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PAPR Reduction using CPM Mappers and SLM in OFDM Systems

Emammer Shafter and Raveendra K. Rao

Abstract—**Orthogonal Frequency Division Multiplexing (OFDM) system with Continuous Phase Modulation (CPM) mappers is considered. Such a system is known to provide superior bit error rate performance by virtue of the memory introduced by the CPM mapper compared to OFDM systems with conventional memoryless mappers such as BPSK and QPSK. In this paper, the ability of CPM mappers in an OFDM system to reduce Peak-to-Average Power Ratio (PAPR) is examined as a function of modulation parameters of mappers and the number of subcarriers used in the system. Firstly, three subclasses of CPM mappers, namely, single-h CPFSK, multi-h CPFSK, and Asymmetric multi-h CPFSK are considered and their PAPR performance is assessed. Next, these mappers in conjunction with SLM technique are considered and their PAPR performance is examined. A comparison of the PAPR reduction capability of CPM mappers relative to memoryless BPSK mappers in an OFDM system is presented. It is noted that, in general, CPM mappers offer superior PAPR performance compared to memoryless mappers in an OFDM system. Also, it is shown that CPM mappers with SLM technique in an OFDM system can offer significant improvement in PAPR performance.**

Index Terms—**Orthogonal Frequency Division Multiplexing (OFDM), Continuous Phase Modulation (CPM), CPFSK, Peak-to-Average Power Ratio (PAPR), and Selective Mapping (SLM).**

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) is a type of multicarrier modulation technique used for transmission of data symbols in parallel on multiple subcarriers that share the system bandwidth. OFDM is used for high data rate wireless transmission because of its high spectral efficiency, robustness to interference and fading inherent in multi-path channels, and ease of efficient hardware implementation using FFT techniques. Thus, OFDM systems have been adopted for a number of applications such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), and IEEE 802.11 wireless local area networks (WLAN)[1], [3]. It is well recognized that one of the major drawbacks of OFDM transmission system, however, is its high Peak-to-Average Power Ratio (PAPR) [4], [6] of the transmitted signal, which may result in nonlinear distortions and hence potentially causing degradations in the performance of the system. The use of large number of subcarriers introduces a high PAPR in OFDM systems. PAPR can defined as the relationship between the maximum power of sample in transmit OFDM symbol and it's average power. When a large number of subcarriers are out of phase, significant PAPR can cause the transmitter's power amplifier (PA) to run within non-linear operating region. This causes significant signal distortion at the output of the power amplifier. Moreover, the high PAPR can cause saturation at the digital-to-analog converter (DAC), leading to saturation of PA. Thus, many PAPR reduction techniques have been developed, such as clipping, coding, tone reservation, tone injection, partial transmit sequence (PTS), active constellation extension (ACE), and selective mapping (SLM) [6], [9]. Among these, SLM is one technique that is easy-to-implement and introduces no distortion in the transmitted signal. A drawback of SLM technique is that it requires side information to be transmitted to the receiver and also its computational complexity increases linearly as the number of phase sequences. In the literature, SLM technique has been extensively examined in OFDM systems with memoryless mappers such as BPSK, QPSK, QAM etc. [10], [11]. The intent of this paper is to introduce mappers with memory in an OFDM system with dual purpose: i) to enhance bit error probability performance of the system; and ii) to reduce PAPR of the transmitted OFDM signal. In particular, we introduce CPM mappers in OFDM system. The advantage of using such a mapper is that it possible to systematically introduce memory amongst adjacent OFDM symbols through an appropriate choice of modulation parameters. It is known that the use of CPM mapper in an OFDM system can enhance the bit error probability performance of the system by virtue of the memory introduced [4], [10]. Thus, in the paper, the PAPR properties of OFDM signals with CPM mapper are examined with and without SLM technique [12]. In particular, three subclasses of CPM mappers are described, single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK in an OFDM system are considered.

The paper is organized as follows: In Section II a general definition of PAPR in an OFDM system is given. In Section III, OFDM system with CPM mapper is briefly discussed and the three subclasses of CPM mappers described. Section IV deals with the description of CPM mapper with SLM technique. In Section V numerical results and their discussion is provided and the paper is concluded in Section VI with suggestions for further work.

