Proceedings of the World Congress on Engineering and Computer Science 2015 Vol I
WCECS 2015, October 21-23, 2015, San Francisco, USA

Study on the Water Lubricated Large-scale Tilting Pad Thrust Bearing by Finite Element Method

Cheng De, Yao Zhen-Qiang, Xue Ya-bo

***Abstract*—In this paper, the finite element method is used to analyze the characteristics of the water-lubricated thrust bearing. The corresponding finite elements are derived from the turbulent Reynolds equation and turbulent energy equation by variation method and Litz-Galerkin method respectively. Based on the finite element model, the pressure field of the lubricating film is obtained. The relationship between the pad load capacity and pad tilting angle is analyzed. The temperature increment of the lubricating film decreases the load capacity of the pad.**

***Index Terms*—Hydrodynamic thrust bearing, water-lubricated, turbulent Reynolds equation**

I. INTRODUCTION

Thrust bearing is a key component of every large hydrogenerator and can also be found in many other rotational machineries like pumps, compressors, engines, marine propulsion systems etc [1]. The large-scale thrust tilting pad bearing which lubricated by water usually used in the canned pump or in the boat. For this kind of thrust bearing, the rotor spinning velocity is very high and the water viscosity is low. The Reynolds number is very high, relatively. The lubricating flow in the large-scale thrust bearing is highly turbulent [2][3] . Characterization of the tilting pad bearings is vital to successful design of high-speed rotating machinery. Theoretical models are particularly important at the design stage of modern high-speed rotating machinery. These models have evolved from full finite element numerical solutions that include analysis of the lubricating flow to the energy balance in the lubricant film. Ettles developed a TEHD analysis of tilting pad bearings. The analysis included a generalized Reynolds equation solution using the turbulence model of Constantinescu and the local calculated Reynolds number to obtain an effective viscosity. Ettles considered the transition region for turbulence in the lubricating flow to be in the range of 1100–1400[4][5][6]. In this paper, the finite element method is used to analysis the characteristics of the water-lubricated thrust bearing. The corresponding finite elements are derived from the turbulent Reynolds equation and turbulent energy equation. Based on the finite element model, the pressure field is obtained. The relationship between the pad load capacity and pad tilting angle is analyzed. The temperature field are calculated, the maximum temperature increment can be approximately 20℃. By considering the thermal influence on the viscosity of the water, the pressure field of the pad is recalculated. The high temperature of the film decreases the load capacity of the pad.

Manuscript received May 20, 2015; revised June 2, 2015. This work is supported by the National Key Basic Research Program (973 Program, grant no. 2015CB057302).

Cheng De is with State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, China (e-mail: 0100209064@sjtu.edu.cn).

Yao Zhen-Qiang is with State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, China (e-mail: zqyao@sjtu.edu.cn).

Xue Ya-bo is with State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, China (e-mail: xueyabo@sjtu.edu.cn).

II. BASIC EQUATIONS

A. Turbulent Reynolds equations

Reynolds equation is the control equation for hydrodynamic lubrication. For small-scale thrust bearing, the laminar flow Reynolds equation is suitable. However, for thrust bearing with big size and high spinning velocity, the Reynolds number in the dynamic lubricating film is very high. Under this condition, the pressure distribution and energy consumption in the lubricating film are quite different from that in the lubricating film with low Reynolds number. The turbulent Reynolds equation need to be used in the analysis of the large-scale thrust bearing. The turbulent coefficients are added in the laminar flow Reynolds equation to describe the turbulent effect in the film. Equation 1 is the modified turbulent Reynolds equation.

$$\frac{\partial}{\partial r}\left(G_r \frac{rh^3}{\mu}\frac{\partial P}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(G_\theta \frac{h^3}{\mu}\frac{\partial P}{\partial \theta}\right)=\frac{r\omega}{2}\frac{\partial h}{\partial \theta}+\frac{1}{12}\frac{\partial}{\partial r}\left(\frac{\rho r^2\omega^2 h^3}{\mu}\right) \quad (1)$$

where, h is the film thickness, ρ is the fluid density, r , θ are polar coordinates, P are the film thickness, μ is the viscosity of the lubrication, ω (rad/s)is the rotor spinning velocity. The boundary condition of the Reynolds equation: the pressure on the boundary of the pad is zero. $G_r=(12+0.0043*\mathrm{Re}^{0.98})^{-1}$, $G_\theta=(12+0.0136*\mathrm{Re}^{0.9})^{-1}$, $\mathrm{Re}=\frac{\rho r\omega h}{\mu}$。The critical Reynolds number are $\mathrm{Re}=1500$。

