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Error Rate Performance of Digital Chirp Communication System over Fading Channels

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***Abstract*—In the paper, new and easy-to-compute closed-form expressions for average symbol error probability of digital M-ary chirp communication system impaired by additive white Gaussian noise and fading are derived. Three fading environments, Rayleigh, Nakagami-m, and generalized-K, that represent most practical wireless channels are considered. The closed-form expressions derived are then used to illustrate the performances of 2-,4-, and 8-ary chirp systems as a function of average received signal-to-noise ratio (SNR), modulation and fading environment parameters. The proposed mathematical analysis can be easily used to design an efficient and reliable M-ary chirp communication system for application over fading channels.**

***Index Terms*—Chirp modulation, Rayleigh fading, Nakagami-m fading, K_G fading, Average error probability.**

I. Introduction

CHIRP modulation, also known as a linear frequency modulation, can be viewed as a type of spread spectrum signaling technique in which a carrier is swept over a wide-band during a given data pulse interval. This spreading technique produces a signal whose bandwidth is much wider than the information bandwidth. Because chirp signals possess properties that are useful in wireless communication, they are expected to be utilized in a variety of future communication systems. In particular, the growing interest in chirp signals is due to the advances in Surface Wave Acoustic (SAW) technology, which offers a rapid close-to-optimum method for both generation and correlation of wide-band chirp pulses [1]. In 2007, IEEE introduced Chirp Spread Spectrum (CSS) physical layer in its standard 802.15.4a [2]. Furthermore, single-chip transceivers for wireless communication in the industrial, scientific, and medical (ISM) band have been developed and are commercially available [3]. In [4], Alsharef and Rao have proposed M-ary signaling technique for chirp modulation and have evaluated its performance over Additive White Gaussian Noise (AWGN) channel for coherent and non-coherent detection. Gupta, Mumtaz, Zaman and Papandreou-Suppappola [5] have evaluated the performance of chirp modulation in a frequency-hopped CDMA over several channel models. The error performance of chirp modulation over frequency selective and non-selective fading channels are investigated in [6]. The performance analysis with closed-form bit error probability expressions for chirp modulation in the maximum ratio combining (MRC) diversity system has been investigated in [7]. Hengstler, Kasilingam and Costa [8] have proposed a novel chirp modulation spread spectrum technique that utilizes antipodal signaling and have derived an expression for its bit error rate and bandwidth efficiency. In [9], Kadri and Rao have evaluated the performance of weak binary chirp signals in ϵ-mixture noise for coherent and non-coherent detection. Phichet, Tran and Tawil have proposed a new system called multi linear chirp frequency hopping code division multiple access (MLC-FH-CDMA) and studied its performance over Nakagami-Rice fading channel [10]. In [11], the authors applied the concept of frequency hopping (FH) spread spectrum technique to the multi-user chirp modulation system and evaluated the performance over multi-path fading channels. In the literature, there are several studies on chirp modulation; however, the impact of the shadowing on chirp signals has not received much attention.

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M-ary chirp signalling provides increased immunity to channel noise. In this paper, chirp modulation for transmission of M-ary data is considered and then expressions for average error probability in closed form are derived for several fading channels. In particular, Rayleigh, Nakagami-m and Generalized-K fading channels are considered in this work. The effect of fading and shadowing on the performance of the receiver are examined for different values of fading parameters c and m. Numerical results are presented to illustrate the effect of channel on the performance of these chirp signals.

The paper is organized as follows. In Section II, M-ary chirp modulation is described and illustrated. Also, optimum coherent receiver and its performance in AWGN channel for M-ary chirp signals are treated. In Section III, symbol error probabilities over Rayleigh, Nakagami-m and Generalized-K fading channels are derived. The paper is concluded in Section IV.

II. M-ary Chirp System Model

A. M-ary Chirp Signalling Technique

The i^{th} modulated M-ary chirp signal is given by [12]:

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}}\ \cos(w_c t + \phi(t, d_i) + \theta), \qquad 0 \le t \le T_s \tag{1}$$

where E_s is the symbol energy in the symbol duration T_s seconds, w_c is the angular carrier frequency, $\phi(t, d_i)$ is the information carrying phase, θ is the starting phase $(t = 0)$ and is assumed to be zero with no loss of generality. The information carrying phase $\phi(t, d_i)$ for chirp modulation is given by:

$$\phi(t, d_i) = \begin{cases} 0, & t \le 0, \quad t > T_s \\ d_i\pi \left\{ h\left(\frac{t}{T_s}\right) - w\left(\frac{t}{T_s}\right)^2 \right\}, & 0 \le t \le T_s \\ d_i\pi q = d_i\pi(h - w), & t = T_s \end{cases} \tag{2}$$

where d_i, $i = 1, 2, ..., M$, takes values from the set $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ and:

