The Reduction of PAPR using CPM Mappers and Partial Transmit Sequence (PTS) Scheme in OFDM Systems

Emammer Shafter and Raveendra K. Rao

***Abstract*—High peak-to-average power ratio of the transmit signal is a major drawback of multicarrier transmission such as OFDM. This paper describes Orthogonal Frequency Division Multiplexing (OFDM) system with Continuous Phase Modulation (CPM) mappers is considered. Such a system is known to provide superior bit error rate performance by virtue of the memory introduced by the CPM mapper compared to OFDM systems with conventional memoryless mappers such as BPSK and QPSK. In this paper, the ability of CPM mappers in an OFDM system to reduce Peak-to-Average Power Ratio (PAPR) is examined as a function of modulation parameters of mappers. Firstly, three subclasses of CPM mappers, namely, single-h CPFSK, multi-h CPFSK, and Asymmetric multi-h CPFSK are considered and their PAPR performance is assessed. Next, these mappers in conjunction with PTS technique are considered and their PAPR performance is examined. Partial transmit sequence (PTS) technique can improve the PAPR statistics of OFDM signals. In the PTS technique, the data block to be transmitted is partitioned into disjoint subblocks and the subblocks are combined using phase factors to minimize PAPR. A comparison of the PAPR reduction capability of CPM mappers relative to memoryless BPSK mappers in an OFDM system is presented. It is noted that, in general, CPM mappers offer superior PAPR performance compared to memoryless mappers in an OFDM system. Also, it is shown that CPM mappers with PTS technique in an OFDM system can offer significant improvement in PAPR performance.**

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Continuous Phase Modulation (CPM), CPFSK, Peak-to-Average Power Ratio (PAPR), and Partial Transmit Sequence (PTS).**

I. Introduction

HIGH peak-to-average power ratio (PAPR) is a well known drawback of orthogonal frequency-division multiplexing (OFDM) system. High peaks of OFDM signals occur when the sinusoidal signals of the subcarriers are added constructively [1]. These high peaks necessitate using larger and expensive linear power amplifiers. Since high peaks occur irregularly and infrequently, this means that power amplifiers will be operating inefficiently [2]. A large number of solutions have been proposed to solve the PAPR problem in OFDM. An overview of different techniques such as clipping, coding, tone reservation, tone injection, active constellation extension, (ACE), selective mapping (SLM) [3], and partial transmit sequence (PTS) can be found in [4], [5], and [6]. Also, Since the initial publication of the PTS scheme, many proposals have been made such as PTS phase optimization[7], [8], techniques to obviate the transmission of side information[9], [10], SLM/PTS combination approaches [11], extensions to MIMO-OFDM [12], and biased subcarrier [13]. In the PTS technique, an original input data block of N symbols is partitioned into some disjoint subblocks. The subcarriers in each subblock are weighted by a phase factor, to generate different signals representing the same information that in original signal. The phase factors are selected such that the PAPR of the combined signal is minimized. The phase factor which minimized the PAPR of combined signal is called the optimum phase factor. The PTS scheme can reduce the PAPR without signal distortion. The PTS requires an exhaustive search over all combinations of allowed phase factors, therefore the search complexity increases exponentially with the number of subblocks. Thus, for larger number of subblocks, the PTS scheme has high computational complexity. PTS technique has been extensively examined in OFDM systems with memoryless mappers such as BPSK, QPSK, QAM etc. [14]. The intent of this paper is to introduce mappers with memory in an OFDM system with dual purpose: i) to enhance bit error probability performance of the system; and ii) to reduce PAPR of the transmitted OFDM signal. In particular, we introduce CPM mappers in OFDM system. The advantage of using such a mapper is that it possible to systematically introduce memory amongst adjacent OFDM symbols through an appropriate choice of modulation parameters. It is known that the use of CPM mapper in an OFDM system can enhance the bit error probability performance of the system by virtue of the memory introduced [15], [16]. Thus, in the paper, the PAPR properties of OFDM signals with CPM mapper are examined with and without PTS technique. In particular, three subclasses of CPM mappers are described, single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK in an OFDM system are considered.