II. DEFINITION OF PAPR

A multicarrier signal is the sum of many independently modulated signals. Denoting the collection of data symbols $C_n, n = 0, 1, , N-1$, as a vector $C = [C_0, C_1, ..., C_{N-1}]^T$, the complex baseband representation of a multicarrier signal can be written as:

$$S(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \ \exp^{j2\pi n \triangle ft}, 0 \le t < NT, \quad (1)$$

where , $\Delta f (= 1/NT)$ is the subcarrier spacing, NT is the data block period, and N is the number of subcarriers in

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the system. The Peak-to-Average Power Ratio (PAPR) of the OFDM signal can then be defined as the ratio of the maximum power to that of the average power, and is given by

$$PAPR = \frac{max|S(t)|^2}{\frac{1}{NT}\int_0^{NT}|S(t)|^2dt}, \ 0 \le t \le NT \quad (2)$$

For computation of PAPR in NL equidistant samples of $S(t)$ will be considered where L is an integer greater than or equal to 1. These L-times oversampled signal samples are represented as a vector $S = [S_0, S_1, ..., S_{NL-1}]^T$ and can be written as

$$S_k = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} C_n \ \exp^{j2\pi kn\Delta fT/L}, k = 0, 1, ...NL-1 \quad (3)$$

It is noted that the sequence S_k can be interpreted as the inverse discrete Fourier transform $(IDFT)$ of data block C with $(L-1)N$ zero padding. In fact, for an accurate measure of the PAPR the signal is samppled with $L = 4$, and the experssion for PAPR is given by

$$PAPR = \frac{max|S_k|^2}{E[|S_k|^2]}, \ 0 \le k \le LN-1 \quad (4)$$

where E[.] is the average power. The sampling rate is the Nyquist rate or a multiple of it. It has been proved that using an oversampling of 4 results in discrete time PAPR that closely matches the continuous time[7].

III. OFDM System with CPM Mapper

The block diagram of a portion of the OFDM transmitter that employs CPM modulator/mapper is shown in Fig. 1. The data stream is fed to the S/P block to get parallel stream of data bits $a_{p,k}$. The CPM mapper/modulator then accepts data bits $a_{p,k}$, $p = 1, 2, 3, ..,$ and $k = 0, 1, ..N-1$, and produces mapped symbols $C_{p,k}$. The suffix p denotes the OFDM symbol number and k the subcarrier number. The parallel output from the IFFT block is then converted to a serial stream by the parallel-to-serial (P/S) block and then the cyclic prefix is added to produce $S_p^{(CP)}$ signal for transmission. Next, we provide descriptions of three types of CPM mappers namely: single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK mappers.

A. OFDM systems with single-h CPFSK Mapper

The parameter h defines the CPFSK mapper and takes values between $0 < h < 1$ and is a ratio of two integers numbers P and Q, i.e., $h = \frac{P}{Q}$.
The quantity h is referred to as the modulation index. The choice of h determines the number of phase states in the mapper. As an example, consider the bits along the kth subcarrier, $a_{1,k}, a_{2,k}, ...$, where $a_{i,k} = \pm 1$ for $i = 1, 2, ...$ of a single-h CPFSK mapper. Then the number of possible phase states, $\theta_{p,k}$ for $h = \frac{1}{2}, \frac{2}{3}$, and $\frac{1}{4}$ would be 4, 3, and 8, respectively. Fig. 2, shows the constellation diagram for $h = \frac{1}{2}$ and Table I shows the possible phases for $h = \frac{1}{2}, \frac{2}{3}$ and $\frac{1}{4}$ [11].

In single-h CPFSK mapper, the value of h is fixed for all OFDM symbols [4]-[10]. The expression for $C_{p,k}$ is given by

$$C_{p,k} = \cos(\theta_{p,k}) + j\sin(\theta_{p,k}) \quad (5)$$

where

$$\theta_{p,k} = a_{p,k} \ \pi \ h + \pi \ h \sum_{q=0}^{p-1} a_{q,k} + \phi \quad (6)$$

and ϕ is the initial phase set equal to zero without lose of generality for a coherent system.