B. Turbulent energy equation

Equation 2 is the energy equation which are derived from the fluid energy conservation law. The temperature distribution can be obtained by solving the energy equation. Considering the turbulent effect in the lubricating film, the turbulent coefficients are added in the energy equation.

$$JC_p\rho g\left[\left(\frac{r\omega h}{2}-\frac{h^3}{12\mu}\frac{\partial P}{r\partial\theta}\right)\frac{1}{r}\frac{\partial T}{\partial\theta}-\frac{h^3}{12\mu}\left(\frac{\partial P}{\partial r}-\rho\omega^2 r\right)\frac{\partial T}{\partial r}\right]-Jkh\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\right]+KT= \frac{\mu\omega^2 r^2}{h}*\tau_c+KT_0+\frac{h^3}{k_x\mu}\left[\left(\frac{\partial P}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial P}{\partial\theta}\right)^2\right]+\frac{h^3}{12\mu}\left[\left(\rho\omega^2 r\right)^2-2\rho\omega^2 r\frac{\partial P}{\partial r}\right] \quad (2)$$

$\tau_c=1+0.0023*\mathrm{Re}^{0.855}$, $k_x=12+0.0136*\mathrm{Re}^{0.9}$. τ_c is bigger than one, which means the energy generation is higher by considering the turbulent effect.

C. oil viscosity

The oil viscosity is known at some specified temperature. The Lagrange interpolation method is used to get the oil viscosity at any temperature.

$$\mu=\sum_{i=1}^{n}\left[\prod_{\substack{j=1\\ i\neq j}}^{n}\frac{T-T_j}{T_i-T_j}\mu_i\right] \quad (3)$$

D. film thickness

The film thickness of one pad is presented by equation 4.

$$h_{ij}=h_0+m_r\left[r\cos\left(\theta-\frac{1}{2}\theta_r\right)-R_0\right]-m_\theta\sin\left(\theta-\frac{1}{2}\theta_r\right)-D_{ij}+W_{ij} \quad (4)$$

Where, h_{ij} is film thickness, h_0 is the film thickness in the middle of the pad. m_θ is the circumferential tilt angle. D_{ij} is the displacement induced by the pad deformation due to the force load and thermal expansion. R_0 is the average radius. W_{ij} is the thrust collar displacement.

III. Finite Element Model

A. Interpolating function and coordinate transformation

Eight nodes isoparametric element is used in the analysis. Figure 1 shows the element and the coordinate transformation.

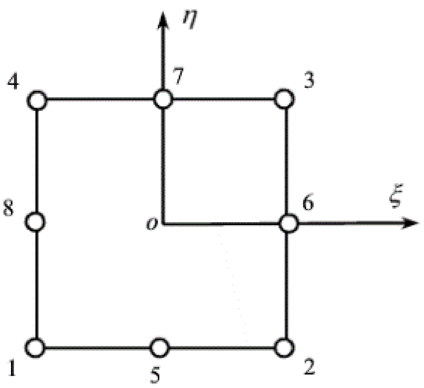


Figure 1 Eight nodes isoparametric element

The parameters at any point in the element can be expressed by the parameters at the eight nodes in the element and the interpolating function, such as the pressure, temperature, viscosity and the film thickness.

$$r=\sum_{i=1}^{8}N_i r_i,\ \theta=\sum_{i=1}^{8}N_i\theta_i,\ P=\sum_{i=1}^{8}N_i P_i,\ T=\sum_{i=1}^{8}N_i T_i,\ h=\sum_{i=1}^{8}N_i h_i,\ \mu=\sum_{i=1}^{8}N_i\mu_i \quad (5)$$

Where i denotes the i th node. N_i is the interpolating function, which is the function of ξ,η.