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$$d_i = \begin{cases} +i, & if\ i\ odd \\ -(i-1), & if\ i\ even \end{cases} \tag{3}$$

for example, for 4-ary chirp modulation, there are 4 possible waveforms $S_1(t), S_2(t), S_3(t), and\ S_4(t)$ corresponding to data $+1, -1, +3, -3$, respectively. In (2), h and w are dimensionless parameters: h represents the initial peak-to-peak frequency deviation divided by the symbol rate $\frac{1}{T_s}$, and w represents the frequency sweep width divided by the symbol rate $\frac{1}{T_s}$. Since $h = (q + w)$, we choose (w, q) to be independent signal modulation parameters. Fig. 1 shows an example of 4-ary chirp modulation (quadrature chirp signals) where the data d_i takes values $\pm1, \pm3$.

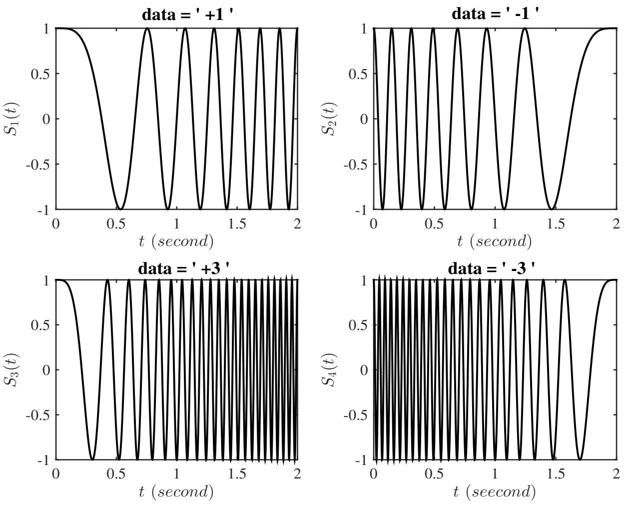

Fig. 1. 4-ary chirp modulated signals as a function of time

B. Optimum Correlator-Receiver

Fig. 2 shows the optimum coherent receiver for detection of M-ary chirp signals in AWGN. This receiver will compute M functions $\Lambda_1, \Lambda_2, \ldots, \Lambda_M$ given by:

$$\Lambda_i = \int_0^{T_s} r(t)\ S_i(t)\ dt, \quad i = 1, 2, ..., M \tag{4}$$

and arrives at an optimum decision based on the largest of these M values. Thus, if:

$$\Lambda_k = \max\{\Lambda_1, \Lambda_2, \ldots, \Lambda_M\} \tag{5}$$

then the receiver decides the transmitted data as:

$$d = \begin{cases} +k, & k\ odd \\ -(k-1), & k\ even \end{cases} \tag{6}$$

C. Symbol Error Rate Performance

Suppose that the transmitted signal is $S_i(t)$. The received signal is given by:

$$r(t) = S_i(t) + n(t), \qquad 0 \le t \le T_s \tag{7}$$

where $n(t)$ is the additive white Gaussian Noise (AWGN) with zero-mean and one-sided power spectral density (PSD)

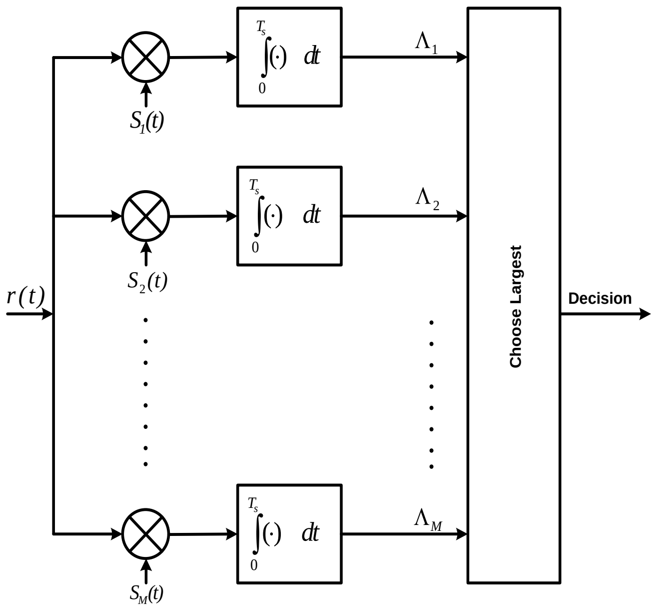

Fig. 2. Optimum coherent receiver for M-ary chirp modulation

of $N_0\ watts/Hz$. The conditional probability of an error given $S_i(t)$ is transmitted is given by:

$$P(\epsilon|S_i) = Prob[\Lambda_1 > \Lambda_i\ \cup \Lambda_2 > \Lambda_i\ \cup ...\ \cup \Lambda_M > \Lambda_i|S_i] \tag{8}$$

