. Several crossover techniques can be performed GA such as linear, arithmetic and heuristic etc. In this paper, arithmetical technique is used. In this technique, a number \(p\) which is named as crossover parameter is produced between 0 and 1 for each mutual gene of selected pairs of chromosomes. If this produced number is bigger than the cross rate, mutual genes are put into process of crossover as
to arithmetically crossover equation (14). For example, if only 3rd genes are in the crossover for \( p = 0.2 \), chromosomes are changed as in Table I and the arithmetically crossover equation is given by:

\[
\text{Gene}_{3,CH1} = p \text{Gene}_{3,CH2} + (1 - p)\text{Gene}_{3,CH1}
\]

\[
\text{Gene}_{3,CH2} = p \text{Gene}_{3,CH1} + (1 - p)\text{Gene}_{3,CH2}
\]

(14)

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>ARITHMETICALLY CROSSOVER SAMPLE OF GA SOLVING SHE EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before crossover</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Chromosome 1</td>
<td></td>
</tr>
<tr>
<td>Chromosome 2</td>
<td></td>
</tr>
</tbody>
</table>

After crossover, mutation occurs. In real-coded GA, in order to perform mutation, a number is produced between 0 and 1 for each gene of each chromosome. If produced number for gene is lower than the mutation rate, gene is reproduced according to limits.

The most important point for the GA is evaluation of each chromosome. The aim of the study is to eliminate desired harmonic orders of output voltage in multilevel inverter, so fitness function must be associated to THD equation which may change based on harmonic orders and the number of harmonic orders desired to be equal zero [9]. For example, if third, fifth, seventh harmonic orders are desired to eliminate in nine-level inverter, fitness function is given by:

\[
\text{Fitness} = \frac{\sum_{m=3,5,7} c_i}{\sum_{k=1}^{4} \cos(a_k)}
\]

(15)

Consequently, the GA calculates the fitness function in each iteration and defines chromosomes as to fitness value. GA must select the chromosomes to minimization of desired harmonic orders. Therefore, the values of fitness functions should be equal to zero.

V. SIMULATION RESULTS

In this study, Selective Harmonic Elimination technique has been applied to seven-level and nine-level inverters. Simulation results have been given to assess the effectiveness of GA based SHE. The SHE equations were solved by GA and simulation was developed in Matlab&Simulink. The fundamental frequency of the output voltage is 50 Hz and switching frequency of the semiconductor power switches in the level module units and H-bridge is 100 Hz and 50 Hz, respectively.

In Table II and Fig. 5, the eliminated harmonic orders are shown for seven-level inverter. Similarly, Table III and Fig. 7 give the results of nine-level inverter. Additionally, Fig. 4 shows the output voltage waveforms of seven-level inverter having 12 V isolated DC source. The output voltage’s maximum rate is 36 V because of the three modules in seven-level inverter.

<table>
<thead>
<tr>
<th>TABLE II.</th>
<th>SEVEN-LEVEL INVERTER % HARMONICS VALUE IN SPWM AND ELIMINATION OF 3rd AND 7th HARMONIC ORDERS IN SHEPWM AND SWITCHING ANGLES OF SHEPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic order</td>
<td>SPWM</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>2.25</td>
</tr>
<tr>
<td>9</td>
<td>3.33</td>
</tr>
<tr>
<td>11</td>
<td>1.68</td>
</tr>
</tbody>
</table>

In Table II, it has been aimed at eliminate 3rd and 7th harmonic orders. While the eliminated harmonic orders are almost equal the zero, 5th harmonic component has increased from % 0.01 to % 4.28 when it is compared to SPWM. To overcome this, 5th harmonic order has been eliminated in nine-level inverter as shown in Table III and also Fig. 7 clearly.

In addition to this, Fig. 6 illustrates the output voltage waveforms of nine-level inverter. The output voltage’s maximum rate is 48 V with four voltage step in positive side. The whole step consists of four voltage step in positive and four voltage step negative side and zero.

Fig 4. Output voltage waveforms in seven-level inverter with SPWM and SHEPWM eliminating 3rd and 7th harmonic orders

Fig 5. Elimination of 3rd and 7th harmonic orders in seven-level inverter
Fig 6. Output voltage waveforms in nine-level inverter with SPWM and SHEPWM eliminating 3rd and 7th harmonic orders

Fig 7. Elimination of 3rd, 5th and 7th harmonic orders in nine-level inverter

Fig 8. Elimination of 5th, 7th and 11th harmonic orders in nine-level inverter

Table III. Nine-level inverter % Harmonics value in SPWM and elimination of 3rd, 5th, 7th harmonic orders in SHEPWM and switching angles of SHEPWM

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>SPWM</th>
<th>SHEPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3rd</td>
<td>5th</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>0.08</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>0.22</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>0.09</td>
</tr>
<tr>
<td>9</td>
<td>1.71</td>
<td>4.48</td>
</tr>
<tr>
<td>11</td>
<td>2.62</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Table IV. Nine-level inverter % Harmonics value in SPWM and elimination of 5th, 7th, 11th harmonic orders in SHEPWM and switching angles of SHEPWM

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>SPWM</th>
<th>SHEPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5th</td>
<td>7th</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>4.71</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>0.14</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>0.12</td>
</tr>
<tr>
<td>9</td>
<td>1.71</td>
<td>3.2</td>
</tr>
<tr>
<td>11</td>
<td>2.62</td>
<td>0.1</td>
</tr>
</tbody>
</table>

VI. Conclusion

GA based selective harmonic elimination in cascaded multilevel inverter with reduced number of switches has been presented. The advantage of proposed inverter topology is to have a structure with the reduced number of switches. Therefore, the hardware cost of proposed multilevel inverter is lower and it is compact. Also, switching techniques applied to conventional multilevel inverter can be used with this topology.

SHE equations of the output voltage of seven-level and nine-level multilevel inverter have been solved by the developed GA software without the use of GA toolbox of MATLAB and applied to proposed inverter topology. The obtained results have clearly proved the effectiveness of the proposed multilevel structure and GA based SHE method.

ACKNOWLEDGMENT

This work was supported by Scientific Research Project Coordinating Office of Selçuk University (SUBAP).

REFERENCES