The transformation of the local coordinates and the global coordinates is:

$$\begin{pmatrix}\frac{\partial N}{\partial\xi}\\ \frac{\partial N}{\partial\eta}\end{pmatrix}=\begin{pmatrix}\frac{\partial r}{\partial\xi} & \frac{\partial\theta}{\partial\xi}\\ \frac{\partial r}{\partial\eta} & \frac{\partial\theta}{\partial\eta}\end{pmatrix}\begin{pmatrix}\frac{\partial N}{\partial r}\\ \frac{\partial N}{\partial\theta}\end{pmatrix}=[J]\begin{pmatrix}\frac{\partial N}{\partial r}\\ \frac{\partial N}{\partial\theta}\end{pmatrix} \quad (6)$$

Where $[J]$ is the JACOB matrix.

B. Variation of the Reynolds function

The Reynolds function's variation is

$$J(\mathrm{P})=\iint\left[\frac{G_r rh^3}{2\mu}\left(\frac{\partial P}{\partial r}\right)^2+\frac{G_\theta h^3}{2\mu r}\left(\frac{\partial P}{\partial\theta}\right)^2-\frac{r\omega h}{2}\frac{\partial P}{\partial\theta}-\frac{\rho r^2\omega^2 h^3}{12\mu}\frac{\partial P}{\partial r}\right]dA \quad (7)$$

where A is the integration area. In the element, $J^e(\mathrm{P})$ is the variation in the element and has the same mathematical form with $J(\mathrm{P})$. According to equation 8, in the element, the pressure $P=\sum_{i=1}^{8}N_i P_i$, then equation $J^e(\mathrm{P})$ can be obtained.

According to the variation principle, the solution of Reynolds equation can make the variation to reach the maximum value. $J^e(\mathrm{P})$ is the function of the pressure $P_i(\mathrm{i}=1,2,...,8)$ on the nodes. When $\delta J^e(\mathrm{P})=0$, $J^e(\mathrm{P}_1,\mathrm{P}_2,...,\mathrm{P}_8)$ will get the maximum value.

$$\frac{\partial J^e(\mathrm{P})}{\partial P_j}=0 \quad (\mathrm{j}=1,2,...,8) \quad (8)$$

According to equation 10, eight linear algebraic equations can be got which shown as equation 11.

$$[K_P]^e_{8\times 8}[P]_{8\times 1}=[Q_P]^e_{8\times 1} \quad (9)$$

Where $[K_P]^e$ is the stiffness matrix, $[Q_P]^e$ is the right column, $[P]$ is the pressure on the nodes to be solved.

For the laminar Reynolds equation, the turbulent coefficients G_r and G_θ are set to be $1/12$.

C. Finite element of the energy equation

The Litz-Galerkin method is used to derive of the finite element of the energy equation.

$$f_1=C_p\rho\left(\frac{\omega rh}{2}-\frac{h^3}{12\mu}\frac{\partial P}{r\partial\theta}\right)\quad f_2=C_p\rho\left(-\frac{h^3}{12\mu}\left(\frac{\partial P}{\partial r}-\rho\omega^2 r\right)\right)\quad f_3=K\quad f_4=-kh$$

$$f_5=\frac{\mu\omega^2 r^2\tau_c}{h}+KT_0+\frac{h^3}{k_x\mu}\left[\left(\frac{\partial P}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial P}{\partial\theta}\right)^2\right]+\frac{h^3}{12\mu}\left[\left(\rho\omega^2 r\right)^2-2\rho\omega^2 r\frac{\partial P}{\partial r}\right]$$

The energy equation can be simplified as:

$$f_1\frac{\partial T}{r\partial\theta}+f_2\frac{\partial T}{\partial r}+f_3 T+f_4\left(\frac{\partial^2 T}{r^2\partial\theta^2}+\frac{\partial^2 T}{\partial r^2}\right)=f_5 \quad (10)$$

Set $\tilde{T}=\sum_{i=1}^{8}N_i T_i$ to be the approximate solution and substituted into equation 10.

$$R=f_1\frac{\partial\tilde{T}}{r\partial\theta}+f_2\frac{\partial\tilde{T}}{\partial r}+f_3\tilde{T}+f_4\left(\frac{\partial^2\tilde{T}}{r^2\partial\theta^2}+\frac{\partial^2\tilde{T}}{\partial r^2}\right)-f_5\neq 0 \quad (11)$$

ISBN: 978-988-19253-6-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

According to the Litz-Galerkin method, the inner product of the residual R and the potential function in the element must be zero.

$$\iint N_i \mathrm{R dA}=0 \qquad (12)$$

Partial integration can be used to solve the equation 15. Eight linear algebraic equations are obtained and are written in matrix form.

$$[K_T]^e_{8\times8}[T]_{8\times1}=[F_T]^e_{8\times1} \qquad (13)$$

where $[K_T]^e$ is the stiffness matrix, $[F_T]^e$ is the right column. $[T]$ is the temperature on the element nodes, which will be determined in the numerical simulation. For laminar Reynolds equation, the coefficients $\tau_c=1, k_x=12$.