The paper is organized as follows: In Section II a Peak-to-Average Power Rario in an OFDM system is given. In Section III, System Description is briefly discussed and the three subclasses of CPM mappers described. PTS technique is given in Section IV. In Section V numerical results and their discussion is provided and the paper is concluded in Section VI with suggestions for further work.

II. Peak-to-Average Power Ratio

A multicarrier signal is the sum of many independently modulated signals. Denoting the collection of data symbols $C_n, n = 0, 1, , N-1$, as a vector $C = [C_0, C_1, ..., C_{N-1}]^T$, the complex baseband representation of a multicarrier signal

Emammer Shafter and Raveendra K. Rao are with the Innovation Centre for Information Engineering (ICIE), Department of Electrical and Computer Engineering, Faculty of Engineering, the University of Western Ontario, London, Ontario, N6A 5B9, Canada (Email:{eshafter, rrao}@uwo.ca)

can be written as:

$$X(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \exp^{j2\pi n \triangle ft}, 0 \leq t < NT, \quad (1)$$

where , $\Delta f(= 1/NT)$ is the subcarrier spacing, NT is the data block period, and N is the number of subcarriers in the system. The Peak-to-Average Power Ratio (PAPR) of the OFDM signal can then be defined as the ratio of the maximum power to that of the average power, and is given by

$$PAPR = \frac{max|X(t)|^2}{\frac{1}{NT}\int_0^{NT} |X(t)|^2 dt}, \ 0 \leq t \leq NT \quad (2)$$

For computation of PAPR in NL equidistant samples of $X(t)$ will be considered where L is an integer greater than or equal to 1. These L-times oversampled signal samples are represented as a vector $X = [X_0, X_1, ..., X_{NL-1}]^T$ and can be written as

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \exp^{j2\pi kn\Delta fT/L}, k = 0, 1, ...NL-1 \quad (3)$$

It is noted that the sequence X_k can be interpreted as the inverse discrete Fourier transform $(IDFT)$ of data block C with $(L-1)N$ zero padding. In fact, for an accurate measure of the PAPR the signal is sampled with $L = 4$, and the experssion for PAPR is given by

$$PAPR = \frac{max|X_k|^2}{E[|X_k|^2]}, \ 0 \leq k \leq LN-1 \quad (4)$$

where E[.] is the average power.

III. System Description

The block diagram of a portion of the OFDM transmitter that employs CPM modulator/mapper is shown in Fig. 1. The data stream is fed to the S/P block to get parallel stream of data bits $a_{p,k}$. The CPM mapper/modulator then accepts data bits $a_{p,k}$, $p = 1, 2, 3, ..,$ and $k = 0, 1, ..N-1$, and produces mapped symbols $C_{p,k}$. The suffix p denotes the OFDM symbol number and k the subcarrier number. The parallel output from the IFFT block is then converted to a serial stream by the parallel-to-serial (P/S) block and then the cyclic prefix is added to produce $X_p^{(CP)}$ signal for transmission. Next, we provide descriptions of three types of CPM mappers namely: single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK mappers.

A. single-h CPFSK Mapper

The parameter h defines the CPFSK mapper and takes values between $0 < h < 1$ and is a ratio of two integers numbers P and Q, i.e., $h = \frac{P}{Q}$.
The quantity h is referred to as the modulation index. The choice of h determines the number of phase states in the mapper. As an example, consider the bits along the kth subcarrier, $a_{1,k}, a_{2,k}, ...,$ where $a_{i,k} = \pm 1$ for $i = 1, 2, ...$ of a single-h CPFSK mapper. Then the number of possible phase states, $\theta_{p,k}$ for $h = \frac{1}{2}, \frac{2}{3}$, and $\frac{1}{4}$ would be 4, 3, and 8, respectively.
In single-h CPFSK mapper, the value of h is fixed for all OFDM symbols [15], [16]. The expression for $C_{p,k}$ is given by

$$C_{p,k} = \cos(\theta_{p,k}) + j\sin(\theta_{p,k}) \quad (5)$$

where

$$\theta_{p,k} = a_{p,k} \ \pi h + \pi h \sum_{q=0}^{p-1} a_{q,k} + \phi \quad (6)$$

and ϕ is the initial phase set equal to zero without lose of generality for a coherent system.