The conditional probability in (8) can be union bounded using the identity:

$$P(\Lambda_1 \cup \Lambda_2\ \cup\ ....\ \cup \Lambda_n) \leqslant \sum_{j=1}^{n} P(\Lambda_j) \tag{9}$$

Thus, (8) can be written as:

$$P(\epsilon|S_i) = \sum_{\substack{j=1 \\ j\neq i}}^{M} Prob[\Lambda_j > \Lambda_i|S_i] \tag{10}$$

Averaging over all possible S_i, the expression for average symbol error probability is given by:

$$P(\epsilon) \leqslant \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} P(S_j) P_r\left[\Lambda_j \geqslant \Lambda_i \mid S_i\right] \tag{11}$$

It is noted that in (11), Λ_i's are Gaussian random variables. Thus, (11) can be written as :

$$P(\epsilon) \leqslant \frac{1}{M} \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} Q\left[\sqrt{\frac{E_s}{N_0}\left(1 - \rho(i,j)\right)}\right] \tag{12}$$

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$

which also can be written as:

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{\left(-\frac{x^2}{2sin^2(\theta)}\right)}\ d\theta \tag{13}$$

and the quantity $\rho(i, j)$ is the normalized correlation between $S_i(t)$ and $S_j(t)$ and is defined as [4]:

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$$\rho(i,j) = \frac{1}{E_s}\int_0^{T_s} S_i(t)\ S_j(t)\ dt, \quad i,j = 1,\ldots,M \tag{14}$$

using (1) in (14), the normalized correlation can be written as:

$$\rho(i,j) = \left[\frac{\cos(\Omega)}{\sqrt{2\zeta w}}\ \mathbb{C} + \frac{\sin(\Omega)}{\sqrt{2\zeta w}}\ \mathbb{S}\right] \tag{15}$$

where

$$\mathbb{C} = \mathbf{C}(u_h) - \mathbf{C}(u_l), \quad \mathbb{S} = \mathbf{S}(u_h) - \mathbf{S}(u_l)$$

$$\Omega = \frac{\pi\zeta h^2}{4w},\ \zeta = |d_i - d_j|$$

$$u_h = \sqrt{\frac{\zeta}{2}}\frac{(w-q)}{\sqrt{w}}, \quad u_l = \sqrt{\frac{\zeta}{2}}\frac{(w+q)}{\sqrt{w}}$$

the value d_i is the data associated with the signal $S_i(t)$ and d_j is the data associated with the signal $S_j(t)$ .The function $\mathbf{C}(.)$ and $\mathbf{S}(.)$ are the Fresnel cosine and sine integral which are given by:

$$\mathbf{C}(u) = \int_0^u \cos(\frac{\pi x^2}{2})\ \ dx, \qquad \mathbf{S}(u) = \int_0^u \sin(\frac{\pi x^2}{2})\ \ dx$$

The performance of the optimum coherent receiver can be evaluated using equations (12) to (15). The optimum modulation parameters (q, w) are those that minimize the symbol error probability given by (12) [4]. These are given in Table I. For all illustrations in the paper these optimum modulations parameters are used.

TABLE I
OPTIMUM VALUES OF q AND w FOR 2-,4-, AND 8- CHIRP SIGNALS

Modulation Size (M)	(q, w)
2	(0.36,1.52)
4	(0.4,2.4)
8	(0.95,0.25)

III. PERFORMANCE OF OPTIMUM RECEIVER OVER FADING CHANNELS

In this section, we consider evaluating the performance of optimum receiver over short-term and long-term fading (shadowing). In particular, closed form expressions for average symbol probability of error for Rayleigh, Nakagami-m and Generalized-K channel models are derived. The received chirp signal over the fading channel can be written as:

$$r(t) = h(t) * S_i(t) + n(t)$$

where $h(t) = \alpha\delta(t)$ is the impulse response of the channel and $n(t)$ is AWGN. Hence, the instantaneous SNR per symbol and the average SNR are described as $\gamma = \alpha^2 E_s/N_0$ and $\bar{\gamma} = \Omega E_s/N_0$, where $\Omega = E\{\alpha^2\}$. The average symbol error probability of M-ary chirp system (P_s) over fading channel is determined by averaging the conditional error probability over the Probability Density Function (PDF) of the fading model and is given by [13]:

$$P_{av} = \int_0^\infty p_\gamma(\gamma) P_e(\gamma)\ \ d\gamma \tag{16}$$

A. Rayleigh Fading Channel

Using (16), the average error probability P_{av} is evaluated for Rayleigh fading channel using:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geqslant 0 \tag{17}$$