TABLE I: Possible phases states for single-h CPFSK mapper for $h = \frac{1}{2}$, $h = \frac{2}{3}$, and $h = \frac{1}{4}$

$h = \frac{P}{Q}$	$\theta_{p,k}$
$\frac{1}{2}$	$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
$\frac{2}{3}$	$0, \frac{2\pi}{3}, \frac{4\pi}{3}$
$\frac{1}{4}$	$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

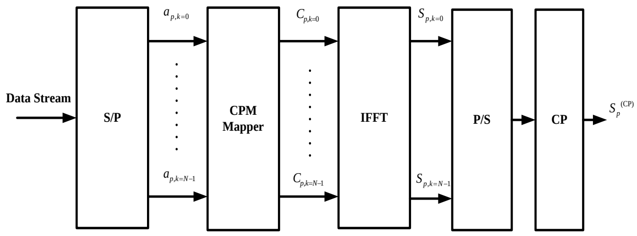

Fig. 1: Portion of OFDM transmitter with CPM mapper

B. OFDM systems with multi-h CPFSK Mapper

In the multi-h CPFSK mapper, we vary the value of h from symbol to symbol. The parameter h is cyclically chosen from a set H_K of K values, $\{h_1, h_2, ..., h_K\}$.
The expression for $\theta_{p,k}$ for this mapper is given by

$$\theta_{p,k} = \begin{cases} a_{p,k}\pi h_{[k]} + \sum_{q=0}^{p-1} a_{q,k}\pi h_{[q]} + \phi, & k > 1 \\ a_{1,k}\pi h_{[1]} + \phi, & k = 1 \end{cases}$$

For illustration, we take the first four symbols for an arbitray kth subcarrier with $H_2 = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and data sequence $a_{p,k} = [+1, +1, -1, +1]$. Assuming the initial phase to be zero, then the number of possible phase states, $\theta_{p,k}$ for $h = \frac{2}{3}$ and $\frac{1}{4}$ would be 4 and 3 respectively. [4]-[10]

C. OFDM systems with Asymmetric multi-h CPFSK

While in multi-h CPFSK, h values are chosen independently of data bits $a_{p,k} = (\pm 1)$, in this case we choose h a function of $a_{p,k}$. That is, the value of h during the ith symbol interval is chosen h_{+i} or h_{-i} accordingly as data is a $+1$ or -1 respectively. For this mapper, the expression for θ_p, k is given by

$$\theta_{p,k} = \begin{cases} a_{p,k}\pi h_{\pm[k]} + \sum_{q=0}^{p-1} a_{q,k}\pi h_{\pm[q]} + \phi, & k > 1 \\ a_{1,k}\pi h_{\pm[1]} + \phi, & k = 1 \end{cases}$$

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This gives additional flexibility to the designers to enhance system performance. Let the h values employed for data a ±1 be $H_{+i} = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and the ones for data -1 be $H_{-i} = \left\{\frac{1}{4}, \frac{2}{3}\right\}$. Then the number of possible phase states, $\theta_{p,k}$ for $H_{+i} = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and $H_{-i} = \left\{\frac{1}{4}, \frac{2}{3}\right\}$ would be 4 and 3 respectively [4]-[10].

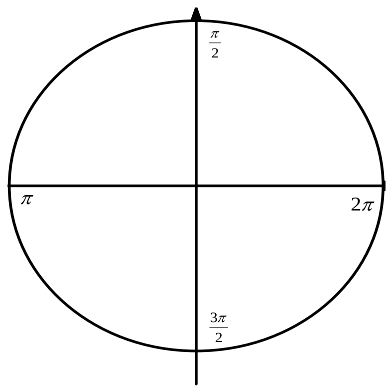

Fig. 2: Constellation diagram of Single-h CPM mapper for h=1/2

IV. CPM Mapper with SLM Technique

The block diagram of the CPM mapper with SLM technique is shown in Fig. 4. The data stream which is generated from the CPM mapper $[C_{p,k=0}, ...C_{p,k=N-1}]^T$ is multiplied with different phase sequences and fed to the IFFT block to produce OFDM symbols as shown in Fig.4. One of these OFDM symbols will have minumum PAPR which is selected and transmitted. Suppose the CPM mapper output is a vector $[C_{p,k=0}, ..., C_{p,k=N-1}]^T$, then this vector is multiplied by U different phase sequences, each of length N, $B^{(u)} = [b_{u,0}, b_{u,1}, ..., b_{u,N-1}]^T, u = 1, 2, ..., U,$ resulting in U modified data blocks. The modified data block for the uth phase sequence is represented as $[C_0 b_{u,0}, C_1 b_{u,1}, ..., C_{N-1} b_{u,N-1}]^T$, $u = 1, 2, ..., U$. The output of the IFFT block for this modified data with an oversampling factor of L is given by

$$S_k^{(u)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n b_{u,n}\, e^{j2\pi kn\Delta fT/L}, u = 1, 2, ..., U. \tag{7}$$