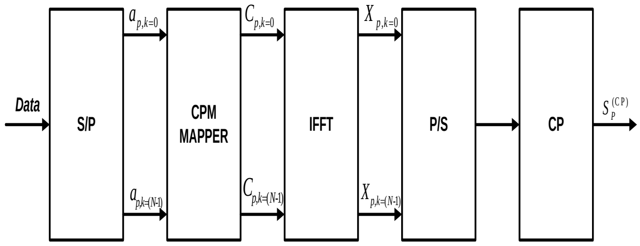

Fig. 1: A portion of the CPM mapper in OFDM transmitter diagram

B. multi-h CPFSK Mapper

In the multi-h CPFSK mapper, we vary the value of h from symbol to symbol. The parameter h is cyclically chosen from a set H_K of K values, $\{h_1, h_2, ..., h_K\}$.
The expression for $\theta_{p,k}$ for this mapper is given by

$$\theta_{p,k} = \begin{cases} a_{p,k}\pi h_{[k]} + \sum_{q=0}^{p-1} a_{q,k}\pi h_{[q]} + \phi, & k > 1 \\ a_{1,k}\pi h_{[1]} + \phi, & k = 1 \end{cases}$$

For illustration, we take the first four symbols for an arbitray kth subcarrier with $H_2 = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and data sequence $a_{p,k} = [+1, +1, -1, +1]$. Assuming the initial phase to be zero, then the number of possible phase states, $\theta_{p,k}$ for $h = \frac{2}{3}$ and $\frac{1}{4}$ would be 4 and 3 respectively. [15]

C. Asymmetric multi-h CPFSK Mapper

While in multi-h CPFSK, h values are chosen independently of data bits $a_{p,k} = (\pm 1)$, in this case we choose h a function of $a_{p,k}$. That is, the value of h during the ith symbol interval is chosen h_{+i} or h_{-i} accordingly as data is a $+1$ or -1 respectively. For this mapper, the expression for θ_p, k is given by

$$\theta_{p,k} = \begin{cases} a_{p,k}\pi h_{\pm[k]} + \sum_{q=0}^{p-1} a_{q,k}\pi h_{\pm[q]} + \phi, & k > 1 \\ a_{1,k}\pi h_{\pm[1]} + \phi, & k = 1 \end{cases}$$

This gives additional flexibility to the designers to enhance system performance. Let the h values employed for data a ± 1 be $H_{+i} = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and the ones for data -1 be $H_{-i} = \left\{\frac{1}{4}, \frac{2}{3}\right\}$. Then the number of possible phase states, $\theta_{p,k}$ for $H_{+i} = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and $H_{-i} = \left\{\frac{1}{4}, \frac{2}{3}\right\}$ would be 4 and 3 respectively [16].

ISBN: 978-988-14047-2-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

IV. PTS Technique

The idea behind the PTS is the data stream which is generated from the CPM mapper is partitioned into disjoint subblocks which are finaly combined to minimize the PAPR [14]. The block diagram of the CPM mapper with PTS technique is shown in Fig. 2. The data stream which is generated from the CPM mapper $[C_{p,k=0},........C_{p,k=N-1}]^T$ is fed to the IFFT block to produce OFDM symbols as shown in Fig. 2. The output of the IFFT block for this data with an oversampling factor of L is given by

$$X_k^{(u)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_n \, e^{j2\pi kn\Delta fT/L}, u = 1, 2, ..., U. \quad (7)$$

The output of the IFFT then is multiplied by U different phase sequences, each of length N, $B^{(u)}$=$[b_{u,0}, b_{u,1}, ..., b_{u,N-1}]^T$, u=$1, 2, ..., U$, resulting in U modified data blocks. The modified data block for the uth phase sequence is represented as $[C_0 b_{u,0}, C_1 b_{u,1}, ..., C_{N-1} b_{u,N-1}]^T$, $u = 1, 2, ..., U$.