Using (12), (13), (16) and changing the order of integration, the P_{av} can be expressed as:

$$P_{av} = \frac{1}{\pi\bar{\gamma}M}\sum_{j=1}^{M}\sum_{\substack{i=1\\ i\neq j}}^{M}\int_0^{\frac{\pi}{2}}\int_0^\infty \exp\left(-\frac{(1-\rho(i,j))\gamma}{2\sin^2(\theta)} - \frac{\gamma}{\bar{\gamma}}\right) d\gamma\ d\theta \tag{18}$$

integrating (18) by using [14] , the symbol error rate can be written as:

$$P_s = \frac{1}{M}\sum_{j=1}^{M}\sum_{\substack{i=1\\ i\neq j}}^{M}\left[1 - \sqrt{\frac{1}{1+\frac{1}{\frac{\Omega E_s}{2N_0}(1-\rho(i,j))}}}\right] \tag{19}$$

Fig. 3 shows the bit error probability for 2-ary chirp over AWGN channel, BPSK, 2-FSK, and DPSK versus the normalized SNR over Rayleigh fading channel. It is noted that the performance of binary chirp is very close to the performance of BPSK modulation over Rayleigh fading channel and better than the performance of 2-FSK and DPSK systems. In Fig. 4 and Fig. 5, the bit error probability performances of the 2-ary chirp system are illustrated as a function of q and w, respectively. It is observed that as the values of the modulation parameters deviate from the optimum values, the performance degrades.

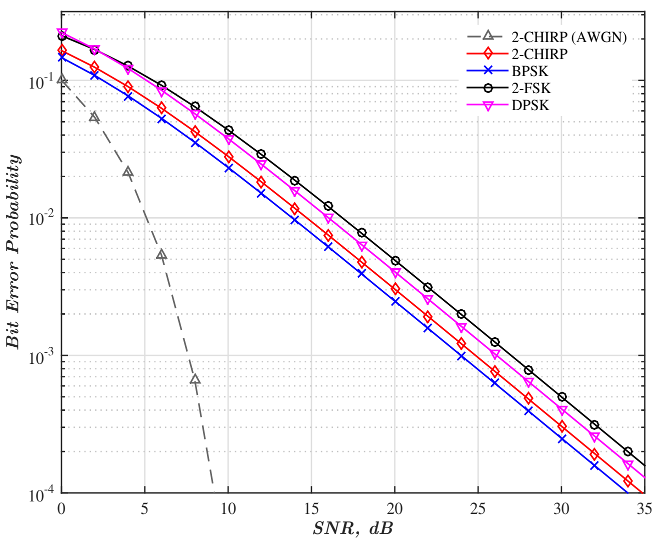

Fig. 3. Performance of 2-ary chirp system over Rayleigh fading channel

Fig. 6 and Fig. 7 shows the symbol error probability performance of 4-ary and 8-ary chirp systems, respectively. For both systems, we have found that the performance of the chirp system is worse than than the performance of 4-PSK and 4-FSK over Rayleigh fading channel.

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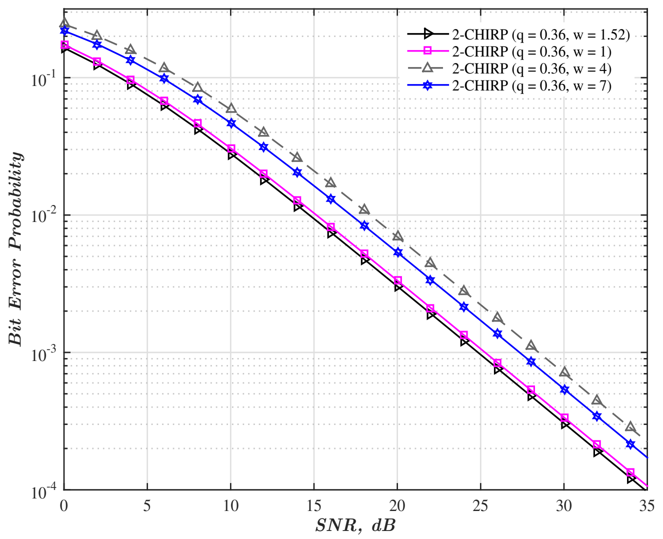