The PAPR for each $\{S_k^{(u)}, k = 0, 1, ..., NL-1\}, u = 1, 2, ..., U,$, block is computed and then the minumum of these is chosen for transmission. That is,

$$\min_{1 \le u \le U} \{S_k^{(u)}, k = 0, 1, ..., NL-1\} \tag{8}$$

It is noted that when using SLM the transmitter needs to convey to the receiver the value of u.

V. Numerical Results

The PAPR performance of CPM mapper in OFDM system has been analyzed using simulations in MATLAB. In Fig.4, the complementary cumulative distribution function (CCDF) has been plotted to show PAPR performances for PBSK and single-h CPFSK ($h = \frac{1}{2}$) mappers for an OFDM system with 256 sub-carriers. Also, in the same figure are shown the performances of these two mappers with SLM. These

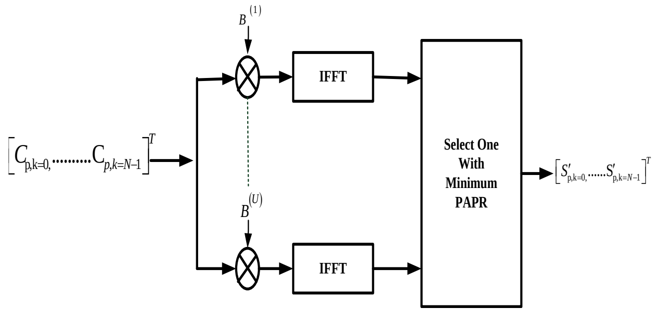

Fig. 3: A block diagram of CPM mapper with SLM technique

plots have been arrived at by examining 10,000 random OFDM symbols. It is obsereved that the OFDM system with single-h CPFSK ($h = \frac{1}{2}$) mapper has a PAPR that exceeds 11.1 dB for less than 0.1 percent of data blocks and for PBSK mapper it is 13.7 dB. However, when SLM is used with these mappers the PAPR reduces to 8.02 dB and 9.2 dB for single-h CPFSK and BPSK mappers, respectively. Thus, it is noted that single-h CPFSK mapper with SLM can offer an improvement in PAPR of nearly 3 dB relative to corresponding system without SLM. Also, it is noted that the improvment in PAPR by using SLM in these two systems are 3.0 dB and 4.5 dB for single-h CPFSK and BPSK mappers, respectively. In Figs.5(a)-5(d), PAPR performances for an OFDM system with 64 subcarriers for $h = \frac{1}{5}, \frac{1}{3}, \frac{1}{2}$,and$\frac{4}{5}$ are shown. Also in these figures performances of BPSK with 64 subcarriers are shown. These figures show that the PAPR performance varies as a function of h. It is noted that $h = \frac{1}{2}$ performs the best among the four h values used. In order to understand the effect of SLM in conjunction with single-h CPFSK mapper, in Figs.6(a) and 6(b) PAPR performance for $h = \frac{1}{2}$ with and witout SLM are plotted as a function of number of subcarriers in the system. It is noted from these figures that single-h CPFSK mapper, in general offers PAPR performance superior to BPSK with and without SLM. Fig.7 shows PAPR performance of multi-h CPFSK mapper for 256 subcarrier OFDM system. The set of modulation parameters used is $(\frac{2}{3}, \frac{1}{4})$. It is noted that this specific multi-h CPFSK mapper performs nearly same as that of BPSK. However, when SLM is used with these systems multi-h CPFSK outperforms BPSK by nearly more than 1dB.