D.Integration of the finite element

In order to get the value of the stiffness matrix and right column, the Gaussian numerical integration are utilized in the local coordinates. The integration transfer is $dA = rdrd\theta = r|J|d\xi d\eta$. Gaussian points and the weight coefficients for numerical integration is shown in table 1. In this paper, n_G is chosen to be 3.

TABLE 1 GAUSSIAN POINTS AND THE WEIGHT COEFFICIENTS

n_G	x_k	A_k
0	0.0000000	2.0000000
1	±0.5773503	1.0000000
2	±0.7745967 0.0000000	0.5555556 0.8888889
3	±0.8611363 ±0.3399810	0.3478548 0.6521452

As the equations within one element are generated by Gaussian integral method, the whole matrix of the model can be assembled according to the connectivity of the elements.

$$[K_P]_{n\times n}[P]_{n\times1}=[F_P]_{n\times1} \qquad (14)$$

where $[K_P]=\sum_{e=1}^{m}[K_p]^e$; $[\mathrm{F}_P]=\sum_{e=1}^{m}[F_p]^e$; m is the number of elements, n is the number of nodes.

Based on the distribution of the pressure in the lubricating film which is obtained by the finite element method, the temperature distribution can be solved. The important parameters of the thrust bearing can be calculated by the following equations.

(a) The pad load: $F=\iint prdrd\theta$ (15)

(b) The center of the pad load:

The moment of the pad load to the origin of global coordinate can be evaluated by the following equation:

$$M_Y=\sum_{i=1}^{m}M^e_{Y_i},\quad M_X=\sum_{i=1}^{m}M^e_{X_i}, \qquad (16)$$

where $M^e_Y=\iint_\Omega \Pr\cos\theta dA$, $M^e_X=\iint_\Omega \Pr\sin\theta dA$

Then the center of the pad load is

$$r_\rho=\sqrt{M_X^2+M_Y^2}/\omega,\quad \theta=\tan^{-1}(\mathrm{M_X}/\mathrm{M}_Y) \qquad (17)$$

IV. CHARACTERISTICS OF THRUST BEARING LUBRICATED BY WATER

A. Work condition of the thrust bearing

There are six pads in a thrust bearing. The thrust bearing's characteristics are listed in table 2.

TABLE 2 THE PARAMETERS ABOUT THE THRUST BEARING

Number of pads N	6
Inner radius R_1	0.18m
Outer radius R_2	0.42m
Pad angle α	50°
Pad thickness Z_b	45mm
Initial film center thickness h_0	40 μm
Lubricant oil density ρ	1000 kg/m^3
Water specific heat capacity Cv	4200J/（kg*℃）
Water viscosity μ	0.001(Pa*s)（20℃）

B. The pressure distribution in the pad

The finite element model are shown in figure 2.