Fig. 4. Performance of 2-ary chirp system over Rayleigh fading channel as a function of $w = 1.52$, 1, 4, 7, for a fixed value of q

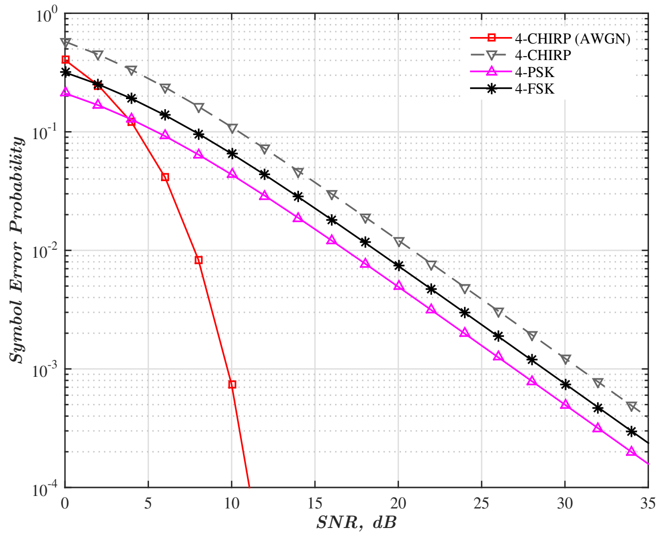

Fig. 6. Performance of 4-ary chirp system over Rayleigh fading channel

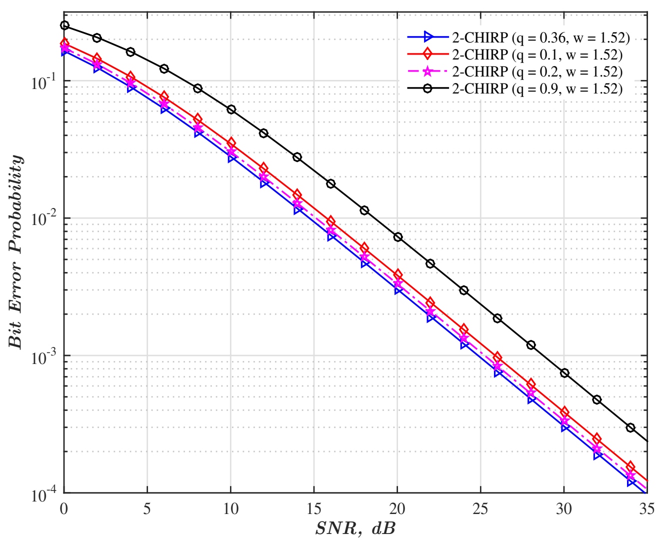

Fig. 5. Performance of 2-ary chirp system over Rayleigh fading channel as a function of $q = 0.36$, 0.1, 0.2, 0.9, for a fixed value of w

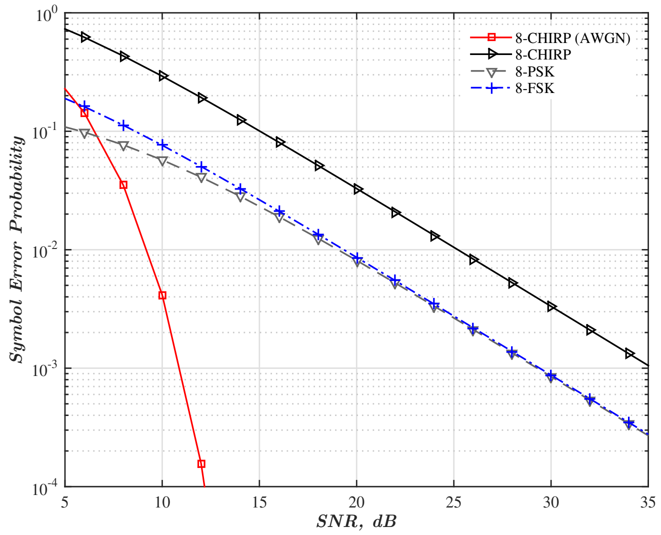

Fig. 7. Performance of 8-ary chirp system over Rayleigh fading channel

B. Nakagami-m Fading Channel

The PDF of Nakagami-m model is given by [13]:

$$p_\gamma(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-m\frac{\gamma}{\bar{\gamma}}}, \quad \gamma \geqslant 0 \qquad (20)$$