The PAPR for each $\{X_k^{(u)}, k = 0, 1, ..., NL-1\}, u = 1, 2, ..., U,$, block is computed and then combined for transmission. That is

$$\grave{C}_n = \sum_{n=0}^{N-1} X_k^{(u)}, k = 0, 1, ..., NL-1 \quad (8)$$

It is noted that when using PTS the transmitter needs to convey to the receiver the value of u.

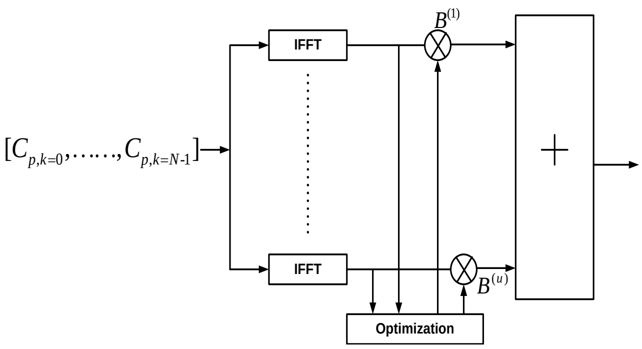

Fig. 2: A block diagram of CPM mapper with PTS technique

V. Numerical Results

We assume an CPM mapper in OFDM system with 128 subcarriers ($N = 128$). Also assume that the number of allowed phase factors is 4 with P= $\{\pm 1, \pm j\}$. We divide the 128 subcarriers into 8 subblocks with 16 contiguous subcarriers. The transmitted signal is oversampled by a factor of 4 ($L = 4$). 10 000 random OFDM blocks were generated to obtain the complementary cumulative density functions (CCDFs) of PAPR. The PAPR performance of CPM mapper in OFDM system has been analyzed using simulations in MATLAB. In Fig. 3, the complementary cumulative distribution function (CCDF) has been plotted to show PAPR performances for PBSK and single-h CPFSK ($h = \frac{1}{2}$) mappers for an OFDM system with 128 subcarriers. Also, in the same figure are shown the performances of these two mappers with PTS. These plots have been arrived at by examining 10,000 random OFDM symbols. It is obsereved that the OFDM system with single-h CPFSK ($h = \frac{1}{2}$) mapper has a PAPR that exceeds 11.1 dB for less than 0.1 percent of data blocks and for PBSK mapper it is 13.7 dB. However, when PTA is used with these mappers the PAPR reduces to 5.02 dB and 10.2 dB for single-h CPFSK and BPSK mappers, respectively. Thus, it is noted that single-h CPFSK mapper with PTS can offer an improvement in PAPR of nearly 6.1 dB relative to corresponding system without PTS. Also, it is noted that the improvment in PAPR by using PTS in these two systems are 6.1 dB and 3.5 dB for single-h CPFSK and BPSK mappers, respectively. Fig.4 shows PAPR performance of multi-h CPFSK mapper for 128 subcarrier OFDM system. The set of modulation parameters used is $(\frac{2}{3}, \frac{1}{4})$. It is noted that this specific multi-h CPFSK mapper performs nearly same as that of BPSK. However, when PTS is used with these systems multi-h CPFSK outperforms BPSK by nearly more than 3 dB.

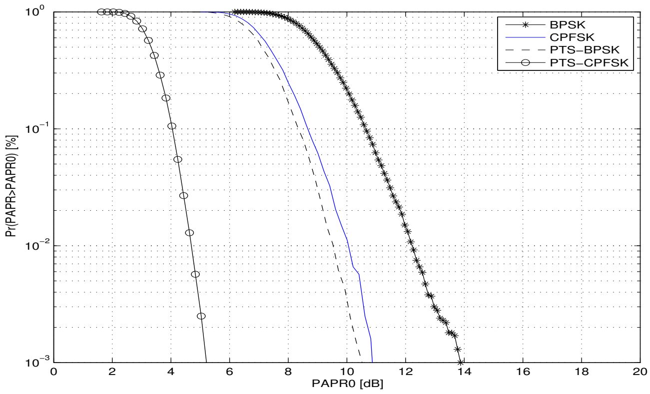

Fig. 3: PAPR Performance of 128 subcarrier OFDM system with single-h CPFSK mapper $(h = \frac{1}{2})$ and PTS