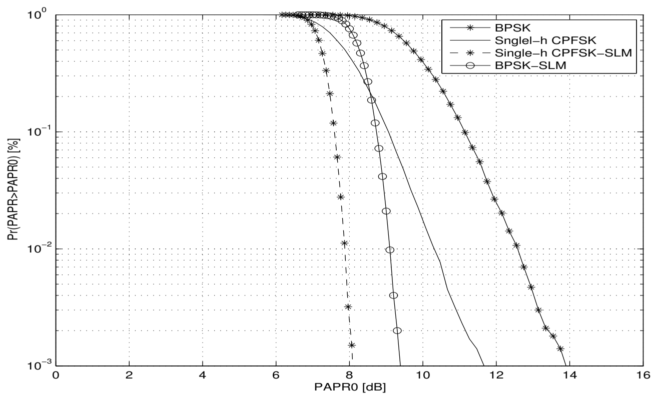

Fig. 4: PAPR Performance of 256 subcarrier OFDM system with single-h CPFSK mapper $(h = \frac{1}{2})$ and SLM

Fig.8, shows CCDFs for BPSK, BPSK with SLM, asymmetric multi-h CPFSK, and asymmetric multi-h CPFSK with

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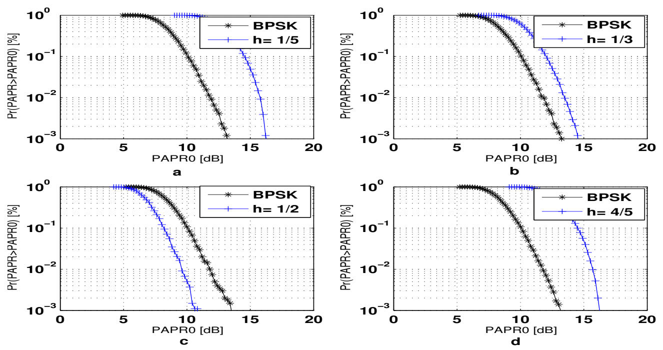

Fig. 5: PAPR Performance of 64 subcarrier OFDM system with single-h CPFSK mapper: (a) $h = \frac{1}{5}$; (b) $h = \frac{1}{3}$; (c) $h = \frac{1}{2}$; and (d) $h = \frac{4}{5}$

TABLE II: Comparision of PAPR performance between single-h CPFSK and BPSK mappers with and without SLM, as a function of number of subcarriers

Number of subcarrier	Mapper			
	PBSK		Single-h CPFSK	
	Without SLM (dB)	With SLM (dB)	Without SLM (dB)	With SLM (dB)
64	13.1	7.9	10.6	6.8
128	13.2	8.6	11.2	7.5
256	13.5	9.3	11.3	8.1
512	14.1	9.9	11.4	8.5

TABLE III: Comparision of PAPR performance between Multi-h CPFSK and BPSK mappers with and without SLM, as a function of number of subcarriers

Number of subcarrier	Mapper			
	PBSK		Multi-h CPFSK	
	Without SLM (dB)	With SLM (dB)	Without SLM (dB)	With SLM (dB)
64	13.2	7.9	12.2	6.9
128	13.6	8.7	13.2	7.5
256	13.8	9.4	13.5	8.1
512	14.3	9.9	14	8.5

TABLE IV: Comparision of PAPR performance between asymmetric multi-h CPFSK and BPSK mappers with and without SLM, as a function of number of subcarriers

Number of subcarrier	Mapper			
	PBSK		Asymmetric Multi-h CPFSK	
	Without SLM (dB)	With SLM (dB)	Without SLM (dB)	With SLM (dB)
64	13.3	7.9	11	6.9
128	13.4	8.6	11.3	7.4
256	13.8	9.4	11.7	8.1
512	14.5	9.9	11.8	8.5

SLM systems. These CCDFs show PAPR performances for 256 subcarrier OFDM system. The modulation parameters used in the asymmetric multi-h CPFSK mapper are $H_{+i} = \{\frac{2}{3}, \frac{1}{4}\}$ and $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$. It is noted that the difference in PAPR performance between asymmetric multi-h CPFSK with and without SLM is nearly 4dB. The difference in PAPR performance between multi-h CPFSK and asymmetric multi-h CPFSK mappers is approximately 1.4 dB, for an OFDM system with 256 subcarriers. The PAPR performance of the three CPM mappers are summarized in Table II to IV, as