m is the fading parameter which can be used to generate several fading channel model ($m = 1$ represents Rayleigh model when there is no line of sight (LOS) component). By expressing the exponential term in (20) as $e^{-m\frac{\gamma}{\bar{\gamma}}} = G^{1,0}_{0,1}\left(\frac{m\gamma}{\bar{\gamma}} \,\middle|\, {- \atop 0}\right)$ and $Q(\sqrt{(1-\rho(i,j))\gamma}) = \frac{1}{\sqrt{\pi}} G^{2,0}_{1,2}\left((1-\rho(i,j))\gamma \,\middle|\, {1 \atop 0,\ 1/2}\right)$ in (12) using [(8.4.3/1),(8.4.14/2)][17] respectively, where $G^{q,p}_{a,b}\left(x \,\middle|\, {\cdots \atop \cdots}\right)$ is the Meijer G-function [16]. (16) yields to the integral:

$$P_{av} = \frac{1}{M\sqrt{\pi}} \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} \int_0^\infty \gamma^{m-1} G^{2,0}_{1,2}\left((1-\rho(i,j))\gamma \,\middle|\, {1 \atop 0,\ 1/2}\right) G^{1,0}_{0,1}\left(\frac{m\gamma}{\bar{\gamma}} \,\middle|\, {- \atop 0}\right) d\gamma \qquad (21)$$

By solving the integral in (21), a closed form expression for the symbol error rate over Nakagami-m channel can be written as:

$$P_s = \frac{1}{M\sqrt{\pi}\Gamma(m)} \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} G^{2,1}_{2,2}\left(\frac{(1-\rho(i,j))}{m} \frac{\Omega E_s}{2N_0} \,\middle|\, {1,\ 1-m \atop 0,\ 1/2}\right) \qquad (22)$$

Figs. 8 to 10 show the performance of 2,4, and 8-ary chirp modulation over Nakagami-m fading channel. For the special case when $m = 1$, Nakagami-m reduces to the well known Rayleigh model. The performance approaches AWGN performance as $m \to \infty$ because the line of sight (LOS) path dominates the received signal. Also, it is noted that as M increases the symbol error rate increases for a fixed value of SNR. This observation can be utilized to design an adaptive modulation technique by changing the modulation order to increase the bandwidth efficiency or to improve the error rate performance to meet a certain quality of service [15] .

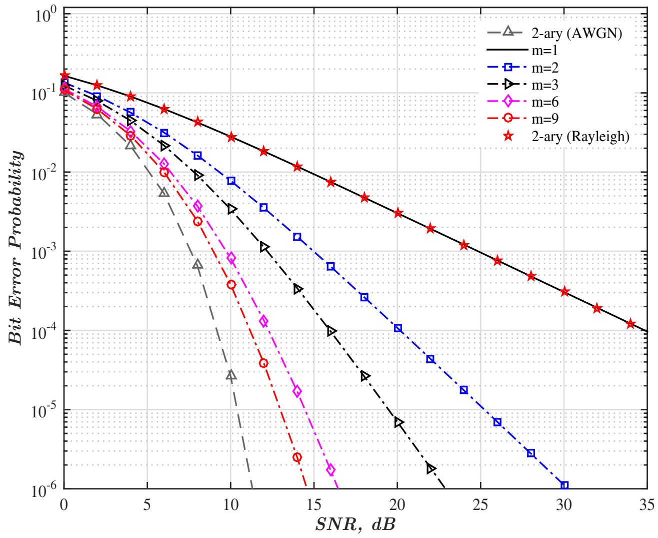

Fig. 8. Performance of 2-ary chirp system over Nakagami-m fading channel as a function of m

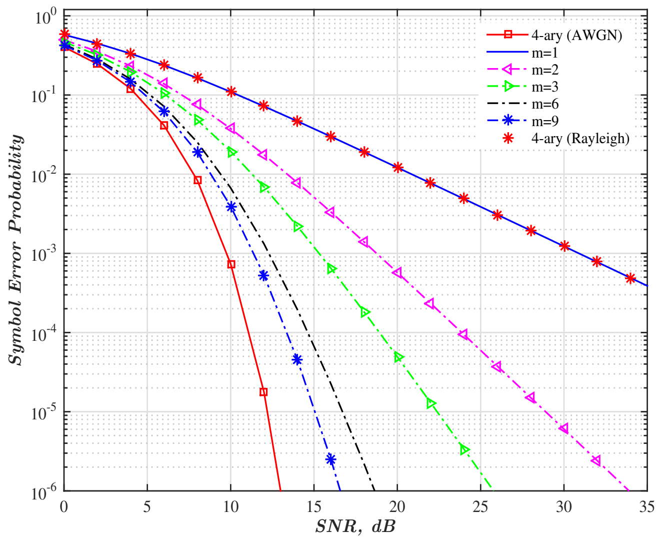

Fig. 9. Performance of 4-ary chirp system over Nakagami-m fading channel as a function of m