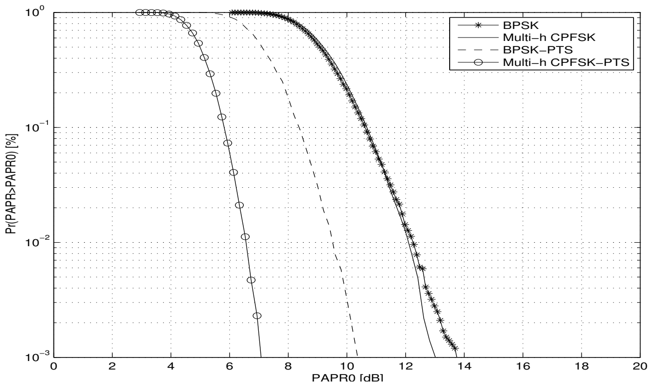

Fig. 4: PAPR Performance of 128 subcarrier OFDM system with$\{\frac{2}{3}, \frac{1}{4}\}$ multi-h CPFSK mapper and SLM

Fig. 5, shows CCDFs for BPSK, BPSK with PTS, asymmetric multi-h CPFSK, and asymmetric multi-h CPFSK with SLM systems. These CCDFs show PAPR performances for 128 subcarrier OFDM system. The modulation parameters used in the asymmetric multi-h CPFSK mapper are $H_{+i} = \left\{\frac{2}{3}, \frac{1}{4}\right\}$ and $H_{-i} = \left\{\frac{1}{4}, \frac{2}{3}\right\}$. It is noted that the difference in PAPR performance between asymmetric multi-h CPFSK with and without PTS is nearly 6 dB. The difference in PAPR performance between multi-h CPFSK and asymmetric multi-h CPFSK mappers is approximately 1.4 dB, for an OFDM

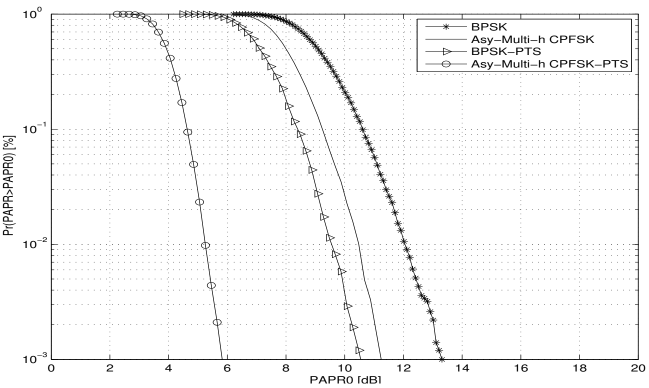

Fig. 5: PAPR Performance of 128 subcarrier OFDM system with $H_{+i} = \{2/3, 1/4\}$ and $H_{-i} = \{\frac{1}{4}, \frac{2}{3}\}$ asymmetric multi-h CPFSK mapper and SLM

system with 128 subcarriers. The results described above show that the PTS technique achieves significant improvement in PAPR performance when we use CPM mappers in conjunction with PTS technique.

VI. Summary and Conclusions

In this paper, three mappers with memory are introduced in an OFDM system. Also, PTS technique has been applied into the system and the results have been studied. Results of simulation of these mappers with PTS technique show that the PAPR reduction of OFDM system, in figures no. 3, 4, and 5 which further results in high performance of wireless communication. In particular, Three subclasses of CPM mappers-single-h CPFSK, multi-h CPFSK, and asymmetric multi-h CPFSK- are considered in an OFDM system. The ability of these mappers to achieve lower PAPR relative to conventional mappers such as BPSK is assessed through extensive simulations. Also, the gains in PAPR that are achievable by these mappers in conjunction with well-known PTS technique are determined. In general, it is observed that CPM mappers with PTS can be very effective in OFDM systems as far as PAPR performance is concerned relative to memory-less mappers. In an extensive study needs to be carried out to detemine the optimum CPM mappers with least values of PAPR. Also, one needs to determine best CPM mappers in OFDM systems that achieve not only least probabality of bit error but also least PAPR. It is worthwhile considering amplitude clipping and filtering, coding, interleaving, tone reservation, other PAPR reduction techniques with CPM mappers in an OFDM system to further reduce PAPR. Also, it would be interesting to obtain analytical bounds on PAPR when CPM mappers are used.