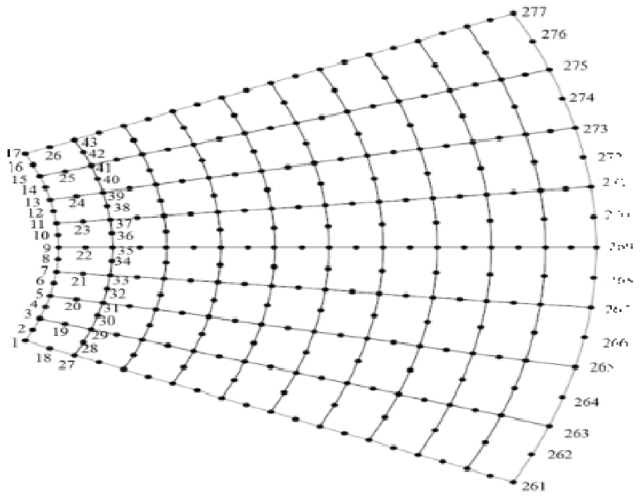


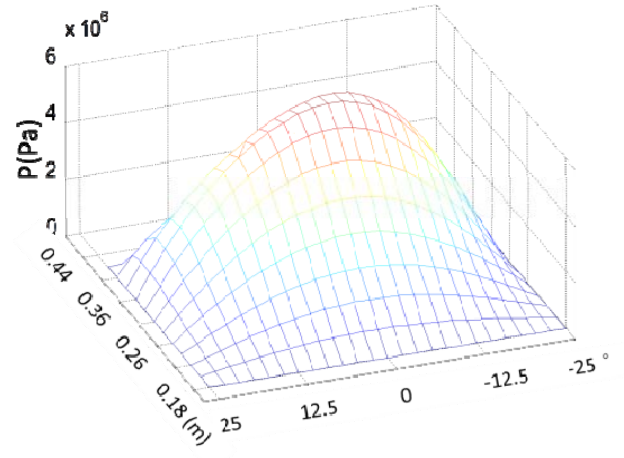
Figure 2 the finite element model of the lubricating film for one pad

In the model, there are 10 elements along the radial direction and 8 elements along the circumferential direction. The total number of the elements is 80. For the first step of the simulation, the pressure distribution are calculated without considering the thermal effect.

The initial average film thickness at the pad center is set to be $h_0=40\mu m$. The range of m_θ is $[-0.005^\circ, 0.007^\circ]$. The range of m_r is $[-0.008^\circ, 0.007^\circ]$. At different pad tilt angle, the pressure distribution are calculated.

Figure 3 is the pressure distribution under specified condition. For bigger m_r, the maximum pressure is bigger, too.

ISBN: 978-988-19253-6-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)



$m_\theta = 0.004^\circ, m_r = -0.008$

Figure 3 the pressure distribution under specified condition

The relationship between the pad load and tilt angle is shown in figure 4.

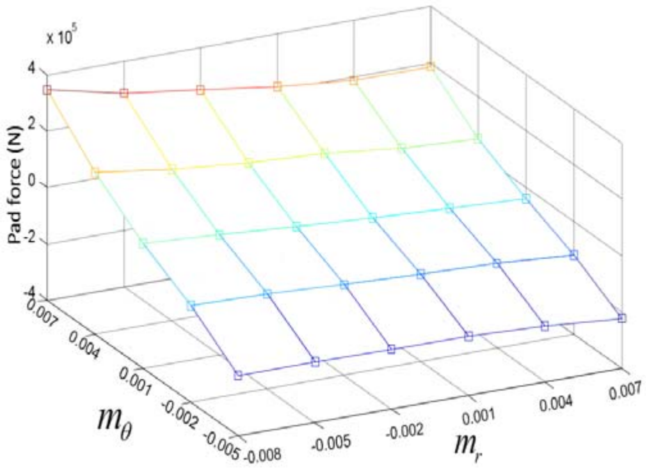


Figure 4 the relation between the pad load and tilt angle

From figure 4, we can concluded that the pad load increases as the m_θ increase. When $m_\theta < 0$, the pressure is negative which means the lubricating film does not exist. When m_r decrease, the pad load increase, due to the inertia effect of the film.

The relationship between the load center and the pad tilt angle.

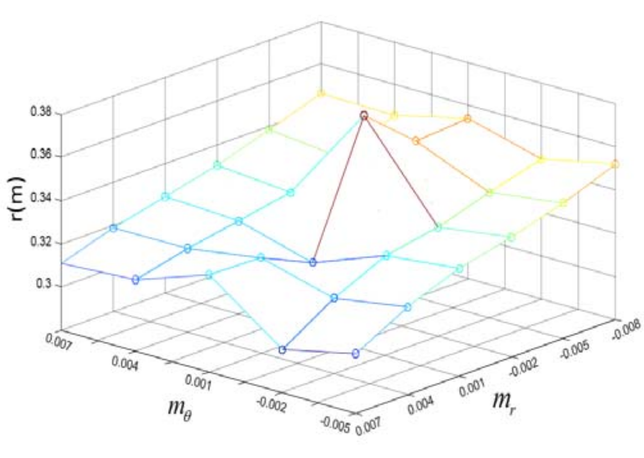


Figure 5 Relationship of the radial coordinate of the load center with the pad tilt angle m_r , m_θ

In figure 5, when m_r increase, the radial coordinate of the load center increases. So when considering the inertia force of the film, the increase of the pad supporting point's radial coordinate can improve the pad's load capacity.