C. Generalized-K Fading and Shadowing Channel

The Generalized-K is a composite model that is used to describe the fading and shadowing channel characteristics. This model represents a wireless channel that subjects to short and long-term fading. The PDF of the Generalized-K model is given by:

$$p_\gamma(\gamma) = 2\left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}} \frac{\gamma^{\frac{c+m-2}{2}}}{\Gamma(c)\Gamma(m)} K_{c-m}\left(2\sqrt{\frac{cm}{\bar{\gamma}}\gamma}\right), \gamma \geqslant 0 \quad (23)$$

Where $K_{c-m}(.)$ is the modified Bessel function of order $c-m$, $\Gamma(.)$ is the Gamma function [16]. The coefficients c and m are the shadowing and fading's parameters; respectively. As m and c increase, the fading and shadowing become less sever. For m and $c \to \infty$, the channel approaches that

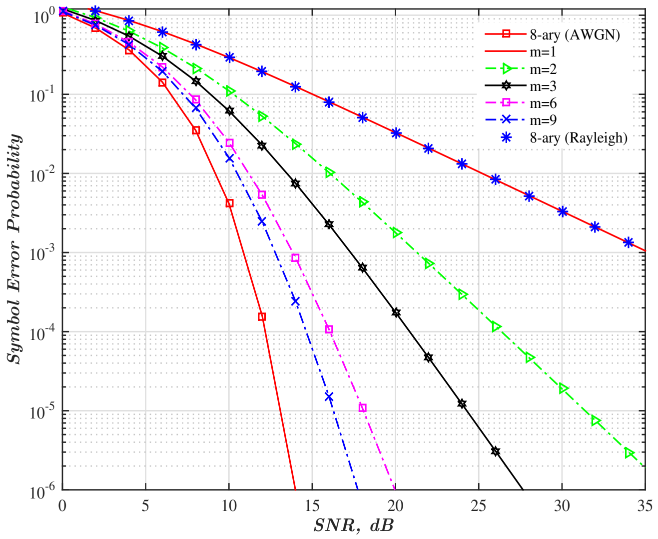

Fig. 10. Performance of 8-ary chirp system over Nakagami-m fading channel as a function of m

of AWGN. using (12), (13), (16), (23) and expressing the modified Bessel function in (23) as $K_{c-m}\left(2\sqrt{\frac{cm}{\bar{\gamma}}\gamma}\right) = \frac{1}{2}G^{2,0}_{0,2}\left(\frac{cm}{\bar{\gamma}}\gamma \,\middle|\, \frac{(c-m)}{2}, \frac{-(c-m)}{2}\right)$ and $Q(\sqrt{(1-\rho(i,j))\gamma}) = \frac{1}{\sqrt{\pi}}G^{2,0}_{1,2}\left((1-\rho(i,j))\gamma \,\middle|\, {1 \atop 0,\ 1/2}\right)$ using [(8.4.23/1),(8.4.14/2)][17]. Thus, the integral of (31) can be expressed in terms of Miejer G-function as:

$$I = \frac{1}{2\sqrt{\pi}} \int_0^\infty \gamma^{\frac{c+m}{2}-1} G^{2,0}_{1,2}\left((1-\rho(i,j))\gamma \,\middle|\, {1 \atop 0,\ 1/2}\right) G^{2,0}_{0,2}\left(\frac{cm}{\bar{\gamma}}\gamma \,\middle|\, \frac{(c-m)}{2}, \frac{-(c-m)}{2}\right) d\gamma \quad (24)$$

average symbol error probability ((P_{av})) of chirp modulation over K_G-channel is obtained in a closed form as:

$$P_s \geqslant \frac{1}{M\sqrt{\pi}\Gamma(c)\Gamma(m)} \cdot \sum_{j=1}^{M}\sum_{\substack{i=1\\ i\neq j}}^{M} G^{2,2}_{3,2}\left(\frac{(1-\rho(i,j))}{m}\frac{\Omega E_s}{2N_0} \,\middle|\, {1,\ 1-c,\ 1-m \atop 0,\ 1/2}\right) \quad (25)$$

Figs. 11 to 13 show the performance of $2, 4$ and 8-ary chirp system over K_G fading channel as a function of c and m. For comparison purposes, symbol error rate for Rayleigh fading and AWGN channel are also plotted. Because of the shadowing effect, the performance over K_G channel is poorer than the performance over AWGN and Rayleigh channel. Several techniques may be used to mitigate the effect of shadowing and fading on the performance. For example, at a symbol error rate of 10^{-4} for 2-ary chirp system, performance deteriorates by 5, 14, and 27 dB for generalized-K with $c = 6, m = 8$, generalized-K with $c = 6, m = 2$, and Rayleigh channel model, respectively, with relative to the performance in AWGN channel. To compensate for this degradation in performance, channel coding and/or diversity techniques could be used. However, these techniques increase the channel bandwidth and system complexity.