References

[1] Baxley, R.J.; Zhou, G.T. ”Comparing Selected Mapping and Partial Transmit Sequence for PAR Reduction”, Broadcasting, IEEE Transactions on, On page(s): 797 - 803 Volume: 53, Issue: 4, Dec. 2007
[2] S. Han and J. Lee,” An overview of peak-to-average power ratio reduction techniques for multicarrier transmission,”*IEEE Trans. Wireless Commun*, vol. 12, no. 2, pp. 56-65, Apr. 2005.
[3] E. shafter, A. Noorwali, and K. Rao, ”OFDM Systems with CPM Mappers for Smart Grid Applications”, to be published in 2015 IEEE Electrical Power and Energy Conference (EPEC2015), Oct. 26-28 2015.
[4] C. Tellambura,” Computation of the ContinuousTime PAR of an OFDM Signal with BPSK Subcarriers,”*IEEE Commun. Lett.*, vol . 5, no. 5, May 2001, pp. 185-87.
[5] Di-xiao Wu ,” A Comparison of Peak Power Reduction SchSelected mapping and partial transmit sequence schemes to reduce PAPR in OFDM systems,”*Image Analysis and Signal Processing (IASP)*, 2011 International Conference on, On page(s): 1-5
[6] Baig, I.; Jeoti, V,” DCT precoded SLM technique for PAPR reduction in OFDM systems,”*Intelligent and Advanced Systems (ICIAS)*, 2010 International Conference on, On page(s): 1-6.
[7] C. L. Wang and Y. Ouyang, Low-complexity selected mapping schemes for peak-to-average power ratio reduction in OFDM systems, IEEE Trans. on Signal Processing, vol. 53, no. 12, pp. 46524660, December 2005.
[8] A. Alavi, C. Tellambura, and I. Fair, PAPR reduction of OFDM signals using partial transmit sequence: An optimal approach using sphere decoding, IEEE Communications Letters, vol. 9, no. 11, pp. 982984, Nov. 2005.
[9] A. D. S. Jayalath and C. Tellambura, SLM and PTS peak-power reduction of OFDM signals without side information, IEEE Trans. on Wireless Communications, vol. 4, no. 5, pp. 20062013, Sept. 2005.
[10] R. J. Baxley and G. T. Zhou, MAP metric for blind phase sequence detection in selected mapping, IEEE Trans. on Broadcasting, vol. 51, no. 4, pp. 565570, December 2005.
[11] Y. H. You, W. G. Jeon, J. H. Paik, and H. K. Song, A simple construction of OFDM-CDMA signals with low peak-to-average power ratio, IEEE Trans. on Broadcasting, vol. 49, no. 4, pp. 403407, Dec. 2003.
[12] M. S. Baek, M. J. Kim, Y. H.You, and H. K. Song, Semi-blind channel estimation and PAR reduction for MIMO-OFDM system with multiple antennas, IEEE Trans. on Broadcasting, vol. 50, no. 4, pp. 414424, Dec. 2004.
[13] Muhammad Ajmal Khan, and Raveendra K. Rao, ”Low complexity PAPR reduction technique for OFDM systems using biased subcarriers”, IEEE Canadian Journal of Electrical and Computer Engineering, under review
[14] J.G. Proakis and M. Salehi,Digital Communication 5^{th} ed, New York; Mc Graw Hill, 2008, Ch3, sec 3.4, pp. 131-147
[15] ImranA. Tasadduq and RaveendraK. Rao,” Detectionof OFDM-CPM signals over multipath channels,”*in Proc. IEEE Int. Conf. Communications (ICC2002).*, vol. 3,Apr. 28May 2, 2002,pp. 1651-1655.
[16] ImranA. Tasadduq and RaveendraK. Rao,” Detectionof OFDM-CPM signals over multipath channels,”*in Proc. IEEE Int. Conf. Communications (ICC2002).*, vol. 3,Apr. 28May 2, 2002,pp. 1651-1655.