C. *The temperature field of the lubricating film*

The inlet temperature of the water is 25℃。By solving the energy equation, the temperature distribution can be obtained. The water temperature can change the water viscosity and influent the pressure distribution. In this paper, for the first iteration, the film temperature is set to be 25℃. And the pressure distribution is calculated. The pressure distribution is used to calculate the film temperature. Then for the second iteration, the temperature distribution are used to modify the water viscosity and get the new pressure distribution. After four iterations, the pressure distribution and temperature distribution are all converged. The temperature field in the middle of the pad are shown in figure 6. The film temperature decrease the load capacity of the pad.

Considering the film temperature, the pressure field changes greatly. The pad load capacity decreases.

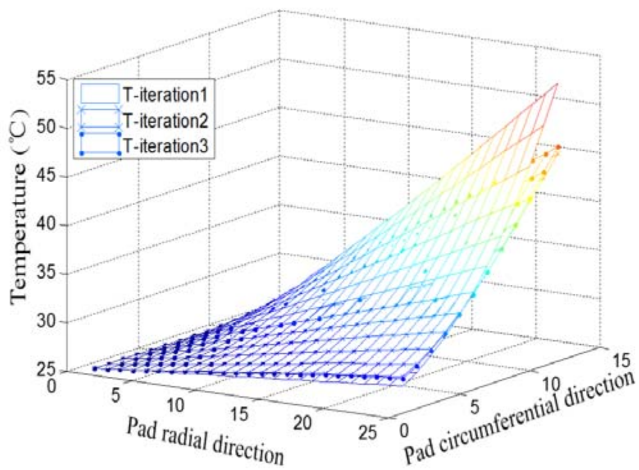


Figure 6 the pad temperature field

$(m_\theta = 0.001^\circ, h_r = 20\mu m)$

After several iterations, the temperature converges. As shown in figure 6, when $m_\theta = 0.001^\circ, h_r = 20\mu m$, the maximum temperature increment in the film is about 20℃.

V. Conclusion

In this paper, the finite element method is used to solve the turbulent Reynolds equation and the turbulent energy equation of the large scale water lubricated tilting pad thrust bearing. In the Reynolds equation, the film inertia effect is considered. The pad pressure field are obtained. The relationship between the pad load capacity and the pad tilting angle are analyzed. The temperature field are calculated. when $m_\theta = 0.001^\circ, h_r = 20\mu m$, the maximum temperature increment can be approximately 20℃. By considering the thermal influence on the viscosity of the water, the pressure field of the pad is recalculated. The high temperature of the film decreases the load capacity of the pad.

ISBN: 978-988-19253-6-7
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

ACKNOWLEDGMENT

This work is supported by the National Key Basic Research Program (973 Program, grant no. 2015CB057302).

REFERENCES

[1] Pajączkowski, P., Schubert, A., Wasilczuk, M., & Wodtke, M. (2014). Simulation of large thrust-bearing performance at transient states, warm and cold start-up. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, 228(1), 96-103.

[2] W.-K. Chen, Linear Networks and Systems (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.

[3] H. Poor, An Introduction to Signal Detection and Estimation. New York: Springer-Verlag, 1985, ch. 4.

[4] B. Smith, "An approach to graphs of linear forms (Unpublished work style)," unpublished.

[5] E. H. Miller, "A note on reflector arrays (Periodical style—Accepted for publication)," Engineering Letters, to be published.

[6] J. Wang, "Fundamentals of erbium-doped fiber amplifiers arrays (Periodical style—Submitted for publication)," IAENG International Journal of Applied Mathematics, submitted for publication.

[7] C. J. Kaufman, Rocky Mountain Research Lab., Boulder, CO, private communication, May 1995.

[8] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces (Translation Journals style)," IEEE Transl. J. Magn.Jpn., vol. 2, Aug. 1987, pp. 740–741 [Dig. 9th Annu. Conf. Magnetics Japan, 1982, p. 301].