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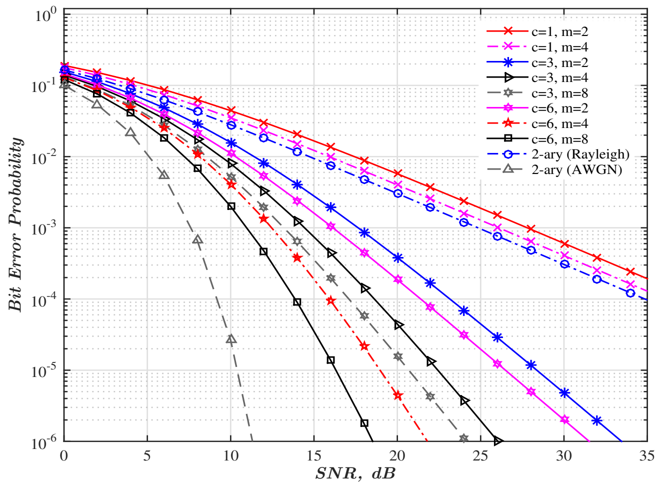

Fig. 11. Performance of 2-ary chirp system over K_G fading channel as a function of c and m

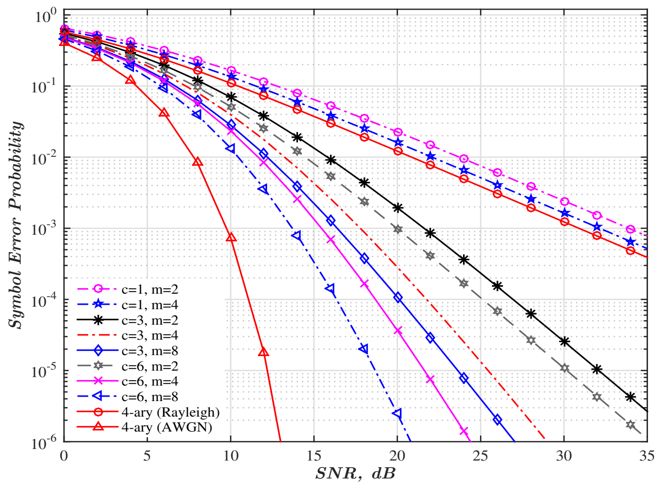

Fig. 12. Performance of 4-ary chirp system over K_G fading channel as a function of c and m

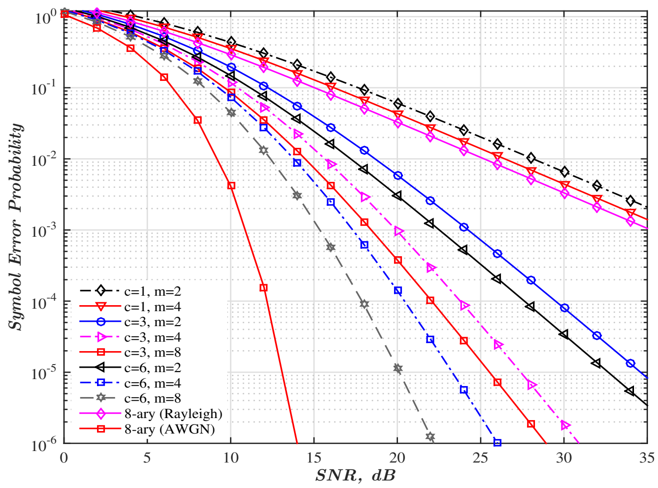

Fig. 13. Performance of 8-ary chirp system over K_G fading channel as a function of c and m

IV. CONCLUSION

In this paper, closed-form SER expressions for M-ary chirp systems over Rayleigh, Nakagami-m and Generalized-K channels have been derived. The SER performances have been plotted as a function of normalized SNR for different modulation orders M and as a function of fading and shadowing parameters. One of the major contributions of this paper is the derivation of closed-form expressions for the performance over Rayleigh, Nakagami-m and Generalized-K channel models for M-ary chirp communication system.

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