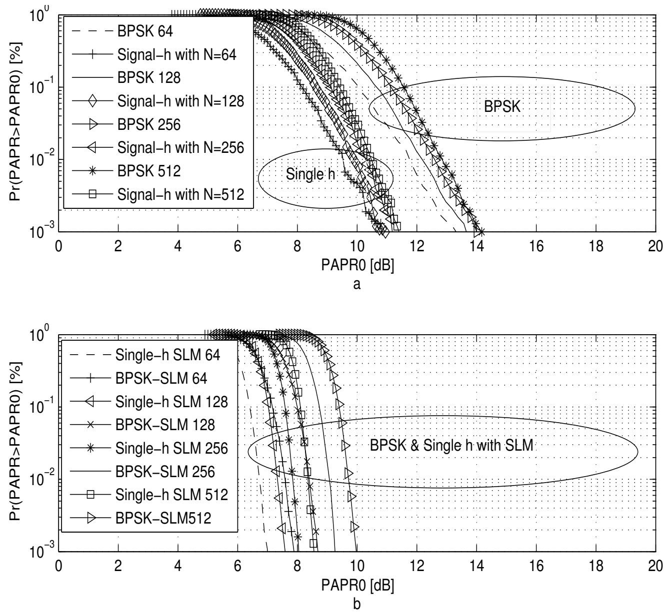

Fig. 6: PAPR Performance as a function of number of subcarrier for single-h CPFSK mapper ($h = \frac{1}{2}$); (a) without SLM; (b) with SLM

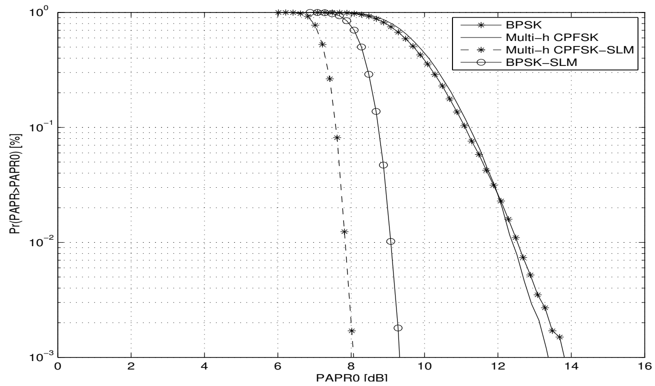

Fig. 7: PAPR Performance of 256 subcarrier OFDM system with$\{\frac{2}{3}, \frac{1}{4}\}$ multi-h CPFSK mapper and SLM

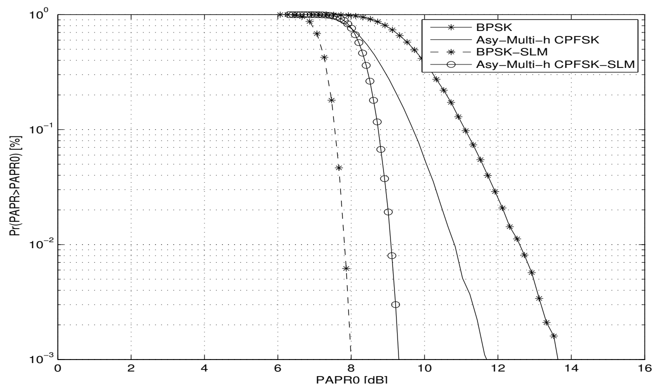

Fig. 8: PAPR Performance of 256 subcarrier OFDM system with $H_{+i} = \{2/3, 1/4\}$ and $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ asymmetric multi-h CPFSK mapper and SLM

a function of number of subcarriers. Also in these Tables performances of corresponding BPSK systems are given.

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VI. CONCLUSIONS

In this paper, mappers with memory are introduced in an OFDM system. In particular, CPM mappers are considered which can be used to introduce memory systematically through an appropiate choice of modulation parameters. Three subclasses of CPM mappers- single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK- are considered in an OFDM system. The ability of these mappers to achieve lower PAPR relative to conventional mappers such as BPSK is assessed through extensive simulations. Also, the gains in PAPR that are achievable by these mappers in conjunction with well-known SLM technique are determined. In general, it is observed that CPM mappers with SLM can be very effective in OFDM systems as far as PAPR performance is concerned relative to memory-less mappers. In an extensive study needs to be carried out to detemine the optimum CPM mappers with least values of PAPR. Also, one needs to determine best CPM mappers in OFDM systems that achieve not only least probabality of bit error but also least PAPR. It is worthwhile considering combined coding and weighting and other PAPR reduction techniques with CPM mappers in an OFDM system to further reduce PAPR. Also, it would be interesting to obtain analytical bounds on PAPR when CPM mappers are used.

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